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Convex Fuzzy Set, Balanced Fuzzy Set, and Absolute Convex Fuzzy Set in a Fuzzy Vector Space

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Abstract: *In this paper, we have studied the absolute convex fuzzy set over a fuzzy vector space. We examine the properties of absolute convex fuzzy set, and established some independent results under the linear mapping from one vector space to another one.*
Keywords *: Fuzzy vector space, Fuzzy subspace, convex fuzzy set, Balanced fuzzy set, and Absolute convex fuzzy set.*

I. Introduction

The concept of fuzzy set was introduced by Zadeh [6], and the notion of fuzzy vector space was defined and established by KATSARAS,A.K and LIU,D.B [2]. Using the definition of fuzzy vector space, balanced fuzzy set and absolute convex fuzzy set over a fuzzy vector space, we established the elementary properties of absolute convex fuzzy set over a fuzzy vector space, using the linear mapping from one space to another one.

II. Preliminaries

FUZZY VECTOR SPACE

Definition 2.1 : Let X be a vector space over K ,where K is the space of real or complex numbers, then the vector space equipped with addition (+) and scalar multiplication defined over the fuzzy set (on X) as below is called a fuzzy vector space.

Addition (+) : Let $A_1,\dots\dots\dots,A_n$ be the fuzzy sets on vector space X , let $f : X^n \to X$, such that $f(x_1,\dots\dots\dots,x_n) = x_1 + \dots\dots\dots + x_n$,we define

$$A_1 + \dots\dots\dots + A_n = f\left(A_1,\dots\dots\dots A_n\right) \text{, by the extension principle}$$

$$\mu_{f(A_1,\dots\dots,A_n)}(y) = \sup_{\substack{x_1,\dots x_n \\ y=f(x_1,\dots x_n)}} \left\{\mu_{A_1}(x_1),\dots\dots\dots,\mu_{A_n}(x_n)\right\}$$

Obviously, when sets $A_1,\dots\dots\dots,A_n$ are ordinary sets , the gradation function used in the sum are taken as characteristic function of the set.

Scalar multiplication (.) : If α is a scalars and B be a fuzzy set on X and $g : X \to X$,such that $g(x) = \alpha x$,then using extension principle we define αB as $\alpha B = g(B)$,where

$$\mu_{g(B)}(y) = \sup_{\substack{y=g(x) \\ y=\alpha x}} \{\mu_B(x)\}, \text{ if } y = \alpha x \text{ holds}$$

$$\mu_{g(B)}(y) = 0 \text{, if } y \neq \alpha x, for any x \epsilon X$$

i.e

$$\mu_{\alpha B}(y) = \sup_{\substack{x \\ y=\alpha x}} \mu_B(x) \text{, if } y \in X$$

$$\mu_{\alpha B}(y) = 0 \text{, if } y \neq \alpha x \text{, for any } x$$

THEOREM 2.1 :*If E and F are vector spaces over K,f is a linear mapping from E to F and A,B are fuzzy sets on E, then*

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(i) $f(A+B)=f(A)+f(B)$

(ii) $f(\alpha A)=\alpha f(A)$, for all scalars α

i.e $f(\alpha A+\beta B)=\alpha f(A)+\beta f(B)$, where α, β, are scalars

Proof: Proof is straight forward.

Definition 2.2*:* *If A is a fuzzy set in a vector space E and x ∈ X, we define x + A as x + A = {x} +A.*

THEOREM 2.2: *If f_x: E → E (vector space) such that $f_x(y) = x + y$, then if B is a fuzzy set in E and A is an ordinary subset of E, the following holds*

$$x+B=f_x(B)$$

$$\mu_{x+B}(z)=\mu_B(z-x)$$

$$A+B=\underset{x\in A}{\cup}(x+B)$$

Proof : Proof is straight forward.

THEOREM 2.3: *If A_1,........,A_n, are fuzzy sets in vector space E and α_1,..........,α_n are scalars*

$\alpha_1 A_1+..........\alpha_n A_n \subset A$, *iffor all $x_{i,\ldots\ldots\ldots}$,$x_{n,}$inE,we have*

$$\mu_A(\alpha_1 x_1+..........+\alpha_n x_n)\geq \min\left\{\mu_{A_1}(x_1),........,\mu_{A_n}(x_n)\right\}$$

Proof : Proof is obvious.

III. Fuzzy Subspace

Definition 3.1*:A fuzzy set F in a vector space E is called fuzzy subspace of E if (i) F + F ⊂ F (ii) α F ⊂ F, for every scalars α.*

THEOREM 3.1:*If F is a fuzzy set in a vector space E, then the followings are equivalent*

(i) *F is a subspace of E*

(ii) *For all scalars k,m, k F + m F ⊂ F*

(iii) *For all scalars k,m , and all x, y ∈ E*

$$\mu_F(kx+my)\geq \min\left\{\mu_F(x),\mu_F(y)\right\}$$

Proof : It is obvious

THEOREM 3.2 :*If E and F are vector spaces over the same field and f is a linear mapping from E to F and A is subspace of E. Then f(A) is a subspace of F and if B is a subspace of F. Then $f^{-1}(B)$ is a subspace of E.*

Proof : Let k, m, be scalars and f is a linear mapping from E to F, then for any fuzzy set A in E

$$kf(A)+mf(A)=f(kA)+f(mA)=f(kA+mA)\subset f(A)$$

As k A + m A⊂ A, since A is a subspace.

∴f(A) is a subspace of F

Also, $\mu_{f^{-1}(B)}(kx+my)=\mu_B(f(kx+my))$

$\mu_{f^{-1}(B)}(kx+my)=\mu_B(kf(x)+mf(y))$, since f is a linear mapping

$\mu_{f^{-1}(B)}(kx+my)\geq \min\left\{\mu_B(f(x)),\mu_B(f(y))\right\}$, as B is a subspace.

$$\mu_{f^{-1}(B)}(kx+my)\geq \min\left\{\mu_{f^{-1}(B)}(x),\mu_{f^{-1}(B)}(y)\right\}$$

i.e*$f^{-1}(B)$, is a subspace of F*

THEOREM 3.3 ***:*** *If A, B, are fuzzy subspace of E and K is a scalars. Then A + B and K A are fuzzy subspaces.*

Proof: Proof is obvious.

IV. Convex Fuzzy Set

Definition 4.1: *A fuzzy set A in a vector space E is said to be convex if for all* $\alpha\epsilon$ [0,1], $\alpha A + (1-\alpha)A \subset A$.

THEOREM 4.1 : Let A be a fuzzy set in a vector space E. Then the following assertions are equivalent

(i) A is convex

(ii) $\mu_A(\alpha x + (1-\alpha)y) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all $x, y \in E$, and for all $\alpha\epsilon$ [0,1],

(iii) For each $\alpha\epsilon$ [0,1], the crisp set $A_\alpha = \{x \in E : \mu_A(x) \geq \alpha\}$, is convex

Proof is obvious.

V. Balanced Fuzzy Set

Definition 5.1: *A fuzzy set A in a vector space E is said to be balanced if α A ⊂ A, for all scalars α with I α I ≤ 1.*

THEOREM 5.1 : *Let A be a fuzzy set in a vector space E. Then the following assertions are equivalent.*

(i) A is balanced

(ii) $\mu_A(\alpha x) \geq \mu_A(x)$, for all scalars α with I α I ≤ 1

(iii) For each α ∈ [0,1], the ordinary set A_α given by

$A_\alpha = \{x \in E : \mu_A(x) \geq \alpha\}$, is balanced

Proof: (i) ⟹ (ii)

Suppose A is balanced i.e α A ⊂ A, for all scalars α with I α I ≤ 1.

i.e $\mu_A(x) \geq \mu_{\alpha A}(x)$, for all scalars α with I α I ≤ 1, taking α x for x

$\therefore \mu_A(\alpha x) \geq \mu_{\alpha A}(\alpha x) = \mu_A(x)$, where α≠0 …………….. (i)

$i.e \mu_A(\alpha x) \geq \mu_A(x)$, for all scalars α,with I α I≤ 1 and x∈ E

If α = 0,from (i)

$$\mu_A(\alpha x) \geq \mu_{\alpha A}(\alpha x) = \mu_{0A}(0x) = \sup_{y \in E} \mu_A(y)$$

$\therefore \mu_A(\alpha x) \geq \mu_A(x)$, where α = 0

Suppose, (ii) ⟹ (iii)

$i.e \mu_A(\alpha x) \geq \mu_A(x)$, for all α with I α I ≤ 1 and x ∈ E

Let $A_\alpha = \{x \in E : \mu_A(x) \geq \alpha\}, \alpha \in [0,1]$

Now, $tA_\alpha = \{tx : x \in A_\alpha\}$,with I t I ≤ 1, let $x \in A_\alpha$

Since $\mu_A(\alpha x) \geq \mu_A(x) \geq \alpha$, with I α I ≤ 1

$tx \in A_\alpha$*, when I t I ≤ 1*

$\therefore tA_\alpha \subset A_\alpha$, with I t I ≤ 1

$\Rightarrow A_\alpha$, is balanced

(iii) ⟹ (i) Let x∈ E, and let $\mu_A(x/k) = \alpha$,where I k I≤ 1

$\therefore \frac{x}{k} \in A_\alpha$, where $A_\alpha = \{y : \mu_A(y) \geq \alpha\}$

Now $kA_\alpha = \{kx : x \in A_\alpha\}$,Since $\frac{x}{k} \in A_\alpha$, $k.\frac{x}{k} \in A_\alpha$ i.e $x \in A_\alpha \because kA_\alpha \subset A_\alpha$, as A_α is balanced

i.e $\mu_{kA}(x) \geq \mu_A\left(x/k\right) = \alpha$

$\therefore \mu_{kA}(x) \leq \mu_A(x)$, for all scalars k with I k I≤ 1, and x ∈ E

$\therefore kA \subset A$, A is balanced.

THEOREM 5.2 ***:*** *Let E, F be vector spaces over k and let f: E → F be a linear mapping . If A is balanced fuzzy set in E. Then f(A) is balanced fuzzy set in F. Similarly $f^{-1}(B)$ is balanced fuzzy set in E, whenever B is balanced fuzzy set in F.*

Proof : Let E, F be vector spaces over k and f : E → F be a linear mapping. Suppose A is balanced fuzzy set in E.
Now α.f (A) = f (αA) ⊂f(A), for all scalars α with I α I ≤ 1
i.e α f(A) ⊂ f(A),hence f(A) is balanced [∵ α A ⊂ A]
Again suppose B is a balanced fuzzy set in F
∴α B ⊂ B, for all scalars α with I α I ≤ 1
Now, let M = α $f^{-1}(B)$, therefore, f(M) = f (α $f^{-1}(B)$) = α f ($f^{-1}(B)$) ⊂ α B ⊂ B
∴ M ⊂ $f^{-1}(B)$, hence α $f^{-1}(B)$ ⊂ $f^{-1}(B)$, therefore $f^{-1}(B)$ is balanced fuzzy set in E.

THEOREM 5.3 ***:*** *If A, B are balanced fuzzy sets in a vector space E over K. Then A + B is balanced fuzzy set in E.*

Proof : Let A,B are balanced fuzzy sets in E. Therefore α A ⊂ A, and α B ⊂ B, for all scalars α with I α I≤ 1, Now α (A + B) = α A + α B ⊂ A + B, hence A + B is balanced fuzzy set in E.

THEOREM 5.4 :*If $\{A_i\}_{i\in I}$,is a family of balanced fuzzy sets in vector spaces E. Then A = ∩ A_i , is balanced fuzzy set in E*

Proof : Since $\{A_i\}_{i\in I}$, is a family of balanced fuzzy sets in E
α A_i⊂ A_i , for all scalars α with I α I≤ 1
that is, $\mu_{A_i}(\alpha x) \geq \mu_{A_i}(x)$, for all scalars α with I α I≤ 1
Now let, A = ∩ A_i

$\mu_A(y) = \inf_{i\in I} \mu_{A_i}(y)$, for all y ∈ E

$\therefore \mu_A(\alpha x) = \inf_{i\in I} \mu_{A_i}(\alpha x)$, take y = α x

$\mu_A(\alpha x) \geq \inf_{i\in I} \mu_{A_i}(x) = \mu_A(x)$, for all scalars α with I α I≤ 1, and x ∈ E

$\therefore A = \bigcap_{i\in I} A_i$, is balanced fuzzy set in E

VI. Absolute Convex Fuzzy Set

Definition 6.1*: A fuzzy set A in a vector space E is said to be absolutely convex if it is both convex and balanced.*

THEOREM 6.1:*Let A be a fuzzy set in a vector space E. Then the following are equivalent*

(i) *A is absolutely convex*

(ii) $\alpha A + \beta A \subset A$ *, for all scalars* α, β *with* $|\alpha| + |\beta| \leq 1$

(iii) $\mu_A(\alpha x + \beta y) \geq \min\{\mu_A(x), \mu_A(y)\}$ *,for all* $x, y \in E$ *and all scalars* α, β *with* $|\alpha| + |\beta| \leq 1$

(iv) *For each* $\alpha \in [0,1]$*,the crisp set* $A_\alpha = \{x \in E : \mu_A(x) \geq \alpha\}$ *is absolutely convex fuzzy set in E.*

Proof : $(i) \Rightarrow (ii)$

Let A is absolutely convex fuzzy set in E i.e A is convex as well as balanced.

$\alpha A \subset A$ ……………………..(I) for all scalars α with $|\alpha| \le 1$

And $\alpha A + (1-\alpha) A \subset A$…………..(II) for all scalars α with $0 \le \alpha \le 1$

Now putting $\alpha = \frac{1}{2}$ in (II) we get

$\frac{1}{2} A + \frac{1}{2} A \subset A$ …………………(III)

Now for all scalars $\alpha^{'}, \beta^{'}$ with $|\alpha^{'}| + |\beta^{'}| \le 1$ we have $|\alpha^{'}| \le 1$ and $|\beta^{'}| \le 1$

From (I) $\alpha^{'} A \subset A$ and $\beta^{'} A \subset A$

Also $\begin{matrix} \frac{1}{2}\alpha^{'} A \subset A \\ \frac{1}{2}\beta^{'} A \subset A \end{matrix}$ ……………………..(a)

Adding (a) we get

$\frac{1}{2}\alpha^{'} A + \frac{1}{2}\beta^{'} A \subset \frac{1}{2} A + \frac{1}{2} A \subset A$ ………. from (III)

Let $\alpha = \frac{1}{2}\alpha^{'}$ and $\beta = \frac{1}{2}\beta^{'}$ then $|\alpha| \le 1$ and $|\beta| \le 1$

$\therefore \alpha A + \beta A \subset A$ for all scalars α, β with $|\alpha| + |\beta| \le 1$

$(ii) \Rightarrow (iii)$ This follows from theorem 3.1

$(iii) \Rightarrow (iv)$ Suppose $\mu_A(\alpha x + \beta y) \ge \min\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in E$ and all scalars α, β with $|\alpha| + |\beta| \le 1$

We take $\alpha \in [0,1]$ and $\beta = 1 - \alpha$ then $|\alpha| + |\beta| = 1$

$\therefore \mu_A(\alpha x + \beta y) \ge \min\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in E$ with $\alpha \in [0,1]$ and $\beta = 1 - \alpha$ ……(b)

Let $A_\alpha = \{x \in E : \mu_A(x) \ge \alpha\}$ $\alpha \in [0,1]$

If $x \in A_\alpha$ and $y \in A_\alpha$

$\Rightarrow \mu_A(x) \ge \alpha$ and $\mu_A(y) \ge \alpha$ ……………………..(c)

$\therefore \mu_A(\alpha x + \beta y) \ge \min\{\mu_A(x), \mu_A(y)\} \ge \alpha$, from (a) & (b)

$\therefore \mu_A(\alpha x + \beta y) \ge \alpha$

$\Rightarrow \alpha x + \beta y \in A_\alpha$ i.e $\alpha x + (1-\alpha) y \in A_\alpha$

$\therefore A_\alpha$ is convex.

Again putting $\alpha = 0, \beta = 0$ in

$\mu_A(\alpha x + \beta y) \ge \min\{\mu_A(x), \mu_A(y)\}$

$\mu_A(0) \ge \min\{\mu_A(x), \mu_A(y)\}$

i.e. $\mu_A(0) \ge \mu_A(x)$, for all $x \in E$

If we put $y = 0$ in $\mu_A(\alpha x + \beta y) \ge \min\{\mu_A(x), \mu_A(y)\}$

$\therefore \mu_A(\alpha x) \ge \min\{\mu_A(x), \mu_A(0)\}$, $|\alpha| \le 1$

$\mu_A(\alpha x) \geq \mu_A(x) \geq \alpha$, for all $x \in A_\alpha$ and for all scalars $|\alpha| \leq 1$

$\therefore \alpha x \in A_\alpha \therefore \alpha A_\alpha \subset A_\alpha$ with $|\alpha| \leq 1$

Hence A_α is balanced.

i.e. A_α is convex as well as balanced , implies that A_α is absolutely convex.

$(iv) \Rightarrow (i)$

Since $A_\alpha = \{x \in E : \mu_A(x) \geq \alpha\}$, is convex for every $\alpha \in [0,1]$. Therefore fuzzy set A is convex by theorem 4.1. Again A_α is balanced for every $\alpha \in [0,1]$. Therefore fuzzy set A is balanced by theorem 5.1. Hence A_α is absolutely convex.

Theorem 6.2 : Every fuzzy subspace F of a vector space E is absolutely convex (i.e. convex as well as balanced)

Proof : Suppose F is a fuzzy subspace

$\therefore F + F \subset F$

And $\alpha F \subset F$ for all scalars α

$\therefore (1-\alpha) F \subset F$

$\Rightarrow \alpha F + (1-\alpha) F \subset F + F \subset F$, with $|\alpha| \leq 1$

Then F is convex, since $\alpha F \subset F$, for all scalars α with $|\alpha| \leq 1$

$\therefore F$ is balanced, therefore F is absolutely convex as it is convex as well as balanced.

Theorem 6.3 :*If A, B are absolutely convex fuzzy sets in a vector space E. Then A + B is absolutely convex fuzzy sets in a vector space E*

Proof : Let A,B are absolutely convex fuzzy sets in a vector space E

i.e. A, B are convex as well as balanced fuzzy sets in a vector space E

Since A, B are convex fuzzy sets in E

$\therefore \alpha A + (1-\alpha) A \subset A$, where $\alpha \in [0,1]$

Also $\alpha B + (1-\alpha) B \subset B$, for all scalars $\alpha \in [0,1]$

Now, $\alpha(A+B) + (1-\alpha)(A+B) = \alpha A + (1-\alpha) A + \alpha B + (1-\alpha) B$

$\Rightarrow \alpha(A+B) + (1-\alpha)(A+B) \subset (A+B)$

$\therefore (A+B)$ is convex fuzzy set in a vector space E.

Also, A,B are balanced fuzzy sets in a vector space E

$\therefore \alpha A \subset A$, for all scalars α with $|\alpha| \leq 1$

And $\alpha B \subset B$, for all scalars α with $|\alpha| \leq 1$

$\therefore \alpha(A+B) = \alpha A + \alpha B \subset A + B$

i.e. $\alpha(A+B) \subset A + B$

$\Rightarrow (A+B)$, is absolutely convex fuzzy sets in a vector space E, since it is convex as well as balanced.

Theorem 6.4 :*If $\{A_i\}_{i \in I}$ is a family of absolutely convex fuzzy sets in a vector space E. Then $A = \bigcap_{i \in I} A_i$ is also absolutely convex fuzzy set in E*

Proof : Since $\{A_i\}_{i\in I}$ is a family of absolutely convex fuzzy sets in a vector space E. This means that $\{A_i\}_{i\in I}$ is a family of convex as well as balanced fuzzy sets in E.

Let $\{A_i\}_{i\in I}$ be a family of convex fuzzy sets in E

Then $\alpha A_i + (1-\alpha) A_i \subset A_i$, for all $\alpha \in [0,1]$

i.e. $\mu_{A_i}(\alpha x + (1-\alpha) y) \geq \min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$

Now let, $A = \underset{i\in I}{\cap} A_i$, Then $\mu_A(y) = \inf \mu_{A_i}(y)$ for all $y \in E$

$\therefore \mu_A(\alpha x + (1-\alpha) y) = \inf_{i\in I} \mu_{A_i}(\alpha x + (1-\alpha) y)$

$\therefore \mu_A(\alpha x + (1-\alpha) y) \geq \inf_{i\in I}\{\min(\mu_{A_i}(x), \mu_{A_i}(y))\} = \min\{\inf_{i\in I} \mu_{A_i}(x), \inf_{i\in I} \mu_{A_i}(y)\}$

$\therefore \mu_A(\alpha x + (1-\alpha) y) \geq \min\{\mu_A(x), \mu_A(y)\}$

Hence $A = \underset{i\in I}{\cap} A_i$ is convex fuzzy set in E.

Also $\{A_i\}_{i\in I}$ is a family of balanced fuzzy sets in E

$\therefore \alpha A_i \subset A_i$, for all scalars α with $|\alpha| \leq 1$

i.e. $\mu_{A_i}(\alpha x) \geq \mu_{A_i}(x)$, for all scalars α with $|\alpha| \leq 1$ ....(i)

Now let $A = \underset{i}{\cap} A_i$

$\therefore \mu_A(y) = \inf_{i\in I} \mu_{A_i}(y)$, for all $y \in E$

$\therefore \mu_A(\alpha x) = \inf_{i\in I} \mu_{A_i}(\alpha x)$, take $y = \alpha x$

From (i) $\mu_A(\alpha x) \geq \inf_{i\in I} \mu_{A_i}(x) = \mu_A(x)$, for all scalars α with $|\alpha| \leq 1$, and $x \in E$

$\therefore A = \cap A_i$ is balanced fuzzy set in E, hence $A = \cap A_i$ is absolutely convex fuzzy set in E.

Theorem 6.5 : Let E, F are fuzzy vector space over K and $f : E \to F$ be a linear mapping

(i) If A is absolutely convex fuzzy set in E. Then $f(A)$ is absolutely convex fuzzy set in F.

(ii) If B is absolutely convex fuzzy set in F. Then $f^{-1}(B)$ is an absolutely convex fuzzy set in E.

Proof (i) : Let A be absolute convex fuzzy set in E, i.e. A is convex as well as balanced fuzzy set in E.

Let $\alpha \in [0,1]$ and A be convex fuzzy set in E, then

$\alpha f(A) + (1-\alpha) f(A) = f(\alpha A + (1-\alpha) A) \subset f(A)$

Hence $f(A)$ is convex fuzzy set in F

Again A is balanced fuzzy set in E

$\therefore \alpha f(A) = f(\alpha A) \subset f(A)$, for all scalars α with $|\alpha| \leq 1$

$\therefore f(A)$ is balanced fuzzy set in F

Therefore $f(A)$ is convex as well as balanced fuzzy set in F, hence $f(A)$ is absolutely convex fuzzy set in F.

(ii) let B is absolute convex fuzzy set in a vector space F implies that B is convex as well as balanced fuzzy set in F

Since B is a convex fuzzy set in F and let $\alpha \in [0,1]$

$M = \alpha f^{-1}(B) + (1-\alpha) f^{-1}(B)$

Then $f(M) = \alpha f\left(f^{-1}(B)\right) + (1-\alpha) f^{-1}(B)$

$f(M) = \alpha B + (1-\alpha) B \subset B$

Hence $M \subset f^{-1}(B)$ is convex fuzzy set in E

Again B is a balanced fuzzy set in F

$\therefore \alpha B \subset B$ for all scalars α with $|\alpha| \leq 1$

Now let $M = \alpha f^{-1}(B)$

$\therefore f(M) = \alpha f\left(f^{-1}(B)\right) \subset \alpha B \subset B$

$\therefore M \subset f^{-1}(B)$

i.e. $\alpha f^{-1}(B) \subset f^{-1}(B)$

$\therefore f^{-1}(B)$ is balanced fuzzy set in E

Therefore, $f^{-1}(B)$ is convex as well as balanced fuzzy set in E i.e., $f^{-1}(B)$ is absolutely convex fuzzy set in E.

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