IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 2 Ver. VI (Mar. - Apr. 2016), PP 08-09
www.iosrjournals.org

Some Results on Associative Ring with Unity

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Abstract*: In this paper we have mainly obtained some theorems related to Associative ring with unity.*
Key words*: Associative ring, ring with unity*

I. Introduction

Quadri Ashraf (5) generalized some results on Associative Rings. They proved that R is an associative semi prime ring in which $(xy)^2-yx^2y$ is centre, then R is commutative ring, then R is Commutative. In this paper, we show that a Associative Ring with unity such that $(yx)x=(xy)x$, $(xy)^2 = xy^2x$, for all x,y in R. Then R is commutative. Throughout the paper Z(R) denotes the centre of non Associative ring R and (x,y)=xy-yx for all x, y in R

Main Results; we prove the following theorems

Theorem 1: Let R be a associate ring with unity 1 such that (yx)x=(xy)x for all x,y in R
Then R is commutative.
Proof: Given condition is (yx)x = (xy)x
Replacing x by x+1 in the above condition ,

then $[y(x+1)](x+1) = [(x+1)y](x+1)$
$(yx+y)(x+1) = (xy+y)(x+1)$
$(yx)x+ yx+yx+y = (xy)x+xy+yx+y$ (by the given condition, cancellation law)
$yx = xy \quad \forall\, x,y \in R$
Hence R is a commutative ring for all x,y.

Theorem 2: Let R be a associate ring with unity 1 such that $(xy)^2 = xy^2x$ for all x,y in R, then R is commutative.
Proof: Given condition is $(xy)^2 = xy^2x$,
Replacing x by x+1 in the given identity, $[(x+1)y]^2 = (x+1) y^2(x+1)$

$$(xy+y)^2 = (xy^2+y^2)(x+1)$$
$$(xy+y)(xy+y) = xy^2x + x y^2 + y^2 x + y^2$$
$$(xy)^2+x y^2 +yxy+ y^2 =x y^2x + x y^2 + y^2x + y^2$$

by the given condition and cancellation law

$$yxy = y^2x$$

Replacing y by y+1 in the above result

$$(y+1)x(y+1)=(y+1)^2x$$
$$(y+1)x(y+1) = (y^2+2y+1)x$$
$$(yx+x)(y+1) = y^2x+2yx+x$$
$$yxy +yx+xy+x = y^2x+2yx+x$$ by the given condition, cancellation law,
$$xy =yx \quad \forall\, x,y \in R$$

Hence R is commutative ring for all x,y.

Theorem 3: If R is a Ring with unity satisfying $(xy – x^ny^m, x)= 0$ for all x,y Є R and fixed integers m>1 , n≥1 then R is commutative
Proof: Given condition $(xy – x^ny^m, x)= 0$ for all x,y Є R and also given that m>1 , n≥1
Let m=2, n=1 ,then given condition $[xy - xy^2 ,x]=0$ for all x, y Є R,That is,$(xy - xy^2) x = x (xy - xy^2)$.
Replacing x by x+1 in the above result,

$$[(x+1)y - (x+1) y^2](x+1) = (x+1)[(x+1)y - (x+1) y^2]$$
$$[xy+y-xy^2-y^2](x+1) = (x+1) [xy+y-xy^2-y^2]$$
$$xyx+xy +yx+y -xy^2x -x y^2-y^2x-y^2 = x^2y+xy-x^2 y^2-x y^2+xy+y-x y^2- y^2$$

by cancellation law

$$(xy-x y^2)x + yx - y^2x = x(xy-xy^2)+xy-xy^2$$
$$yx - y^2x = xy - xy^2$$ (From the given condition)

$yx = xy \quad \forall\, x,y \in R$ (by the theorem $y^2x = xy^2$)

Hence R is a commutative ring for all x, y

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