

CASP-CUSUM Schemes Based on Truncated Burr Distribution Using Lobbato Integration Method

B. Sainath¹, Dr. P. Mohammed Akhtar², G.Venkatesulu³ and Dr. B.R.Narayana
Murthy⁴

^{1,3} Research scholars & ² Professor, Dept. of Statistics, Sri Krishnadevaraya University, Ananthapuramu-515003 (A.P.)

⁴ Lecturers in Statistics Govt. (U.G&P.G) College, Ananthapuramu-515001 (A.P.)

Abstract: Acceptance sampling plans are introduced mainly to accept or reject the lots of finished products. There are several techniques available to control the quality. Some of the techniques are popularly used where testing involves destruction, for instance, in the manufacturing of crackers, bullets, batteries, bulbs and so on, it is impossible to go for 100% inspection. In this paper we optimized CASP-CUSUM Schemes based on the assumption that the continuous variable under consideration follows a Truncated Burr Distribution. The Burr Distribution is continuous distribution generally used in Life-time Analysis of products, particularly in estimating reliability by considering its distribution. Optimization of CASP-CUSUM Schemes is suggested based on numerical results obtained by changing the values of the parameters of the Burr distribution. We used the Lobatto Integration Method to solve the truncated integral equations.

Keywords: CASP-CUSUM Schemes, type-C, OC Curve, ARL, Truncated Burr distribution.

I. Introduction

Acceptance sampling is concerned with inspection and decision making regarding lots of product by the techniques of quality of assurance in 1930's and 1940's Acceptance Sampling Plans were one of the important statistical tools on the field of Statistical Quality Control. These techniques require specification of a probability model describing the life of the products. In this process the Burr distribution has been studied.

Procedures of statistical quality control are traditionally attributed to two main areas: acceptance sampling and statistical process control. The main aim of the oldest procedures of acceptance sampling, known as *acceptance sampling plans*, is to inspect certain items (products, documents, etc.) submitted for inspection in lots or batches. First acceptance sampling plans, proposed by one of the fathers of SQC, Harold Dodge, were designed for the inspection of lots submitted in sequences (lot-by-lot inspection). The only aim of those plans, known as Dodge-Romig LTPD plans or Dodge-Romig AOQL plans, was to "screen" the inspected series of lots, and to reject lots of supposedly "spotty" quality. In case of stable production processes, i.e. processes characterized by constant probabilities of producing nonconforming items, high quality requirements can be achieved by occasional screening of rejected lots.

In past research the term "quality" defines in different dimensions, particularly with regards to consumer point of view, system designer's point of view etc. Particularly in consumer's point of view durability, safety, low-cost, the degree of satisfaction etc are the major characteristics which determine the quality of a product. Where as in producer's point of view the degree of profit and the degree of low cost of production are the major properties that determine the quality. The term quality is closely associated with reliability of the product.

There are several techniques for controlling the quality Acceptance Sampling Plans are introduced to decide the case of lots of finished products-either acceptance or rejection. These are mainly used to test the level of destruction. For example in manufacture of crackers, bullets,..etc, 100% inspection is not possible. Techniques of Acceptance Sampling Plans may not have direct impact in controlling the quality of product. In improving the quality of the product, they have much indirect effect. A product may be continuously rejected for lack of quality, then, the producer will try hard to improve the quality of the product. In case quality of the product not being improved, the user will go for a product of better quality.

Hawkins, D. M. [3] proposed a fast accurate approximation for ARL's of a CUSUM Control Charts. This approximation can be used to evaluate the ARL's for Specific parameter values and the out of control ARL's of location and scale CUSUM Charts.

Vardeman.S, Di-ou Ray [9] was introduced CUSUM control charts under the restriction that the values are regard to quality is exponentially distributed. Further the phenomena under study is the occurrence of rate of rare events and the inter arrival times for a homogenous poisson process are identically independently distributed exponential random variables.

Lonnie. C. Vance, [6] consider Average Run Length of cumulative Sum Control Charts for controlling for normal means and to determine the parameters of a CUSUM Chart. To determine the parameters of CUSUM Chart the acceptable and rejectable quality levels along with the desired respective ARL's are consider.

Hawkins, D. M. [3] proposed a fast accurate approximation for ARL's of a CUSUM Control Charts. This approximation can be used to evaluate the ARL's for Specific parameter values and the out of control ARL's of location and scale CUSUM Charts.

Kakoty. S., Chakravaborthy A.B. [5] proposed CASP-CUSUM charts under the assumption that the variable under study follows a Truncated Normal Distribution. Generally truncated distributions are employed in many practical phenomena where there is a constraint on the lower and upper limits of the variable under study. For example, in the production engineering items, the sorting procedure eliminates items above or bellows designated tolerance limits. It is worthwhile to note that any continuous variable be first approximated as an exponential variable.

Muhammed Riaz, Nasir Abbas and Ronald J.M.M Does [7] proposed two Runs rules schemes for the CUSUM Charts. The performance of the CUSUM and EWMA Charts are compared with the usual CUSUM and weighted CUSUM, the first initial response CUSUM compared with usual EWMA Schemes. This comparison stated that the proposed schemes perform better for small and moderate shifts.

Mohammed Akhtar. P and Sarma K.L.A.P [1] proposed an optimization of CASP-CUSUM Schemes based on truncated two parametric Gamma distribution and evaluate L (0) L' (O) and probability of Acceptance and also Optimized CASP-CUSUM Schemes based numerical results.

In this study, we proposed acceptance sampling method based on the truncated Burr distribution under the assumption that the variable under study distributed according to truncated Burr distribution under this assumption we determined an appropriate measures for CUSUM schemes by using Lobatto Method of Integration. Thus it is more worthwhile to study some interesting characteristics of this distribution. The characteristics and applications of the Burr distribution were discussed.

The Burr Distribution is a continuous probability distribution for a non-negative random variable. It is also known as the Sing- Maddala distribution and is one of a number of different distributions sometimes called the 'generalized log-logistic distribution'.

A continuous random variable X assuming non-negative values is said to have Burr Distribution with parameters c, k > 0, its probability density function is given by:

$$f(x; c, k) = ck \frac{x^{c-1}}{(1 + x^c)^{k+1}} \quad \dots\dots (1.1)$$

The random variable X is said to follow a truncated Burr Distribution as:

$$f_B(x) = \frac{ck \frac{x^{c-1}}{(1 + x^c)^{k+1}}}{1 - \frac{1}{(1 + B^c)^k}} \quad \dots\dots (1.2)$$

Where B is the truncated point of the Burr Distribution.

There are several different parameterizations of the distribution in use. When $c = 1$, The Burr distribution becomes the Parato Type-II distribution, when $k = 1$, the Burr distribution is a special case of the Champernowne distribution. The Burr distribution has appeared in the literature under different names. The relationship between the Burr distribution and the various other distributions, namely, the Lomax, the Compound Weibull, the Weibull-Exponential, the Logistic, the Weibull and the Kappa family of distributions is summarized.

II. Description Of The Plan And Type- C Oc Curve

Battie [2] has suggested the method for constructing the continuous acceptance sampling plans. The procedure, suggested by him consists of a chosen decision interval namely, "Return interval" with the length h', above the decision line is taken. We plot on the chart the sum $S_m = \sum (X_i - k_1)X_i'$ ($i = 1, 2, 3, \dots\dots$) are distributed independently and k_1 is the reference value. If the sum lies in the area of normal chart, the product is accepted and if it lies of the return chart, then the product is rejected, subject to the following assumptions.

1. When the recently plotted point on the chart touches the decision line, then the next point to be plotted at the maximum, i.e., $h+h'$
2. When the decision line is reached or crossed from above, the next point on the chart is to be plotted from the baseline.

When the CUSUM falls in the return chart, network or a change of specification may be employed rather than outright rejection.

The procedure in brief is given below.

1. Start plotting the CUSUM at 0.
2. The product is accepted when $S_m = \sum (X_i - k) < h$; when $S_m < 0$, return cumulative to 0.
3. When $h < S_m < h+h'$ the product is rejected: when S_m crossed h , i.e., when $S_m > h+h'$ and continue rejecting product until $S_m > h+h'$ return cumulative to $h+h'$

The type-C, OC function, which is defined as the probability of acceptance of an item as function of incoming quality, when sampling rate is same in acceptance and rejection regions. Then the probability of acceptance $P(A)$ is given by

$$P(A) = \frac{L(0)}{L(0) + L'(0)} \quad \dots\dots (2.1)$$

Where $L(0)$ = Average Run Length in acceptance zone and

$L'(0)$ = Average Run Length in rejection zone.

Page E.S. [8] has introduced the formulae for $L(0)$ and $L'(0)$ as

$$L(0) = \frac{N(0)}{1 - P(0)} \quad \dots\dots (2.2)$$

$$L'(0) = \frac{N'(0)}{1 - P'(0)} \quad \dots\dots (2.3)$$

Where $P(0)$ = Probability for the test starting from zero on the normal chart,

$N(0)$ = ASN for the test starting from zero on the normal chart,

$P'(0)$ = Probability for the test on the return chart and

$N'(0)$ = ASN for the test on the return chart

He further obtained integral equations for the quantities

$P(0)$, $N(0)$, $P'(0)$, $N'(0)$ as follows:

$$P(z) = F(k_1 - z) + \int_0^h P(y) f(y + k_1 - z) dy, \quad \dots\dots (2.4)$$

$$N(z) = 1 + \int_0^h N(y) f(y + k_1 - z) dy, \quad \dots\dots (2.5)$$

$$P'(z) = \int_{k_1+z}^B f(y) dy + \int_0^h P'(y) f(-y + k_1 + z) dy \quad \dots\dots (2.6)$$

$$N'(z) = 1 + \int_0^h N'(y) f(-y + k_1 + z) dy, \quad \dots\dots (2.7)$$

$$F(x) = 1 + \int_A^h f(x) dx :$$

$$F(k_1 - z) = 1 + \int_A^{k_1-z} f(y) dy$$

and z is the distance of the starting of the test in the normal chart from zero.

III. Method Of Solution

We first express the integral equation (2.4) in the form

$$F(X) = Q(X) + \int_c^d R(x,t)F(t)dt \quad \dots\dots (3.1)$$

where

$$F(X) = P(z),$$

$$Q(X) = F(k - z),$$

$$R(X, t) = f(y + k - z)$$

Let the integral $I = \int_c^d f(x)dx$ be transformed to

$$I = \frac{d-c}{2} \int_c^d f(y)dy = \frac{d-c}{2} \sum a_i f(t_i) \quad \dots\dots (3.2)$$

Where $y = \frac{2x - (c-d)}{d-c}$ where a_i 's and t_i 's respectively the weight factor and abscissa for the Gass-Chibyshev polynomial, given in Jain M.K. and et al [4] using (3.1) and (3.2), (2.4) can be written as

$$F(X) = Q(X) \frac{d-c}{2} \sum a_i R(x, t_i) F(t_i) \quad \dots\dots (3.3)$$

Since equation (3.3) should be valid for all values of x in the interval (c, d), it must be true for $x=t_i, i = 0(1)n$ then obtain.

$$F(t_i) = Q(t_i) + \frac{d-c}{2} \sum a_j R(t_j, t_i) F(t_j) \quad j = 0(1)n \quad \dots\dots (3.4)$$

Substituting

$$F(t_i) = F_i, Q(t_i) = Q_i, i = 0(1)n, \text{ in (3.4), we get}$$

$$F_0 = Q_0 + \frac{d-c}{2} [a_0 R(t_0, t_0) F_0 + a_1 R(t_0, t_1) F_1 + \dots\dots\dots a_n R(t_0, t_n) F_n]$$

$$F_1 = Q_1 + \frac{d-c}{2} [a_0 R(t_1, t_0) F_0 + a_1 R(t_1, t_1) F_1 + \dots\dots\dots a_n R(t_1, t_n) F_n]$$

.....

$$F_n = Q_n + \frac{d-c}{2} [a_0 R(t_n, t_0) F_0 + a_1 R(t_n, t_1) F_1 + \dots\dots\dots a_n R(t_n, t_n) F_n] \quad \dots\dots (3.5)$$

In the system of equations except $F_i, i= 0,1,2,\dots\dots\dots n$ are known and hence can be solved for F_i , we solved the system of equations by the method of Iteration.

For this we write the system (3.5) as

$$[1 - T a_0 R(t_0, t_0)] F_0 = Q_0 + T [a_0 R(t_0, t_0) F_0 + a_1 R(t_0, t_1) F_1 + \dots\dots\dots a_n R(t_0, t_n) F_n]$$

$$[1 - T a_1 R(t_1, t_1)] F_1 = Q_1 + T [a_0 R(t_1, t_0) F_0 + a_1 R(t_1, t_1) F_1 + \dots\dots\dots a_n R(t_1, t_n) F_n]$$

.....

$$[1 - T a_n R(t_n, t_n)] F_n = Q_n + T [a_0 R(t_n, t_0) F_0 + a_1 R(t_n, t_1) F_1 + \dots\dots\dots a_n R(t_n, t_n) F_n] \quad \dots\dots (3.6)$$

Where $T = \frac{d-c}{2}$

To start the Iteration process, let us put $F_1 = F_2 = \dots = F_n = 0$ in the first equation of (3.6), we then obtain a rough value of F_0 . Putting this value of F_0 and $F_1 = F_2 = \dots = F_n = 0$ on the second equation, we get the rough value F_1 and so on. This gives the first set of values F_i $i= 0,1,2,\dots,n$ which are just the refined values of F_i $i= 0,1,2,\dots,n$. The process is continued until two consecutive sets of values are obtained up to a certain degree of accuracy. In the similar way solutions $P'(0)$, $N(0)$, $N'(0)$ can be obtained.

IV. Computation Of ARL'S P (A)

We developed computer programs to solve these equations by using Lobatto Method of Integration and we get the following results given in the Tables (4.1) to (4.16).

TABLE- 4.1

Values of ARL's AND TYPE-C OC CURVES when $c=2, k=3, k_1=2, h=0.1, h'=0.1$

B	L(0)	L'(0)	P(A)
2.9	130.07643	1.1438599	0.9912829
2.8	134.63844	1.1438880	0.9915756
2.7	140.78049	1.1439230	0.9919399
2.6	149.30469	1.1439668	0.9923963
2.5	161.63007	1.1440217	0.9929717
2.4	180.52048	1.1440915	0.9937022
2.3	212.17067	1.1441805	0.9946362
2.2	273.89825	1.1442947	0.9958396
2.1	439.48712	1.1444426	0.9974027
2.0	2082.76489	1.1446353	0.9994507

TABLE- 4.2

Values of ARL's AND TYPE-C OC CURVES when $c=2, k=3, k_1=2, h=0.15, h'=0.15$

B	L(0)	L'(0)	P(A)
2.9	117.18018	1.1981601	0.9898785
2.8	120.97939	1.1981912	0.9901930
2.7	126.06363	1.1982299	0.9905846
2.6	133.06259	1.1982783	0.9910750
2.5	143.06744	1.1983393	0.9916936
2.4	158.13802	1.1984166	0.9924787
2.3	182.70578	1.1985151	0.9934829
2.2	228.29968	1.1986414	0.9947771
2.1	337.31949	1.1988049	0.9964586
2.0	895.33490	1.1990176	0.9986626

TABLE- 4.3

Values of ARL's AND TYPE-C OC CURVES when $c=2, k=3, k_1=2, h=0.2, h'=0.2$

B	L(0)	L'(0)	P(A)
2.8	106.20831	1.2395940	0.9884633
2.7	110.22456	1.2396272	0.9888787
2.6	115.70068	1.2396687	0.9893991
2.5	123.42341	1.2397208	0.9900554
2.4	134.83228	1.2397870	0.9908888
2.3	152.86931	1.2398711	0.9919546
2.2	184.62001	1.2399789	0.9933285
2.1	252.563.98	1.2401180	0.9951138
2.0	486.06720	1.2402987	0.9974548

TABLE- 4.4

Values of ARL's AND TYPE-C OC CURVES when
 $c=2, k=3, k_1=2, h=0.25, h'=0.25$

B	L(0)	L'(0)	P(A)
2.8	91.62343	1.2681966	0.9863476
2.7	94.68713	1.2682160	0.9867833
2.6	98.82321	1.2682402	0.9873292
2.5	104.57600	1.2682706	0.9880176
2.4	112.90941	1.2683089	0.9888918
2.3	125.69627	1.2683576	0.9900101
2.2	147.11438	1.2684197	0.9914517
2.1	188.78175	1.2684995	0.9933255
2.0	299.53790	1.2686023	0.9957827
1.9	1314.68787	1.2687353	0.9990359

TABLE- 4.5

Values of ARL's AND TYPE-C OC CURVES when
 $c=2, k=3, k_1=4, h=0.1, h'=0.1$

B	L(0)	L'(0)	P(A)
4.0	2133.46509	1.1437469	0.9994642
3.9	2295.62134	1.1437505	0.9995020
3.8	2521.66650	1.1437546	0.9995466
3.7	2855.63647	1.1437595	0.9995996
3.6	3390.29297	1.1437656	0.9996628
3.5	4366.95068	1.1437725	0.9997382
3.4	6680.35400	1.1437811	0.9998288
3.3	18466.36328	1.1437913	0.9999381

TABLE- 4.6

Values of ARL's AND TYPE-C OC CURVES when
 $c=2, k=3, k_1=4, h=0.15, h'=0.15$

B	L(0)	L'(0)	P(A)
3.7	1032.93848	1.1980486	0.9988415
3.6	1097.31189	1.1980551	0.9989094
3.5	1185.69678	1.1980630	0.9989906
3.4	1312.61084	1.1980724	0.9990881
3.3	1507.33105	1.1980838	0.9992058
3.2	1837.46399	1.1980976	0.9993484
3.1	2505.60181	1.1981144	0.9995220
3.0	4515.17383	1.1981348	0.9997347
2.9	333724.03125	1.1981601	0.9999964

TABLE- 4.7

Values of ARL's AND TYPE-C OC CURVES when
 $c=2, k=3, k_1=4, h=0.2, h'=0.2$

B	L(0)	L'(0)	P(A)
3.4	599.01300	1.2394922	0.9979351
3.3	637.69720	1.2395020	0.9980600
3.2	691.83472	1.2395138	0.9982116
3.1	771.58246	1.2395281	0.9983961
3.0	898.28186	1.2395457	0.9986220
2.9	1125.67822	1.2395673	0.9989001
2.8	1638.92163	1.2395940	0.9992442
2.7	3788.10962	1.2396272	0.9996729

TABLE-4.8

Values of ARL's AND TYPE-C OC CURVES when
 $c=2, k=3, k_1=4, h=0.25, h'=0.25$

B	L(0)	L'(0)	P(A)
3.2	373.47791	1.2681496	0.9966159
3.1	396.24057	1.2681581	0.9968097
3.0	428.16718	1.2681683	0.9970469
2.9	475.29526	1.2681810	0.9973389
2.8	550.22369	1.2681966	0.9977005
2.7	684.43610	1.2682402	0.9981505
2.6	985.23639	1.2682402	0.9987144
2.5	2206.78760	1.2682706	0.9994256

TABLE- 4.9

Values of ARL's AND TYPE-C OC CURVES when
 $c=2, k=3, k_1=6, h=0.1, h'=0.1$

B	L(0)	L'(0)	P(A)
4.1	3312.27905	1.1437439	0.9996548
4.0	3651.60229	1.1437469	0.9996869
3.9	4153.80957	1.1437505	0.9997247
3.8	4959.56738	1.1437546	0.9997694
3.7	6438.54248	1.1437595	0.9998224
3.6	9991.09082	1.1437656	0.9998856
3.5	29361.46289	1.1437725	0.9999610

TABLE- 4.10

Values of ARL's AND TYPE-C OC CURVES when
 $c=2, k=3, k_1=6, h=0.15, h'=0.15$

B	L(0)	L'(0)	P(A)
3.7	1303.49500	1.1980486	0.9990817
3.6	1407.71045	1.1980551	0.9991497
3.5	1556.56519	1.1980630	0.9992309
3.4	1783.07739	1.1980724	0.9993286
3.3	2162.27710	1.1980838	0.9994462
3.2	2913.64819	1.1980976	0.9995890
3.1	5048.28125	1.1981144	0.9997627
3.0	49046.61719	1.1981348	0.9999756

TABLE- 4.11

Values of ARL's AND TYPE-C OC CURVES when
 $c=2, k=3, k_1=6, h=0.2, h'=0.2$

B	L(0)	L'(0)	P(A)
3.4	683.96472	1.2394922	0.9981911
3.3	734.86682	1.2395020	0.9983161
3.2	807.70392	1.2395138	0.9984677
3.1	918.60046	1.2395281	0.9986525
3.0	1104.07007	1.2395457	0.9988785
2.9	1468.89441	1.2395673	0.9991568
2.8	2484.38403	1.2395940	0.9995013
2.7	17751.67773	1.2396272	0.9999302

TABLE- 4.12

Values of ARL's AND TYPE-C OC CURVES when
 $c=2, k=3, k_1=6, h=0.25, h'=0.25$

B	L(0)	L'(0)	P(A)
3.2	405.95563	1.2681496	0.9968858
3.1	433.00653	1.2681581	0.9970798
3.0	471.43665	1.2681683	0.9973172
2.9	529.23541	1.2681810	0.9976095
2.8	623.83154	1.2681966	0.9979712
2.7	802.22668	1.2682160	0.9984216
2.6	1249.41382	1.2682402	0.9989859
2.5	4194.95410	1.2682706	0.9996977

TABLE- 13

Values of ARL's AND TYPE-C OC CURVES when
 $c=2, k=3, k_1=8, h=0.1, h'=0.1$

B	L(0)	L'(0)	P(A)
4.2	3238.40625	1.1437414	0.9996470
4.1	3510.66309	1.1437439	0.9996743
4.0	3895.16211	1.1437469	0.9997064
3.9	4470.62549	1.1437505	0.9997442
3.8	5418.00049	1.1437546	0.9997889
3.7	7236.36865	1.1437595	0.9998420
3.6	12053.22754	1.1437656	0.9999051
3.5	58832.07813	1.1437725	0.9999806

TABLE- 14

Values of ARL's AND TYPE-C OC CURVES when
 $c=2, k=3, k_1=8, h=0.15, h'=0.15$

B	L(0)	L'(0)	P(A)
3.7	1334.07751	1.1980486	0.9991028
3.6	1443.44580	1.1980551	0.9991707
3.5	1600.37549	1.1980630	0.9992520
3.4	1840.80286	1.1980724	0.9993496
3.3	2248.08423	1.1980838	0.9994674
3.2	3071.01660	1.1980976	0.9996100
3.1	5542.17432	1.1981144	0.9997839
3.0	357013.75000	1.1981348	0.9999967

TABLE- 15

Values of ARL's AND TYPE-C OC CURVES when
 $c=2, k=3, k_1=8, h=0.2, h'=0.2$

B	L(0)	L'(0)	P(A)
3.4	692.59308	1.2394922	0.9982135
3.3	744.83673	1.2395020	0.9983386
3.2	819.76453	1.2395138	0.9984903
3.1	934.17389	1.2395281	0.9986749
3.0	1126.64478	1.2395457	0.9989010
2.9	1509.12585	1.2395673	0.9991793
2.8	2601.69507	1.2395940	0.9995238
2.7	26189.97461	1.2396272	0.9999527

TABLE- 16

Values of ARL's AND TYPE-C OC CURVES when
 $c=2, k=3, k_1=8, h=0.25, h'=0.25$

B	L(0)	L'(0)	P(A)
3.2	409.08060	1.2681496	0.9969096
3.1	436.55048	1.2681581	0.9971035
3.0	475.65646	1.2681683	0.9973409
2.9	534.53955	1.2681810	0.9976332
2.8	631.21484	1.2681966	0.9979949
2.7	814.52466	1.2682160	0.9984454
2.6	1279.50281	1.2682402	0.9990098
2.5	4554.59766	1.2682706	0.9997216

V. Numerical Results And Conclusions

At the hypothetical values of the parameters c, k, k_1, h and h' given at the top of each table, we determine optimum truncated point B at which $P(A)$ the probability of accepting an item is maximum and also obtained ARL's values which represents the acceptance zone $L(0)$ and rejection zone $L'(0)$ values. The values of truncated point B of random variable $X, L(0), L'(0)$ and the values for Type-C Curve, i.e. $P(A)$ are given in columns I, II, III, and IV respectively.

From the above tables 4.1 to 4.16 we made the following conclusions

1. From the table 4.1 to 4.4, it is observed that the value of $L(0)$ and $P(A)$ are increased as the value of truncated point decreases. Thus the truncated point of the random variable and the various parameters for CASP-CUSUM are related.
2. And also we observe that it can be minimize the truncated point B by increasing value of k_1
3. From table 4.1 to 4.4, it is observed that truncated point B of the random variable X decreases from 2.9 to 1.9 as $h \rightarrow 0.25$, while the value of $L(0)$ decrease from 2082.76489 to 1314.68787 and rejection zone values changes from 1.1446353 to 1.2687353 where as the probability of acceptance $P(A)$ changes from 0.9994507 to 0.9990359. Thus hypothetical value h and truncated point B inversely related, while the values $L(0), L'(0)$ and $P(A)$ are positively related.
4. From table 4.5 to 4.8, it is observed that at the maximum level of probability of acceptance $P(A)$ the truncated point B from 4.0 to 2.5 as the value of h changes from 0.1 to 0.25.
5. From table 4.5 to 4.8, it is observed that the size of acceptance zone is changes from 18466.36328 to 2206.78760. Thus the optimal truncated point and size of the acceptance zone are positively related.
6. From the table 4.9 to 4.12, it is observed that the value of $L(0)$ and $P(A)$ are increases as the value of truncated point decreases. Thus the truncated point of the random variable and the various parameters for CASP-CUSUM are related.

7. And also we observed that it can be minimize the truncated point B by increasing value of k_1
8. From the table 4.9 to 4.12, it is observed that the truncated point B changes from 4.1 to 2.5 as $h \rightarrow 0.25$
9. From the table 4.13 to 4.16, it is observed that the truncated point B changes from 4.2 to 2.5 and P (A) is as $h \rightarrow 0.25$ maximum **0.9999967**. Thus truncated point B and h are inversely related and h and P (A) are positively related.
10. From Table 4.1 to 4.16 it can be observed that the optimal truncated point B is more influenced by C, however the value of h and h' tends to 0.25.
11. The various relations exhibited among the ARL's and Type-C OC Curves with the parameters of the CASP-CUSUM based on the above table 4.1 to 4.16 are observed from the following Table 5.1.

B	c	k	k_1	h	h'	L(0)	L'(0)	P(A)
2.0	2	3	2	0.1	0.1	2082.76489	1.1446353	0.9994507
2.0	2	3	2	0.15	0.15	895.33490	1.1990176	0.9986626
2.0	2	3	2	0.2	0.2	486.06720	1.2402987	0.9974548
1.9	2	3	2	0.25	0.25	1314.68787	1.2687353	0.9990359
3.3	2	3	4	0.1	0.1	18466.36328	1.1437913	0.9999381
2.9	2	3	4	0.15	0.15	333724.03125	1.1981601	0.9999964
2.7	2	3	4	0.2	0.2	3788.10962	1.2396272	0.9996729
2.5	2	3	4	0.25	0.25	2206.78760	1.2682706	0.9994256
3.5	2	3	6	0.1	0.1	29361.46289	1.1437725	0.9999610
3.0	2	3	6	0.15	0.15	49046.61719	1.1981348	0.9999756
2.7	2	3	6	0.2	0.2	17751.67773	1.2396272	0.9999302
2.5	2	3	6	0.25	0.25	4194.95410	1.2682706	0.9996977
3.5	2	3	8	0.1	0.1	58832.07813	1.1437725	0.9999806
3.0	2	3	8	0.15	0.15	357013.75000	1.1981348	0.9999967
2.7	2	3	8	0.2	0.2	26189.97461	1.2396272	0.9999527
2.5	2	3	8	0.25	0.25	4554.59766	1.2682706	0.9997216

By observing the Table 5.1, we can conclude that the optimum CASP-CUSUM Schemes which have the values of ARL and P (A) reach their maximum i.e., 357013.75000 0.9999967 respectively, is

$$\left[\begin{array}{l} B = 3.0 \\ c = 2 \\ k = 3 \\ k_1 = 8 \\ h = 0.15 \\ h' = 0.15 \end{array} \right]$$

On similar lines we can obtain CASP-CUSUM Schemes when a particular parameter is fixed at a point, for example, if we fixed the value of $k_1 = 2$ in that case only the maximum value of probability of acceptance P(A)= 0.9990359, is

$$\left[\begin{array}{l} B = 1.9 \\ c = 2 \\ k = 3 \\ k_1 = 2 \\ h = 0.25 \\ h' = 0.25 \end{array} \right]$$

References

- [1]. Akhtar, P. Md. and Sarma, K.L.A.P. (2004). "Optimization of CASP-CUSUM Schemes based on Truncated Gamma Distribution". Bulletin of Pure and applied sciences, Vol-23E (No.2):215-223.
- [3]. Beattie, B.W. (1962). "A Continuous Acceptance Sampling procedure based upon a cumulative Sums Chart for number of defective". Applied Statistics, Vol. 11 (No.2): 137-147.

- [4]. Hawkins, D.M. (1992). "A Fast Accurate Approximation for Average Lengths of CUSUM Control Charts". *Journal on Quality Technology*, Vol. 24(No.1): 37-43.
- [5]. Jain, M.K. Iyengar, S.R.K. and Jain, R.K. "Numerical Methods of Scientific and Engineering Computations", Willy Eastern Ltd., New Delhi.
- [6]. Kakoty, S and Chakravaborthy, A.B., (1990), "A Continuous Acceptance Sampling
- [7]. Plan for Truncated Normal distribution based on Cumulative Sums", *Journal of National Institution for Quality and Reliability*, Vol.2 (No.1): 15-18
- [8]. Lonnie, C. Vance. (1986). "Average Run Length of CUSUM Charts for Controlling Normal means". *Journal of Quality Technology*, Vol.18:189-193.
- [9]. Muhammad Riaz, Nasir, Abbas and and Ronald, J.M.M. Does, (2011). "Improving the performance of CUSUM charts", *Quality and Reliability Engineering International*, Vol.27:415-424.
- [10]. Page, E.S., (1954) "Continuous Inspection Schemes", *Biometrika*, Vol. XLI, pp. 104-114.
- [11]. Vardeman, S. and Di-ou Ray. (1985). "Average Run Lengths for CUSUM schemes where observations are Exponentially Distributed", *Technometrics*, vol. 27 (No.2): 145-150.
- [12]. Narayana Muthy, B. R, Akhtar, P. Md and Venkataramudu, B.(2012) "Optimization of CASP-CUSUM Schemes based on Truncated Log-Logistic Distribution".
- [13]. *Bulletin of Pure and applied Sciences*, Vol-31E (Math&Stat.): Issue (No.2) pp243-255.
- [14]. Narayana Muthy, B. R, Akhtar, P. Md and Venkataramudu, B. (2013) "Optimization of CASP-CUSUM Schemes based on Truncated Rayleigh Distribution".
- [15]. *International Journal of Engineering and Development*, Volum 6, Issue 2, pp. 37-44
- [16]. B.Sainath, P.Mohammed Akhtar, G.Venkatesulu, and Narayana Muthy, B. R, (2015)
- [17]. "Optimization of CASP-CUSUM Schemes based on Truncated Burr
- [18]. Distribution". *Bulletin of Pure and applied Sciences*, Vol-34E (Math&Stat.): Issue (No.2) pp.47-60