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Two-Warehouse Inventory Model for Deteriorating Items with Variable Holding Cost, Time-Dependent Demand and Shortages

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Abstract: *In his paper a deterministic two warehouse inventory model for single instantaneous deteriorating items has been developed under assumption that the inventory cost (including holding cost and deterioration cost) in RW (Rented Warehouse) is higher than those in OW (Owned Warehouse) due to better preservation facilities in RW. Also considering equal deterioration in both warehouses has been discussed as a special case of the model with other cases as well. The demand and holding cost, both are taken as time dependent.*

Keywords: *Two warehouses, deterioration, Time-dependent Demand, Variable holding cost*

I. Introduction

Basically inventory management and control system deals with demand and supply chain problems. The business is totally based on demand and supply of goods either finished or raw materials. To fulfil the demand of consumer or supplier, it is necessary to have the items demanded at any time and for same purpose the sufficient space is required to stock the goods to fulfil the demands. The space used to stock the goods is termed as ware-house. In the traditional models it is assumed that the demand and holding cost are constant and goods are supplied instantly under infinite replenishment policy, when demanded but as time passed away many researchers considered that demand may vary with time, due to price and on the basis of other factors and holding cost also may vary with time and depending on other factors. Many models have been developed considering various time dependent demand with shortages and without shortage. All those models that consider demand variation in response to inventory level, assume that the holding cost is constant for the entire inventory cycle. In studies of inventory models, unlimited warehouse capacity is often assumed. However, in busy marketplaces, such as super markets, corporation markets etc. the storage area for items may be limited. Another case, of inadequate storage area, can occur when a procurement of a large amount of items is decided. That could be due to, an attractive price discount for bulk purchase which is available or, when the cost of procuring goods is higher than the other inventory related costs or, when demand for items is very high or, when the item under consideration is a seasonal product such as the yield of a harvest or, when there are some problems in frequent procurement. In this case these items cannot be accommodated in the existing store house known as own ware house. Hence, in order to store the excess items, an additional warehouse, which may be located nearby own ware-house.Hartely[1] was first who discussed an inventory model with two storage system**.** To reduce the inventory costs, it is necessary to consume the goods of the RW at the earliest due to more holding cost.

Buzacott (1975) developed the first EOQ model taking inflationary effects into account. In this model, a uniform inflation was assumed for all the associated costs and an expression for the EOQ was derived by minimizing the average annual cost. Misra (1975, 1979) investigated inventory systems under the effects of inflation. Bierman and Thomas (1977) suggested the inventory decision policy under inflationary conditions. An economic order quantity inventory model for deteriorating items was developed by Bose et al. (1995). Authors developed inventory model with linear trend in demand allowing inventory shortages and backlogging. The effects of inflation and time-value of money were incorporated into the model. Hariga and Ben-Daya (1996) then discussed the inventory replenishment problem over a fixed planning horizon for items with linearly time-varying demand under inflationary conditions. Ray and Chaudhuri (1997) developed a finite time-horizon deterministic economic order quantity inventory model with shortages, where the demand rate at any instant depends on the on-hand inventory at that instant. The effects of inflation and time value of money were taken into account. The effects of inflation and time-value of money on an economic order quantity model have been discussed by Moon and Lee (2000). The two-warehouse inventory models for deteriorating items with constant demand rate under inflation were developed by Yang (2004). The shortages were allowed and fully backlogged in the models. Some numerical examples for illustration were provided. Models for ameliorating / deteriorating items with time-varying demand pattern over a finite planning horizon were proposed by Moon et al. (2005).

The effects of inflation and time value of money were also taken into account. An inventory model for deteriorating items with stock-dependent consumption rate with shortages was produced by Hou (2006). Model was developed under the effects of inflation and time discounting over a finite planning horizon. Jaggi et al. (2007) presented the optimal inventory replenishment policy for deteriorating items under inflationary conditions using a discounted cash flow (DCF) approach over a finite time horizon. Shortages in inventory were allowed and completely backlogged and demand rate was assumed to be a function of inflation. Two stage inventory problems over finite time horizon under inflation and time value of money was discussed by Dey et al. (2008).

The deterioration of goods is realistic phenomenon and needs to be controlled during storage periods of deteriorating items. Generally, deterioration is defined as the damage, spoilage, dryness, vaporization, etc. that results in decrease of usefulness of the original one. In many literatures, deterioration phenomenon was taken into account into single warehouse only while developing single warehouse system as well as two storage policies. Assuming the deterioration in both warehouses**,** Sarma (1983) extended his earlier model to the case of infinite replenishment rate with shortages. Pakkala and Achary (12992) extended the two-warehouse inventory model for deteriorating items with finite replenishment rate and shortages, taking time as discrete and continuous variable, respectively. In these models mentioned above the demand rate was assumed to be constant. Subsequently, the ideas of time varying demand and stock dependent demand considered by some authors, such as Goswami and Chaudhary (1998) in that model they were not consider the deterioration and shortages were allowed and backlogged. Many researchers have considered that the holding cost during storage period is always constant but it is not realistic but due to the time value of money it is not assumed that holding cost will remain constant forever and thus holding cost may vary with time. Maya Gyan and A.K.Pal (2009) developed a two ware house inventory model for deteriorating items with stock dependent demand rate and holding cost. T.P.M. Pakkala, K.K. Achary (1992) developed a deterministic inventory model for deteriorating items with two warehouses and finite replenishment rate. U. Dave (1988) produces an EOQ models with two levels of storage, K. D. Rathod and P. H. Bhathawa (2013) developed an Inventory model with inventory-level dependent demand rate, variable holding cost and shortages. T.A. Murdeshwar, Y.S. Sathe (1985) developed some aspects of lot size model with two levels of storage. R.P. Tripathi (2013) developed an Inventory model with different demand rate and different holding cost. Mishra (2001) et. al. developed an inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging. A.K. Bhunia, M. Maiti (1998) considering time dependent demand as linear function of time produces A two-warehouse inventory model for deteriorating items with a linear trend in demand and shortages. S. Kar. A.K. Bhunia, M. Maiti (2001) considering fix time horizon developed a Deterministic inventory model with two levels of storage, a linear trend in demand and a fixed time horizon. K.V.S. Sarma (1987) developed a deterministic order level inventory model for deteriorating items with two storage facilities. In all cases of developed model it is assumed that the rented ware-house is equipped with better facilities but due to advance technologies and in the competitive business environment every businessman want to minimize deterioration rate of goods hence has an well-equipped ware-house and therefore the consideration of equal deterioration rate in the both ware-houses of two storages system can't be ignored.

In this paper a deterministic Inventory model for single deteriorating items with two level of storage and time dependent demand with partially backordering shortage is developed. Stock is transferred RW to OW under continuous release pattern and the transportation cost is not taken into account. The deterioration rates in both the warehouses are constant but different due to the different preservation procedures as discussed in traditional models as discussed above, in this paper the same rate of deterioration is considered and a special case has been discussed with some other cases of the model and compared their total relevant inventory costs. The numerical example is presented to demonstrate the development of the model.

II. Assumption and notations

The mathematical model of two warehouse inventory model for Non-Instantaneous deteriorating items is based on the following notation and assumptions

Assumption

1 Replenishment rate is infinite.
2 Lead time is negligible i.e. zero.
3 Holding cost vary with time.
4 The time horizon of the inventory system is infinite.
5 Goods of RW are consumed first due to the more holding cost in RW than in OW.
6 RW has unlimited capacity while OW is of limited capacity.
7 Demand vary with time and is linear function of time and given by $d(t)=at$; where $a>0$;

8 Deterioration rate in both ware houses are different and a fraction of on hand inventory deteriorates per unit time in both the warehouse.
9 Shortages are allowed and demand is partially backlogged at constant rate in the beginning of next replenishment cycle.
10 The unit inventory cost (Holding cost + deterioration cost) in RW>OW

Notations

C: Cost of Ordering per Order.
η_1 :Capacity of RW.
η: Capacity of OW.
T: The length of replenishment cycle.
Q: Maximum Inventory level per cycle to be ordered.
t_1: The time at which inventory level reaches to zero in RW.
t_2: The time at which inventory level reaches to zero in OW.
O_h: The holding cost per unit time in OW i.e. $O_h = h_2 t$; where h_2 is positive constant.
R_h: The holding cost per unit time in RW i.e. $R_h = (h_1+1)t$ where $(h_1+1)>0$ and $R_h > O_h$.
s_c: The shortages cost per unit per unit time.
L_c: The lost sales cost per unit per unit time.
$\Psi^{r1}(t)$:The level of inventory in RW during time interval $[0\ \ t_1]$.
$\Psi^{o1}(t)$:The level of inventory in OW during time interval $[0\ \ t_1]$.
$\Psi^{o2}(t)$:The level of inventory in OW during time interval $[t_1\ t_2]$.
$\Psi^{o3}(t)$:The level of inventory in OW during time interval $[t_2\ t_3]$.
$\Psi^{S}(t)$: Determine the inventory level during time interval $[t_2\ \ T]$.
$\alpha\gamma$: Deterioration rate in RW and OW Such that $0 < \alpha\gamma < 1$.
$(\beta+1)$: Deterioration rate in OW Such that $0 < (\beta+1) < 1$.
P_c: Purchase cost per unit of item.
$\Psi^{TC}(t_1, t_2, T)$: The total relevant inventory cost per unit time of inventory system.

III. Mathematical formulation of model and analysis

In the beginning of the cycle at t=0 a lot size of Q units of inventory enters into the system in which backlogged units are cleared and the remaining units are kept into two warehouses according to the capacity of ware-houses.

In the time interval $[0\ t_1]$ the inventory level decreases in RW due to both, demand and deterioration and is governed by differential equation

$$\frac{d\Psi^{r}1(t)}{dt} = -\alpha\gamma\ \Psi^{r1}(t) - a\,t \quad ; \qquad 0 \le t \le t_1 \tag{1}$$

Since demand is fulfilled from the goods kept in RW during the time interval $[0t_1]$ and therefore, in OW inventory level decreases only due to deterioration and inventory level is governed by differential equation

$$\frac{d\Psi^{o1}(t)}{dt} = -(\alpha\gamma)\Psi^{01}(t); \qquad 0 \le t \le t_1 \tag{2}$$

During the time period $[t_1 t_2]$ the inventory remains in OW and therefore its level depleted due to common effect of demand and deterioration and level of inventory is governed by differential equation

$$\frac{d\Psi^{o2}(t)}{dt} = \Psi^{02}(t) - a\,t\,; \qquad 0 \le t \le t_1 \tag{3}$$

Now at $t=t_2$ the inventory level vanishes but demand is continue thus shortages occur in the time interval $[t_2T]$ and the part of shortages quantity supplied to the customers at the beginning of the next replenishment cycle. The shortages is governed by the differential equation

$$\frac{d\Psi^{4S}(t)}{dt} = -b(a\,t); \ \ 0 \le t \le \mathrm{T} \tag{4}$$

Now inventory level at different time intervals is given by solving the above differential equations (1) to (4) with boundary conditions

$\Psi^{r1}(t) = 0\ at\ t = t_1$;
$\Psi^{o1}(t) = 0\ at\ t = 0$;
$\Psi^{o1}(t) = 0\ at\ t = t_2$ and
$\Psi^{s}(t) = 0\ at\ t = t_2$, Respectively as

$$\Psi^{r1}(t) = \frac{a}{(\alpha\gamma)^2}\left\{(1-(\alpha\gamma)\,t) - (1-(\alpha\gamma)\,t_1)e^{(\alpha\gamma)(t_1-t)}\right\}; \qquad 0 \le t \le t_1 \tag{5}$$

$$\Psi^{o1}(t) = \eta e^{(\alpha\gamma)\,t} \quad ; \qquad 0 \le t \le t_1 \tag{6}$$

$$\Psi^{o2}(t) = \frac{a}{(\alpha\gamma)^2}\left\{(1-(\alpha\gamma)\,t) - (1-(\alpha\gamma)\,t_2)e^{(\alpha\gamma)(t_2-t)}\right\}\ ; \qquad t_1 \le t \le t_2 \tag{7}$$

$\Psi^{s}(t) = \frac{b\,a}{2}\,(t_2^2 - t^2);\qquad t_2 \le t \le T$ (8)

At $t = 0$; $\eta_1 = \Psi^{r1}(0)$

$\eta_1 = \frac{a}{(\alpha\gamma)^2}\{((\alpha\gamma)\,t_1 - 1)e^{(\alpha\gamma)\,t_1}\}$ (9)

Maximum amount of inventory backordered during time interval [t_2T] at $t = T$is given by

$B_{max} = -\Psi^{s}(t) = \frac{b\,a}{2}\,(t_2^2 - T^2)$ (10)

The maximum amount of Inventory to be ordered at the end of cycle length is given as

$Q_{max} = \eta + \eta_1 + B_{max}$

$= \eta + \frac{bt_1^2}{2} + \frac{b}{\alpha^2}\{((\alpha\gamma)t_2\text{-}1)e^{(\alpha\gamma)(t_2-t_1)} - ((\alpha\gamma)t_1 - 1)\} + \frac{b}{6}\{T^3 \text{ -}2t_3^3 \text{ -}3t_3^2\text{T}\};$ (11)

At the time $t = T$ replenishment cycle restarts. The objective of the model is to minimize thetotal inventory cost as low as possible.

Next the total relevant inventory cost per cycle consists of the following elements:

1. Cost of ordering per cycle: C_o
2. Inventory holding cost per unit per unit of time in RW: R_h
3. Inventory holding cost per unit per unit of time in OW: O_h
4. Purchase cost: PC
5. Shortages cost: SC
6. Lost sales cost: LC

Above cost are given as follows:

$R_h = \int_0^{t_1} \Psi^{r1}(t)\, R_h\, dt$

$= \frac{a(h_1+1)}{(\alpha\gamma)^2}\{(\frac{t_1^2}{2} - (\alpha\gamma)\frac{t_1^3}{2} - (1-(\alpha\gamma)t_1)e^{(\alpha\gamma)t_1}(\frac{1-e^{-(\alpha\gamma)t_1}}{(\alpha\gamma)^2} - \frac{t_1 e^{-(\alpha\gamma)t_1}}{(\alpha\gamma)})\}$ (12)

$O_h = \int_0^{t_1} \Psi^{o1}(t) O_h\, dt + \int_{t_1}^{t_2} \Psi^{o2}(t) O_h\, dt$

$= \eta\, h_2(\frac{1-e^{-(\beta+1)t_1}}{(\beta+1)^2} - \frac{t_1 e^{-(\beta+1)t_1}}{(\beta+1)}) + \frac{ah_2}{\beta^2}\left\{\frac{(t_2^2-t_1^2)}{2} - \frac{(\beta+1)}{3}(t_2^3 - t_1^3) - (1-(\beta+1)t_2)\left(\left(\frac{e^{-(\beta+1)t_1} - e^{-(\beta+1)t_1}}{(\beta+1)^2}\right) + \right.\right.$

$t1e-(\beta+1)t1(\beta+1)-t2e-(\beta+1)t2(\beta+1)e-(\beta+1)t2$

(13)

PC = P_c* Q_{max}

$= P_c(\eta + \frac{bt_1^2}{2} + \frac{b}{\alpha^2}\{((\alpha\gamma)t_2\text{-}1)e^{(\alpha\gamma)(t_2-t_1)} - ((\alpha\gamma)t_1 - 1)\} + \frac{b}{6}\{T^3 \text{ -}2t_3^3 \text{ -}3t_3^2\text{T}\})$ (14)

SC = $S_c \int_{t_2}^{T}(-\Psi^{s}(t))dt$

$= \frac{abS_c}{2}\left(\frac{T^3}{3} + \frac{2t_2^3}{3} - Tt_2^2\right)$ (15)

LC = $L_c \int_{t_2}^{T}(1-b)\, a\, t\, dt$

$= \frac{aL_c}{2}\left(\frac{(T^2-t_1^2)}{2} - b\,\frac{(T^3-t_1^3)}{3}\right)$ (16)

Therefore, Total relevant Inventory cost per unit per unit of time is denoted and given by

Ψ^{TC} (t_1, t_2 T) $= \frac{1}{T}$ [Ordering cost + Inventory holding cost in RW + Inventory holding cost in OW+ Purchase cost+ Shortage cost + Lost sales cost]

T^{IC} (t_1, t_3, T) $= \frac{1}{T}[C_o + R_h + O_h$ +PC+SC+LC] (17)

Substituting values from equations (12) to (16) in equation (17) we get

$I^{TC}(t_1,t_2T) = \frac{1}{T}[C_o + \frac{a(h_1+1)}{\alpha^2}\{(\frac{t_1^2}{2} - (\alpha\gamma)\frac{t_1^3}{2} - (1-(\alpha\gamma)t_1)e^{(\alpha\gamma)t_1}(\frac{1-e^{-(\alpha\gamma)t_1}}{(\alpha\gamma)^2} - \frac{t_1 e^{-(\alpha\gamma)t_1}}{(\alpha\gamma)})\} + \eta\, h_2(\frac{1-e^{-(\beta+1)t_1}}{(\beta+1)^2} -$

$\frac{t_1 e^{-(\beta+1)t_1}}{(\beta+1)}) + \frac{ah_2}{(\beta+1)^2}\left\{\frac{(t_2^2-t_1^2)}{2} - \frac{(\beta+1)}{3}(t_2^3 - t_1^3) - (1-(\beta+1)t_2)\left(\left(\frac{e^{-(\beta+1)t_1} - e^{-(\beta+1)t_1}}{(\beta+1)^2}\right) + \left(\frac{t_1 e^{-(\beta+1)t_1}}{(\beta+1)} - \right.\right.\right.$

$t2e-(\beta+1)t2(\beta+1)e-(\beta+1)t2+\ Pc(\eta+bt122+\ \ ba2\{((\alpha\gamma)t_2\text{-}1)e(\alpha\gamma)(t2-t1) - ((\alpha\gamma)t1-1)\}+b6\{T3$ -

$2t_3^3 \text{ -}3t_3^2\text{T}\}) + \frac{abS_c}{2}\left(\frac{T^3}{3} + \frac{2t_2^3}{3} - Tt_2^2\right) + \frac{aL_c}{2}\left(\frac{(T^2-t_1^2)}{2} - b\,\frac{(T^3-t_1^3)}{3}\right)]$

(18)

The optimal values of t_1, $t_2$and T denoted as $t_2^*, t_3^*, T*$ respectively can be obtained after solving the following equations

$\frac{\partial\Psi^{TC}}{\partial t_1} = 0$, $\frac{\partial\Psi^{TC}}{\partial t_2} = 0$ and $\frac{\partial\Psi^{TC}}{\partial T} = 0$ (19)

Now the special cases of model are discussed as follows:

Case-1: Model with same deterioration rate$(\beta+1)$in both ware houses

If we take $(\alpha\gamma)$= $(\beta+1)$in the proposed model, the model becomes the case with same deterioration rate and corresponding inventory model can be obtained as

$$\Psi^{TC}(t_1,t_2T)=\frac{1}{T}[C_o+\frac{a(h_1+1)}{(\alpha\gamma)^2}\{(\frac{t_1^2}{2}-(\alpha\gamma)\frac{t_1^3}{2}-(1-(\beta+1)t_1)e^{(\beta+1)t_1}(\frac{1-e^{-(\beta+1)t_1}}{(\alpha\gamma)^2}-\frac{t_1e^{-(\beta+1)t_1}}{(\alpha\gamma)})\}+\eta\, h_2(\frac{1-e^{-(\beta+1)t_1}}{(\beta+1)^2}-\frac{t_1e^{-(\beta+1)t_1}}{(\beta+1)})+\frac{ah_2}{(\alpha\gamma)^2}\left\{\frac{(t_2^2-t_1^2)}{2}-\frac{(\beta+1)}{3}(t_2^3-t_1^3)-(1-(\beta+1)t_2)\left(\left(\frac{e^{-(\beta+1)t_1}-e^{-(\beta+1)t_1}}{(\beta+1)^2}\right)+\left(\frac{t_1e^{-(\beta+1)t_1}}{(\beta+1)}-\frac{t_2e^{-(\beta+1)t_2}}{(\beta+1)}\right)\right)e^{-(\beta+1)t_2}\right\}+P_c(\eta+\frac{bt_1^2}{2}+\frac{b}{(\alpha\gamma)^2}\{((\beta+1)t_2-1)e^{(\beta+1)(t_2-t_1)}-((\beta+1)t_1-1)\}+\frac{b}{6}\{T^3-2t_3^3-3t_3^2T\})+\frac{abS_c}{2}\left(\frac{T^3}{3}+\frac{2t_2^3}{3}-Tt_2^2\right)+\frac{aL_c}{2}\left(\frac{(T^2-t_1^2)}{2}-b\frac{(T^3-t_1^3)}{3}\right)] \quad (20)$$

IV. Numerical Example

In order to illustrate the above solution procedure, consider an inventory system with the following base data chosen randomly in appropriate units: C_o =5500, η =400, b=0.07, (h_1+1)=9, h_2=6, a=28, $(\alpha\gamma)$=0.32, (β+1)=0.57, S_c=59, L_c=36, P_c=55.The values of decision variables are computed for the model and also for the models of special cases. The computational optimal solutions of the models are shown in Table-1.

Analysis of results and Numerical comparison between model and its special cases

Using the same value of parameters as given in numerical example we obtain the total relevant inventory costs of the model for three special cases are given in table-1.The observation is made as follows:

1. The convexity of the graph displayed in Fig. 3 shows that the model has unique point where inventory cost total is minimal. Also the graph drawn between cycle length and total inventory cost in Fig.2 shows that at the optimal values of decision variables obtained on solving equation (19) ,the inventory cost is minimum and as the length of cycle increases or decreases the inventory cost will increases.
2. From the table it is observed that the model with deterioration rate as given for OW and shortages is least expensive than the other models and the model without shortages agrees most expensive state of affairs. So among other two models with different rate of deterioration in both ware-houses, the original model is the flexible model and it corresponds to the less expensive circumstances.

Table-1:

Model	t_1^*	t_2^*	T^*	Total relevant cost
$\Psi^{TC}(t_1,t_2\,T)$	8.2626	9.7582	30.9978	7279.98
Case 1	9.7597	7.8950	27.8546	7925.67

V. Sensitivity analysis

Sensitivity analysis is performed on every parameter of the model. The analysis is carried out by changing the value of only one parameter at a time by increasing and decreasing by 50% keeping the value of rest parameters unchanged. The change in the values of decision variables t_2^*,t_3^* ,T^* and the corresponding change in the total relevant inventory cost is is shown in Table-2.The observation made from the table-2 is given as follows:

(1) The total relevant inventory cost $\Psi^{TC}(t_2^*,t_3^*,T^*)$ increases as the value of C_o, η,h_2a,P_c,L_c, and S_c increases, however the value of $\Psi^{TC}(t_2^*,t_3^*,T^*)$ is decreases as b,$(\alpha\gamma)$ and $(\beta+1)$ increases.

(2) The increase in the value of holding costh_1 in RW does not affect the value of total relevant inventory cost which shows that the model is more useful in case of deteriorating items.

(3) The total relevant inventory cost is highly sensitive in change of values of parameters a,h_2, $(\alpha\gamma)$, $(\beta+1)$, L_c and moderately sensitive in S_c,P_c , η and C_o, the others are slightly sensitive .Thus when decision is to be made, it is necessary to pay attention on the fluctuation of parameters which highly affect the total relevant inventory cost.

(4) The ordering cycle length is also sensitive to the parameters a,h_2, $(\alpha\gamma)$, $(\beta+1)$, L_c, P_c *and* η. As the value of these parameters changed, the length of order cycle is also changed and hence total inventory cost is affected.

Table-2:

2.1 Parameter C_o

Parameter	Changes in the value of the decision variables			Change in the value of total inventory cost
	t_2^*	t_3^*	T^*	$\Psi^{TC}(t_1^*, t_2^*, T^*)$
(3270)	7.2919	9.8768	30.7166	7284.64
(920)	7.2928	9.8753	30.8112	7235.74

2.2 Parameter η

Parameter	Changes in the value of the decision variables			Change in the value of total inventory cost
	t_2^*	t_3^*		t_2^*
(400)	7.4213	6.6453	53.8293	7850.42
(300)	8.7429	7.2453	46.3540	8240.17

2.3 Parameter $(h_1 + 1)$

Parameter	Changes in the value of the decision variables			Change in the value of total inventory cost
	t_2^*	t_3^*		t_2^*
(9.5)	8.2919	5.0981	30.7754	6157.76
(4.5)	8.2919	5.0981	30.7754	6157.76

2.4 Parameter h_2

Parameter	Changes in the value of the decision variables			Change in the value of total inventory cost
	t_2^*	t_3^*	T^*	$\Psi^{TC}(t_1^*, t_2^*, T^*)$
(8.0)	8.1478	7.0513	36.6185	8407.19
(4)	8.1975	6.7487	23.3524	5428.71

2.5 Parameter a

Parameter	Changes in the value of the decision variables			Change in the value of total inventory cost
	t_2^*	t_3^*	T^*	$\Psi^{TC}(t_1^*, t_2^*, T^*)$
(32)	6.6809	9.1077	17.9226	6413.05
(7.0)	4.2860	8.6646	26.1534	3156.84

2.6 Parameter b

Parameter	Changes in the value of the decision variables			Change in the value of total inventory cost
	t_2^*	t_3^*	T^*	$\Psi^{TC}(t_1^*, t_2^*, T^*)$
(0.027)	7.1839	7.9529	12.1497	7025.52
(0.009)	8.1508	7.4916	29.6742	8274.05

2.7 Parameter $(\alpha\gamma)$

Parameter	Changes in the value of the decision variables			Change in the value of total inventory cost
	t_2^*	t_3^*	T^*	$\Psi^{TC}(t_1^*, t_2^*, T^*)$
(0.17)	6.5127	8.9538	27.3450	6433.25
(0.08)	14.3485	14.2459	343.133	7428.21

2.8 Parameter $(\beta + 1)$

Parameter	Changes in the value of the decision variables			Change in the value of total inventory cost
	t_2^*	t_3^*	T^*	$\Psi^{TC}(t_1^*, t_2^*, T^*)$
(0.447)	5.7279	7.2019	21.1904	5061.91
(0.093)	7.5180	24.8014	70.1056	31441.70

2.9 Parameter S_c

Parameter	Changes in the value of the decision variables			Change in the value of total inventory cost
	t_2^*	t_3^*	T^*	$\Psi^{TC}(t_1^*, t_2^*, T^*)$
(12.5)	7.1256	5.9581	50.0484	4217.42
(6.5)	7.1473	7.0113	41.6477	5913.28

2.10 Parameter L_c

Parameter	Changes in the value of the decision variables			Change in the value of total inventory cost
	t_2^*	t_3^*	T^*	$\Psi^{TC}(t_1^*, t_2^*, T^*)$
(44)	5.2172	9.7819	36.0740	7029.76
(9)	5.1187	7.0758	49.0520	5512.47

2.11 Parameter P_c

Parameter	Changes in the value of the decision variables			Change in the value of total inventory cost
	t_2^*	t_3^*	T^*	$\Psi^{TC}(t_1^*,t_2^*,T^*)$
(22.7)	5.2135	6.1217	25.0115	4102.11
(7.9)	9.8426	7.4357	47.5722	7192.45

VI. Conclusions

In this paper, we proposed a deterministic two-ware house inventory model for instantaneous single deteriorating item with linear time-dependent demand and linearly increasing time dependent holding cost, infinite replenishment rate, with the objective of minimizing the total relevant inventory cost. Shortages are allowed and partially backlogged at fraction of constant rate. Also different cases have been discussed one assuming equal deterioration in both ware-houses and others with fully backlogged shortages and without shortages. Furthermore the proposed model can be used in inventory control of certain instantaneous deteriorating items such as food grains, seasonal vegetables, medicines fashionable items etc. and can be further extended by incorporating with other and probabilistic demand pattern and time dependent deterioration rates.

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