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The Shape Parameter of Burr Type XDistribution Bayesian AnalysesUnder Singly Type II Censored Samples

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***Abstract**: The Bayesian analysis of the Burr type X distribution (Exponentiated Rayleigh) has been considered in the research. The Gamma, Exponential, Chi-Squared and Jeffrey prior have been assumed for posterior analysis. The estimation has been made under singly type II censored samples. The Bayes estimation has been obtained under eight different loss functions (Squared error, Quadratic,Weighted, Linear exponential,Precautionary, Entropy, De Groot and non-Linear exponential loss functions). The simulation study has been conducted to compare by mean square error (MSE) for the performance of various estimators.*
***Keyword:** Bayesian Analyses,Exponentiated Rayleigh Distribution, Burr type X distribution, Loss function, Prior, Posterior, (Squared error, Quadratic, Weighted, Linear exponential, Precautionary, Entropy, De Groot and non-Linear exponential) loss functions.*

I. Introduction.

Burr [2] introduction twelve different forms of cumulative distribution function for modeling data. Among those twelve distribution functions, Burr Type X and Burr Type XII received the maximum attention. Surles and Padgett [14] observed that the Burr Type X distribution (Exponentiated Rayleigh distribution) can be used quite effectively in modelling strength data and also modelling general lifetime data. Several aspects of the one parameter (scale parameter=1) Burr Type X distribution were studies by Sartawi and Abu-Salih [10], Jaheen [7], Ahmed, Fakhry and Jaheen [1], Raqab [9], Hassan, Albadri, Ibrahim and Ameen [5], Feroze and Aslam [4] and Sindhu and Aslam [11]. The cumulative distribution function (CDF), and the probability density function (pdf) of the Burr Type X distribution with shape parameter $(\lambda > 0)$ are respectively as follows:[4][5][8]

$F(x;\lambda) = \left(1 - e^{-x^2}\right)^{\lambda} \quad ; \quad x > 0, \quad \lambda > 0$ … (1)

$f(x;\lambda) = 2\lambda x e^{-x^2}\left(1 - e^{-x^2}\right)^{\lambda-1} \quad ; \quad x > 0\,,\ \lambda > 0$ … (2)

The random number X has been generated by inverse function method, which is for uniform random U.

$X = \left(-\ln\left(1 - U^{\frac{1}{\lambda}}\right)\right)^{\frac{1}{2}}$ … (3)

The problem of estimating the unknown parameters in statistical distributions used to study a certain phenomenon is one of the important problems facing constantly those who are interested in applied statistics. This paper considers the estimations of the unknown parameters of the Burr Type X distribution. This distribution is an important distribution in statistics and operations research. It is applied in several areas such as health, agriculture, biology, and other sciences. The main aim of this is to consider the Bayesian analysis of the unknown parameters under different priors (information and non-information) and loss functions under singly type II censored samples.

II. Likelihood function

Suppose (n)items are put on a life-testing experiment and only first(r) failure times been observed, that is $(x_1 < x_2 < \cdots < x_r < \cdots < x_n)$ and remaining $(n - r)$ items are still working. Under the assumptions that the lifetimes of the items are independently and identically distributionBurr Type X random variable, the likelihood function of the observed data can be written as: [4]

$$L(\lambda|\underline{x}) = \left(\frac{n!}{(n-r)!}\right)\prod_{i=1}^{r} f(x_i,\lambda)\left(1 - F(x_r)\right)^{n-r}$$

$L(\lambda/\underline{x}) = Q\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\lambda^r e^{-\lambda\varphi_j(x)}$….. (4)

Such that:

$$Q = \frac{n!}{(n-r)!}2^r e^{(\sum_{i=1}^{r}\ln x_i - \sum_{i=1}^{r}{x_i}^2 + \ln(1-e^{{x_i}^2})^{-1}}$$

$$\varphi_j(x) = \left(\sum_{i=1}^{n} ln(1 - e^{x_i{}^2})^{-1} + j\, ln\left(1 - e^{-x_r^2}\right)^{-1} \right)$$

III. Bayesian Estimators forsingly Type II censored samples using different Priors and Loss functions.

In this section Bayesian Estimators of the shape parameter for four different prior functionsand under eight different loss functions has been determined.

● Types of loss function using in this paper.

If $\hat{\lambda}$ represent of estimator for the shape parameter $\hat{\lambda}$

1 - **Squared error loss function**$(\boldsymbol{slf})$: the squared error loss function defined as: [13]

$L(\hat{\lambda},\lambda) = c(\hat{\lambda} - \lambda)^2$; $\hat{\lambda}_{slf} = E(\lambda)$ … (5)

2 - **Quadratic loss function**$(\boldsymbol{qlf})$: the quadratic loss function defined as: [13]

$L(\hat{\lambda},\lambda) = \left(\frac{\hat{\lambda}-\lambda}{\lambda}\right)^2$; $\hat{\lambda}_{qlf} = \frac{E(\lambda^{-1})}{E(\lambda^{-2})}$ … (6)

3 - **Weighted loss function**$(\boldsymbol{wlf})$: the weighted loss function defined as: [4]

$L(\hat{\lambda},\lambda) = \frac{(\hat{\lambda}-\lambda)^2}{\lambda}$; $\hat{\lambda}_{wlf} = \frac{1}{E(\lambda^{-1})}$ … (7)

4 - **Linear exponential loss function**$(\boldsymbol{LINX})$: the $(LINX)$ loss function defined as: [6]

$L(\hat{\lambda},\lambda) = \left(e^{c(\hat{\lambda}-\lambda)} - c(\hat{\lambda}-\lambda) - 1\right)$; $\hat{\lambda}_{llf} = -\frac{1}{c} ln\, E\left(e^{-c\lambda}\right)$ … (8)

5 - **Precautionary loos function**$(\boldsymbol{plf})$: the Precautionary loss function defined as: [4]

$L(\hat{\lambda},\lambda) = \frac{(\hat{\lambda}-\lambda)^2}{\hat{\lambda}}$; $\hat{\lambda}_{plf} = \sqrt{E(\lambda^2)}$ … (9)

6 - **Entropy loss function**$(\boldsymbol{Elf})$: the entropy loss function defined as: [6]

$L(\hat{\lambda},\lambda) = \left((\hat{\lambda}/\lambda)^t - t\, ln(\hat{\lambda}/\lambda) - 1\right)$; $\hat{\lambda}_{Elf} = \left(E(\lambda^{-t})\right)^{-1/t}$ … (10)

7 - **De Groot loss function**$(\boldsymbol{Dlf})$: the De Groot loss function defined as: [6]

$L(\hat{\lambda},\lambda) = \left(\frac{\lambda-\hat{\lambda}}{\hat{\lambda}}\right)^2$; $\hat{\lambda}_{Dlf} = \frac{E(\lambda^2)}{E(\lambda)}$ … (11)

8 - **Non- Linear exponential loss functions**$(\boldsymbol{NLINEX})$: the $(NLINX)$ loss function defined as: [12]

$$L(\hat{\lambda},\lambda) = \left(e^{c(\hat{\lambda}-\lambda)} + c(\hat{\lambda}-\lambda)^2 - c(\hat{\lambda}-\lambda) - 1\right)$$

$\hat{\lambda}_{Nllf} = -\frac{1}{c+2}\left(ln\, E\left(e^{-c\lambda}\right) - 2E(\lambda)\right)$ … (12)

● Posterior distributions with different prior.

The posterior density function of the shape parameter for the given random X is well known as:

$p(\lambda|\underline{x}) = \frac{L(\lambda|\underline{x})\,.\,p(\lambda)}{\int_0^\infty L(\lambda|\underline{x})\,.\,p(\lambda)\,d\lambda}$ ... (13)

For Bayesian estimation, we specify four different prior distributions for the shape parameter, and which can be obtained four different posterior distributions under doubly type II censored samples, as follows:

1– The **Non-information** prior, for any parameterλ, with pdf as: $p(\lambda) = \frac{1}{\lambda}\lambda > 0$

By equation (13) the posterior distribution under the assumption Non-information prior is:

$$p_J(\lambda/\underline{X}) = \frac{Q\sum_{j=0}^{n-r}(-1)^j \binom{n-r}{j} \lambda^r e^{-\lambda\varphi_j(x)} \left(\frac{1}{\lambda}\right)}{\int_0^\infty Q\sum_{j=0}^{n-r}(-1)^j \binom{n-r}{j} \lambda^r e^{-\lambda\varphi_j(x)} \left(\frac{1}{\lambda}\right) d\lambda}$$

$\therefore p_J(\lambda/\underline{X}) = \frac{\sum_{j=0}^{n-r}(-1)^j \binom{n-r}{j} \lambda^{r-1} e^{-\lambda\left(\varphi_j(x)\right)}}{\sum_{j=0}^{n-r}(-1)^j \binom{n-r}{j} \frac{\Gamma(r)}{\left(\varphi_j(x)\right)^r}}$ ... (14)

2– The **Chi-Square** prior is assumed to be:$p(\lambda) = \frac{\lambda^{(d/2)-1} e^{-\frac{\lambda}{2}}}{\Gamma(d/2)\ 2^{(d/2)}}$; $\lambda, d > 0$

By equation (13) the posterior distribution under the assumption Chi-Square prior is:

$$p_{Ch}(\lambda/\underline{X}) = \frac{Q\sum_{j=0}^{n-r}(-1)^j \binom{n-r}{j} \lambda^r e^{-\lambda\varphi_j(x)} \left(\frac{\lambda^{(d/2)-1} e^{-\frac{\lambda}{2}}}{\Gamma(d/2)\ 2^{(d/2)}}\right)}{\int_0^\infty Q\sum_{j=0}^{n-r}(-1)^j \binom{n-r}{j} \lambda^r e^{-\lambda\varphi_j(x)} \left(\frac{\lambda^{(d/2)-1} e^{-\frac{\lambda}{2}}}{\Gamma(d/2)\ 2^{(d/2)}}\right) d\lambda}$$

$$p_{Ch}(\lambda|\underline{x}) = \frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\lambda^{r+\frac{d}{2}-1}e^{-\lambda\left(\varphi_j(x)+\frac{1}{2}\right)}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma\left(r+\frac{d}{2}\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{\left(r+\frac{d}{2}\right)}}\right)} \quad \ldots (15)$$

3 – The **Exponential** prior, for any parameterλ, as:$p(\lambda) = be^{-b\lambda}$; $\quad b > 0$

By equation (13) the posterior distribution under the assumption Exponential prior is:

$$p_E(\lambda/\underline{X}) = \frac{Q\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\lambda^r e^{-\lambda\varphi_j(x)}\left(be^{-b\lambda}\right)}{\int_0^\infty Q\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\lambda^r e^{-\lambda\varphi_j(x)}\left(be^{-b\lambda}\right)d\lambda}$$

$$\therefore p_E(\lambda|\underline{x}) = \frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\lambda^{r-1}e^{-\lambda\left(\varphi_j(x)+b\right)}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+1)}{\left(\varphi_j(x)+b\right)^{(r+1)}}\right)} \quad \ldots (16)$$

4 – The **Gamma** prior, assumed to be:$p(\lambda) = \frac{b^a\lambda^{a-1}}{\Gamma(a)}e^{-\lambda b}$; $a, b > 0$

By equation (13) the posterior distribution under the assumption Gamma prior is:

$$p_G(\lambda/\underline{X}) = \frac{Q\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\lambda^r e^{-\lambda\varphi_j(x)}\left(\frac{b^a\lambda^{a-1}}{\Gamma(a)}e^{-\lambda b}\right)}{\int_0^\infty Q\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\lambda^r e^{-\lambda\varphi_j(x)}\left(\frac{b^a\lambda^{a-1}}{\Gamma(a)}e^{-\lambda b}\right)d\lambda}$$

$$p_G(\lambda|\underline{x}) = \frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\lambda^{r+a-1}e^{-\lambda\left(\varphi_j(x)+b\right)}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+a)}{\left(\varphi_j(x)+b\right)^{(r+a)}}\right)} \quad \ldots (17)$$

3-1. Bayesian Estimators forsingly Type II censored samples under Non-information Prior usingdifferent loss functions.

$$\hat{\lambda}_{1_S} = \frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+1)}{\left(\varphi_j(x)\right)^{r+1}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r)}{\left(\varphi_j(x)\right)^{r}}}\ ; \qquad \hat{\lambda}_{1_Q} = \frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r-1)}{\left(\varphi_j(x)\right)^{r-1}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r-2)}{\left(\varphi_j(x)\right)^{r-2}}};$$

$$\hat{\lambda}_{1_W} = \frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r)}{\left(\varphi_j(x)\right)^{r}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r-1)}{\left(\varphi_j(x)\right)^{r-1}}}; \qquad \hat{\lambda}_{1_L} = \frac{-1}{c}ln\left(\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r)}{\left(\varphi_j(x)+c\right)^{r}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r)}{\left(\varphi_j(x)\right)^{r}}}\right);$$

$$\hat{\lambda}_{1_P} = \sqrt{\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+2)}{\left(\varphi_j(x)\right)^{r+2}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r)}{\left(\varphi_j(x)\right)^{r}}}}\ ; \qquad \hat{\lambda}_{31_E} = \sqrt[t]{\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r)}{\left(\varphi_j(x)\right)^{r}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r-t)}{\left(\varphi_j(x)\right)^{r-t}}}};$$

$$\hat{\lambda}_{1_N} = \left(\frac{-1}{c+2}\right)\left(\left(ln\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r)}{\left(\left(\varphi_j(x)\right)+c\right)^{r}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r)}{\left(\varphi_j(x)\right)^{r}}}\right) - 2\left(\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+1)}{\left(\varphi_j(x)\right)^{r+1}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r)}{\left(\varphi_j(x)\right)^{r}}}\right)\right);$$

$$\hat{\lambda}_{1_D} = \frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+2)}{\left(\varphi_j(x)\right)^{r+2}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+1)}{\left(\varphi_j(x)\right)^{r+1}}}$$

3-2. Bayesian Estimators forsingly Type II censored samples under Chi-Square Prior usingdifferent loss functions.

$$\hat{\lambda}_{2_S} = \frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma\left(r+\frac{d}{2}+1\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{\left(r+\frac{d}{2}+1\right)}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma\left(r+\frac{d}{2}\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{\left(r+\frac{d}{2}\right)}}\right)}; \quad \hat{\lambda}_{2_Q} = \frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma\left(r+\frac{d}{2}-1\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{\left(r+\frac{d}{2}-1\right)}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma\left(r+\frac{d}{2}-2\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{\left(r+\frac{d}{2}-2\right)}}\right)};$$

$$\hat{\lambda}_{2_W}=\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma\left(r+\frac{d}{2}\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{\left(r+\frac{d}{2}\right)}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma\left(r+\frac{d}{2}-1\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{\left(r+\frac{d}{2}-1\right)}}\right)};\quad \hat{\lambda}_{2_L}=\frac{-1}{c}\ln\left(\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma\left(r+\frac{d}{2}\right)}{\left(\varphi_j(x)+\frac{1}{2}+c\right)^{r+\frac{d}{2}}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma\left(r+\frac{d}{2}\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{r+\frac{d}{2}}}}\right);$$

$$\hat{\lambda}_{2_P}=\sqrt{\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma\left(r+\frac{d}{2}+2\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{\left(r+\frac{d}{2}+2\right)}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma\left(r+\frac{d}{2}\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{\left(r+\frac{d}{2}\right)}}\right)}};\quad \hat{\lambda}_{2_E}=\sqrt[t]{\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma\left(r+\frac{d}{2}\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{r+\frac{d}{2}}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma\left(r+\frac{d}{2}-t\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{r+\frac{d}{2}-t}}}};$$

$$\hat{\lambda}_{2_N}=\left(\frac{-1}{c+2}\right)\left(\left(ln\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma\left(r+\frac{d}{2}\right)}{\left(\left(\varphi_j(x)\right)+\frac{1}{2}+c\right)^{r+\frac{d}{2}}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma\left(r+\frac{d}{2}\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{r+\frac{d}{2}}}}\right)-2\left(\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma\left(r+\frac{d}{2}+1\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{r+\frac{d}{2}+1}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma\left(r+\frac{d}{2}\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{r+\frac{d}{2}}}}\right)\right);$$

$$\hat{\lambda}_{2_D}=\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma\left(r+\frac{d}{2}+2\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{r+\frac{d}{2}+2}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma\left(r+\frac{d}{2}+1\right)}{\left(\varphi_j(x)+\frac{1}{2}\right)^{r+\frac{d}{2}+1}}\right)}$$

3-3. Bayesian Estimators under for Type II censored samples under Exponential Prior usingdifferent loss functions.

$$\hat{\lambda}_{3_S}=\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+2)}{\left(\varphi_j(x)+b\right)^{(r+2)}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+1)}{\left(\varphi_j(x)+b\right)^{(r+1)}}\right)};\quad \hat{\lambda}_{3_Q}=\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r)}{\left(\varphi_j(x)+b\right)^{(r)}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r-1)}{\left(\varphi_j(x)+b\right)^{(r-1)}}\right)};$$

$$\hat{\lambda}_{3_W}=\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+1)}{\left(\varphi_j(x)+b\right)^{(r+1)}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r)}{\left(\varphi_j(x)+b\right)^{r}}\right)};\quad \hat{\lambda}_{3_L}=\frac{-1}{c}\ln\left(\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+1)}{\left(\varphi_j(x)+b+c\right)^{r+1}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+1)}{\left(\varphi_j(x)+b\right)^{r+1}}}\right);$$

$$\hat{\lambda}_{3_P}=\sqrt{\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+3)}{\left(\varphi_j(x)+b\right)^{(r+3)}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+1)}{\left(\varphi_j(x)+b\right)^{(r+1)}}\right)}};\quad \hat{\lambda}_{3_E}=\sqrt[t]{\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+1)}{\left(\varphi_j(x)+b\right)^{r+1}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r-t+1)}{\left(\varphi_j(x)+b\right)^{r-t+1}}}};$$

$$\hat{\lambda}_{3_N}=\left(\frac{-1}{c+2}\right)\left(\left(ln\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+1)}{\left(\left(\varphi_j(x)\right)+b+c\right)^{r+1}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+1)}{\left(\varphi_j(x)+b\right)^{r+1}}}\right)-2\left(\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+2)}{\left(\varphi_j(x)+b\right)^{r+2}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+1)}{\left(\varphi_j(x)+b\right)^{r+1}}}\right)\right);$$

$$\hat{\lambda}_{3_D}=\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+3)}{\left(\varphi_j(x)+b\right)^{r+3}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+2)}{\left(\varphi_j(x)+b\right)^{r+2}}\right)}$$

3-4. Bayesian Estimators for singly Type II censored samples under Gamma Prior usingdifferent loss functions.

$$\hat{\lambda}_{4_S}=\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+a+1)}{\left(\varphi_j(x)+b\right)^{(r+a+1)}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+a)}{\left(\varphi_j(x)+b\right)^{(r+a)}}\right)}\ ;\quad \hat{\lambda}_{4_Q}=\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+a-1)}{\left(\varphi_j(x)+b\right)^{(r+a-1)}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+a-2)}{\left(\varphi_j(x)+b\right)^{(r+a-2)}}\right)};$$

$$\hat{\lambda}_{4_W}=\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+a)}{\left(\varphi_j(x)+b\right)^{(r+a)}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+a-1)}{\left(\varphi_j(x)+b\right)^{(r+a-1)}}\right)}\ ;\quad \hat{\lambda}_{4_L}=\frac{-1}{c}\ln\left(\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+a)}{\left(\varphi_j(x)+b+c\right)^{r+a}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+a)}{\left(\varphi_j(x)+b\right)^{r+a}}}\right);$$

$$\hat{\lambda}_{4_P}=\sqrt{\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+a+2)}{\left(\varphi_j(x)+b\right)^{(r+a+2)}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+a)}{\left(\varphi_j(x)+b\right)^{(r+a)}}\right)}}\ ;\quad \hat{\lambda}_{4_E}=\sqrt[t]{\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+a)}{\left(\varphi_j(x)+b\right)^{r+a}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+a-t)}{\left(\varphi_j(x)+b\right)^{r+a-t}}}}\ ;$$

$$\hat{\lambda}_{4_N}=\left(\frac{-1}{c+2}\right)\left(\left(\ln\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\left(\frac{\Gamma(r+a)}{\left(\left(\varphi_j(x)\right)+b+c\right)^{r+a}}\right)}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+a)}{\left(\varphi_j(x)+b\right)^{r+a}}}\right)-2\left(\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+a+1)}{\left(\varphi_j(x)+b\right)^{r+a+1}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+a)}{\left(\varphi_j(x)+b\right)^{r+a}}}\right)\right);$$

$$\hat{\lambda}_{4_D}=\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+a+2)}{\left(\varphi_j(x)+b\right)^{r+a+2}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{\Gamma(r+a+1)}{\left(\varphi_j(x)+b\right)^{r+a+1}}}$$

IV. Simulation results and Conclusions.

In this section, the results presented of some of numerical experiments to compare the performance of the Bayes estimators for shape parameter under four prior distributions and eight loss functions proposed in the previous sections, applying Monte Carlo simulations to comer the performance of different estimators, mainly with respect to their mean squared error (MSE) for different sample size $(n = 10, 20, 30, 50, 75\ and\ 100)$ and two values of the shape parameters$(1.5\ and\ 3)$. The results of (MSE) are computed over (1000) replications for two different cases $\{\boldsymbol{case}\mathbf{I}: (a = 3,\ b = 0.8,\ d = 3,\ c = 1,\ t = 2); \boldsymbol{case}\mathbf{II}: (a = 4, b = 1.2,\ d = 4,$ *c=1 , t=2* and recorded in sections*3−1), (3−2),(3−3),(3−4*. And for the purpose of conducting was a determined fixed value $(r = 40\%\ from n)$.

Table (1).The value of (mean) for Bayesian est. when$\lambda = 1.5$

n	Jeffery priors							
	BS	**BQ**	**BW**	**BP**	**BD**	**BL**	**BE**	**BNL**
10	1.6746	1.3081	1.4892	1.7671	1.8646	1.5215	1.3957	1.6236
20	1.5628	1.3862	1.4738	1.6074	1.6532	1.4932	1.4293	1.5396
30	1.5598	1.4405	1.4998	1.5899	1.6205	1.5129	1.4699	1.5442
50	1.5260	1.4650	1.4955	1.5412	1.5566	1.5027	1.4801	1.5183
75	1.5217	1.4813	1.5014	1.5318	1.5420	1.5063	1.4913	1.5166
100	1.5193	1.4889	1.5041	1.5269	1.5345	1.5078	1.4965	1.5155
	Chi-Square priors$(d = 3)$							
10	1.7699	1.4333	1.5999	1.8546	1.9433	1.6230	1.5143	1.7210
20	1.6203	1.4499	1.5345	1.6632	1.7072	1.5511	1.4916	1.5972
30	1.6000	1.4834	1.5414	1.6293	1.6592	1.5531	1.5122	1.5843
50	1.5477	1.4876	1.5177	1.5627	1.5778	1.5244	1.5026	1.5400
75	1.5364	1.4961	1.5163	1.5464	1.5565	1.5210	1.5062	1.5312
100	1.5303	1.5002	1.5153	1.5379	1.5454	1.5189	1.5077	1.5265
	Exponential priors $(b = 0.8)$							
10	1.5952	1.2820	1.4372	1.6737	1.7560	1.4721	1.3574	1.5542
20	1.5356	1.3717	1.4531	1.5768	1.6191	1.4726	1.4118	1.5146
30	1.5424	1.4291	1.4855	1.5709	1.5999	1.4985	1.4570	1.5278
50	1.5187	1.4591	1.4889	1.5335	1.5485	1.4961	1.4739	1.5112
75	1.5170	1.4771	1.4971	1.5270	1.5370	1.5019	1.4871	1.5565
100	1.5159	1.4859	1.5009	1.5234	1.5309	1.5044	1.4933	1.5120

	Gamma priors $(a = 3\ \ , b = 0.8)$							
10	1.9197	1.5952	1.7560	2.0012	2.0862	1.7672	1.6737	1.8689
20	1.9197	1.5952	1.7560	2.0012	2.0862	1.7672	1.6737	1.8689
30	1.6580	1.5424	1.5999	1.6871	1.7167	1.6099	1.5709	1.6420
50	1.5783	1.5187	1.5485	1.5932	1.6082	1.5548	1.5335	1.5705
75	1.5570	1.5170	1.5370	1.5670	1.5770	1.5415	1.5270	1.5518
100	1.5459	1.5159	1.5309	1.5534	1.5609	1.5342	1.5234	1.5420
	Chi-Square priors $(d = 4)$							
10	1.8562	1.5162	1.6845	1.9417	2.0312	1.7009	1.5981	1.8044
20	1.6636	1.4920	1.5772	1.7068	1.7512	1.5921	1.5340	1.6398
30	1.6295	1.5124	1.5706	1.6590	1.6890	1.5815	1.5412	1.6135
50	1.5628	1.5026	1.5327	1.5777	1.5929	1.5393	1.5176	1.5549
75	1.5464	1.5062	1.5263	1.5564	1.5665	1.5309	1.5162	1.5412
100	1.5379	1.5077	1.5228	1.5454	1.5530	1.5264	1.5152	1.5341
	Exponential priors $(b = 1.2)$							
10	1.4921	1.2017	1.3457	1.5646	1.6407	1.3856	1.2717	1.4566
20	1.4839	1.3267	1.4048	1.5234	1.5639	1.4255	1.3652	1.4644
30	1.5066	1.3965	1.4513	1.5343	1.5625	1.4650	1.4237	1.4927
50	1.5005	1.4415	1.4710	1.5151	1.5299	1.4784	1.4562	1.4931
75	1.5049	1.4653	1.4851	1.5148	1.5247	1.4900	1.4752	1.4999
100	1.5067	1.4769	1.4918	1.5142	1.5217	1.4956	1.4843	1.5030
	Gamma priors $(a = 4\ \ , b = 1.2)$							
10	1.9451	1.6407	1.7917	2.0214	2.1007	1.8008	1.7146	1.8970
20	1.7269	1.5639	1.6449	1.7679	1.8099	1.6567	1.6039	1.7035
30	1.6757	1.5625	1.6188	1.7042	1.7331	1.6281	1.5904	1.6598
50	1.5888	1.5299	1.5594	1.6035	1.6183	1.5654	1.5446	1.5810
75	1.5643	1.5247	1.5445	1.5742	1.5841	1.5488	1.5346	1.5592
100	1.5515	1.5217	1.5366	1.5589	1.5664	1.5400	1.5291	1.5477

Table (2).The value of (mean) for Bayesian est. when$\lambda = 3$

n	Jeffery priors							
	BS	**BQ**	**BW**	**BP**	**BD**	**BL**	**BE**	**BNL**
10	3.3493	2.6163	2.9784	3.5341	3.7292	2.8096	2.7915	3.1694
20	3.1256	2.7723	2.9477	3.2147	3.3063	2.8650	2.8586	3.0388
30	3.1197	2.8811	2.9997	3.1797	3.2409	2.9402	2.9398	3.0599
50	3.0521	2.9299	2.9909	3.0825	3.1132	2.9608	2.9603	3.0217
75	3.0435	2.9625	3.0029	3.0636	3.0839	2.9825	2.9826	3.0231
100	3.0386	2.9778	3.0082	3.0537	3.0689	2.9927	2.9929	3.0233
	Chi-Square priors$(d = 3)$							
10	3.2370	2.6295	2.9307	3.3892	3.5485	2.8008	2.7760	3.0916
20	3.0998	2.7770	2.9374	3.1810	3.2643	2.8638	2.8561	3.0212
30	3.1051	2.8806	2.9923	3.1615	3.2190	2.9372	2.9359	3.0491
50	3.0487	2.9302	2.9895	3.0782	3.1080	2.9602	2.9597	3.0192
75	3.0418	2.9621	3.0021	3.0616	3.0816	2.9821	2.9820	3.0219
100	3.0375	2.9777	3.0076	3.0525	3.0675	2.9925	2.9926	3.0225
	Exponential priors $(b = 0.8)$							
10	2.8064	2.2641	2.5334	2.9416	3.0833	2.4653	2.3950	2.6927
20	2.8721	2.5698	2.7201	2.9480	3.0259	2.6667	2.6439	2.8037
30	2.9455	2.7313	2.8379	2.9992	3.0540	2.7933	2.7841	2.8947
50	2.9653	2.8489	2.9072	2.9943	3.0235	2.8808	2.8779	2.9372
75	2.9857	2.9071	2.9467	3.0053	3.0252	2.9280	2.9268	2.9664
100	2.9954	2.9361	2.9657	3.0102	3.0250	2.9514	2.9509	2.9807
	Gamma priors $(a = 3\ \ , b = 0.8)$							
10	3.3641	2.8064	3.0833	3.5036	3.6489	2.9467	2.9416	3.2250
20	3.1814	2.8721	3.0259	3.2591	3.3387	2.9495	2.9480	3.1041
30	3.1635	2.9455	3.0540	3.2182	3.2739	2.9974	2.9992	3.1081
50	3.0817	2.9653	3.0235	3.1107	3.1400	2.9939	2.9943	3.0525
75	3.0645	2.9857	3.0252	3.0840	3.1038	3.0050	3.0053	3.0446
100	3.0547	2.9954	3.0250	3.0695	3.0844	3.0099	3.0102	3.0398
	Chi-Square priors $(d = 4)$							
10	3.3921	2.7795	3.0832	3.5457	3.7062	2.9322	2.9274	3.2388
20	3.1818	2.8570	3.0184	3.2635	3.3473	2.9382	2.9366	3.1006
30	3.1619	2.9363	3.0486	3.2763	2.9901	3.2186	2.9919	3.1046
50	3.0783	2.9599	3.0191	3.1376	2.9890	3.1078	2.9893	3.0486
75	3.0616	2.9821	3.0219	3.1015	3.0017	3.0815	3.0019	3.0416
100	3.0525	2.9926	3.0226	3.0824	3.0072	3.0674	3.0076	3.0374
	Exponential priors $(b = 1.2)$							
10	2.5139	2.0334	2.2723	2.6333	2.7583	2.2407	2.1495	2.4228
20	2.7007	2.4195	2.5594	2.7712	2.8434	2.5206	2.4884	2.6407
30	2.8199	2.6170	2.7180	2.8708	2.9227	2.6814	2.6670	2.7737

50	2.8967	2.7829	2.8398	2.9250	2.9535	2.8160	2.8112	2.8698
75	2.9389	2.8616	2.9004	2.9583	2.9777	2.8830	2.8809	2.9203
100	2.9600	2.9013	2.9306	2.9746	2.9893	2.9170	2.9160	2.9456
	Gamma priors $(a = 4 \ \ , b = 1.2)$							
10	3.2556	2.7583	3.0055	3.3796	3.5084	2.8926	2.8793	3.1346
20	3.1333	2.8434	2.9876	3.2060	3.2804	2.9190	2.9146	3.0618
30	3.1308	2.9227	3.0263	3.1831	3.2362	2.9738	2.9740	3.0785
50	3.0672	2.9535	3.0103	3.0955	3.1241	2.9818	2.9818	3.0388
75	3.0551	2.9777	3.0165	3.0744	3.0937	2.9968	2.9970	3.0357
100	3.0478	2.9893	3.0186	3.0625	3.0772	3.0036	3.0039	3.0331

Table (3). The value of (MSE) for Bayesian est. when$\lambda = 1.5$

n	Jeffery								**Best**
	BS	BQ	BW	BP	BD	BL	BE	BNL	
10	0.3876	0.2541	0.2821	0.4693	0.5765	0.2394	0.2584	0.3302	L
20	0.1533	0.1307	0.1336	0.1695	0.1905	0.1217	0.1301	0.1414	L
30	0.1048	0.0899	0.0936	0.1132	0.1237	0.0895	0.0908	0.0991	L
50	0.0497	0.0464	0.0471	0.0517	0.0542	0.0461	0.0465	0.0484	L
75	0.0318	0.0300	0.0305	0.0327	0.0339	0.0301	0.0301	0.0311	Q
100	0.0243	0.0231	0.0235	0.0249	0.0256	0.0233	0.0233	0.0240	Q
	Chi-Square priors$(d = 3)$								
10	0.3895	0.2136	0.2696	0.4728	0.5770	0.2379	0.2332	0.3323	Q
20	0.1566	0.1168	0.1290	0.1763	0.2005	0.1203	0.1209	0.1432	Q
30	0.1096	0.0861	0.0943	0.1200	0.1324	0.0911	0.0892	0.1029	Q
50	0.0511	0.0453	0.0473	0.0537	0.0568	0.0465	0.0460	0.0495	Q
75	0.0326	0.0296	0.0307	0.0338	0.0352	0.0304	0.0301	0.0318	Q
100	0.0249	0.0230	0.0237	0.0256	0.0265	0.0236	0.0233	0.0244	Q
	Exponential priors $(b = 0.8)$								
10	0.2382	0.1972	0.1910	0.2818	0.3418	0.1668	0.1876	0.2099	L
20	0.1211	0.1126	0.1098	0.1321	0.1470	0.1009	0.1095	0.1133	L
30	0.0910	0.0818	0.0830	0.0975	0.1059	0.0794	0.0816	0.0866	L
50	0.0465	0.0443	0.0445	0.0482	0.0503	0.0434	0.0441	0.0454	L
75	0.0304	0.0291	0.0293	0.0312	0.0322	0.0289	0.0291	0.0298	L
100	0.0235	0.0226	0.0228	0.0240	0.0247	0.0226	0.0226	0.0232	Q.L,E
	Gamma priors $(a = 3 \ \ , b = 0.8)$								
10	0.5045	0.2382	0.3418	0.6072	0.7296	0.3072	0.2818	0.4318	Q
20	0.1882	0.1211	0.1470	0.2144	0.2452	0.1397	0.1321	0.1706	Q
30	0.1279	0.0910	0.1059	0.1415	0.1571	0.1034	0.0975	0.1191	Q
50	0.0560	0.0465	0.0503	0.0595	0.0634	0.0499	0.0482	0.0538	Q
75	0.0349	0.0304	0.0322	0.0366	0.0384	0.0321	0.0312	0.0340	Q
100	0.0263	0.0235	0.0247	0.0273	0.0284	0.0246	0.0240	0.0257	Q
	Chi-Square priors $(d = 4)$								
10	0.4746	0.2338	0.3214	0.5750	0.6972	0.2842	0.2687	0.4036	Q
20	0.1765	0.1210	0.1408	0.2002	0.2287	0.1323	0.1289	0.1603	Q
30	0.1201	0.0893	0.1010	0.1323	0.1467	0.0982	0.0942	0.1122	Q
50	0.0537	0.0460	0.0490	0.0568	0.0603	0.0484	0.0473	0.0518	Q
75	0.0338	0.0301	0.0315	0.0352	0.0369	0.0313	0.0307	0.0329	Q
100	0.0256	0.0233	0.0242	0.0265	0.0275	0.0242	0.0237	0.0251	Q
	Exponential priors $(b = 1.2)$								
10	0.1745	0.2038	0.1667	0.1953	0.2292	0.1431	0.1802	0.1608	L
20	0.1036	0.1132	0.1020	0.1093	0.1185	0.0928	0.1061	0.0991	L
30	0.0811	0.0806	0.0777	0.0852	0.0910	0.0737	0.0784	0.0782	L
50	0.0439	0.0440	0.0431	0.0450	0.0466	0.0418	0.0433	0.0431	L
75	0.0291	0.0288	0.0286	0.0297	0.0305	0.0281	0.0286	0.0287	L
100	0.0228	0.0224	0.0223	0.0231	0.0236	0.0221	0.0223	0.0225	L
	Gamma priors $(a = 4 \ \ , b = 1.2)$								
10	0.4883	0.2292	0.3331	0.5842	0.6971	0.3046	0.2739	0.4211	Q
20	0.1900	0.1185	0.1471	0.2167	0.2477	0.1410	0.1309	0.1724	Q
30	0.1308	0.0910	0.1075	0.1449	0.1610	0.1054	0.0983	0.1217	Q
50	0.0571	0.0466	0.0510	0.0609	0.0651	0.0507	0.0485	0.0549	Q
75	0.0356	0.0305	0.0327	0.0374	0.0393	0.0326	0.0315	0.0345	Q
100	0.0267	0.0236	0.0250	0.0278	0.0290	0.0250	0.0242	0.0261	Q

Table (4). The value of (MSE) for Bayesian est. when$\lambda = 1.5$

1Xn	Jeffery								**Best**
	BS	BQ	BW	BP	BD	BL	BE	BNL	
10	1.5504	1.0163	1.1282	1.8771	2.3058	0.7338	1.0335	1.1836	L
20	0.6134	0.5227	0.5346	0.6780	0.7621	0.4261	0.5204	0.5315	L
30	0.4191	0.3594	0.3742	0.4528	0.4949	0.3220	0.3631	0.3784	L

50	0.1988	0.1856	0.1884	0.2068	0.2168	0.1750	0.1860	0.1888	L
75	0.1270	0.1198	0.1218	0.1308	0.1356	0.1157	0.1204	0.1223	L
100	0.0974	0.0926	0.0940	0.0997	0.1025	0.0902	0.0931	0.0945	L
	Chi-Square priors$(d = 3)$								
10	0.9317	0.7225	0.7271	1.1082	1.3464	0.5360	0.7003	0.7453	L
20	0.4781	0.4278	0.4256	0.5250	0.5874	0.3552	0.4199	0.4222	L
30	0.3636	0.3185	0.3279	0.3913	0.4264	0.2862	0.3199	0.3307	L
50	0.1860	0.1745	0.1767	0.1933	0.2025	0.1646	0.1747	0.1770	L
75	0.1216	0.1152	0.1168	0.1253	0.1297	0.1111	0.1156	0.1173	L
100	0.0943	0.0897	0.0911	0.0965	0.0992	0.0875	0.0902	0.0915	L
	Exponential priors $(b = 0.8)$								
10	0.5824	0.9022	0.6656	0.5995	0.6591	0.6153	0.7680	0.5614	NL
20	0.3760	0.4753	0.4022	0.3808	0.3982	0.3765	0.4334	0.3651	NL
30	0.2988	0.3276	0.3014	0.3064	0.3203	0.2823	0.3117	0.2875	L
50	0.1686	0.1774	0.1695	0.1707	0.1746	0.1632	0.1726	0.1651	L
75	0.1130	0.1155	0.1126	0.1143	0.1163	0.1094	0.1137	0.1110	L
100	0.0887	0.0892	0.0881	0.0896	0.0910	0.0859	0.0884	0.0873	L
	Gamma priors $(a = 3 \ , b = 0.8)$								
10	0.9024	0.5824	0.6591	1.0851	1.3191	0.4642	0.5995	0.7087	L
20	0.4706	0.3760	0.3982	0.5256	0.5949	0.3240	0.3808	0.4077	L
30	0.3667	0.2988	0.3203	0.3991	0.4385	0.2745	0.3064	0.3290	L
50	0.1875	0.1686	0.1746	0.1965	0.2073	0.1610	0.1707	0.1768	L
75	0.1229	0.1130	0.1163	0.1273	0.1326	0.1099	0.1143	0.1177	L
100	0.0952	0.0887	0.0910	0.0979	0.1011	0.0869	0.0896	0.0920	L
	Chi-Square priors $(d = 4)$								
10	1.1122	0.7004	0.8038	1.3416	1.6357	0.5462	0.7259	0.8630	L
20	0.5255	0.4199	0.4449	0.5867	0.6640	0.3574	0.4254	0.4536	L
30	0.3916	0.3199	0.3424	0.4261	0.4681	0.2924	0.3278	0.3510	L
50	0.1933	0.1747	0.1804	0.2024	0.2134	0.1664	0.1767	0.1824	L
75	0.1253	0.1156	0.1188	0.1297	0.1349	0.1122	0.1168	0.1201	L
100	0.0965	0.0902	0.0925	0.0992	0.1024	0.0884	0.0911	0.0933	L
	Exponential priors $(b = 1.2)$								
10	0.5882	1.1692	0.8200	0.5187	0.4779	0.8026	0.9845	0.6400	D
20	0.3679	0.5626	0.4454	0.3447	0.3315	0.4406	0.4998	0.3839	D
30	0.2808	0.3618	0.3109	0.2737	0.2721	0.3052	0.3340	0.2842	D
50	0.1629	0.1877	0.1720	0.1609	0.1604	0.1697	0.1790	0.1636	D
75	0.1096	0.1195	0.1130	0.1090	0.1091	0.1116	0.1159	0.1095	P
100	0.0861	0.0909	0.0876	0.0860	0.0863	0.0865	0.0890	0.0858	P
	Gamma priors $(a = 4 \ , b = 1.2)$								
10	0.6381	0.4779	0.4931	0.7583	0.9171	0.3763	0.4694	0.5162	L
20	0.3868	0.3315	0.3374	0.4278	0.4811	0.2845	0.3290	0.3410	L
30	0.3209	0.2721	0.2853	0.3471	0.3795	0.2487	0.2758	0.2907	L
50	0.1752	0.1604	0.1645	0.1830	0.1925	0.1527	0.1617	0.1660	L
75	0.1174	0.1091	0.1117	0.1213	0.1261	0.1058	0.1100	0.1127	L
100	0.0919	0.0863	0.0882	0.0943	0.0973	0.0844	0.0870	0.0889	L

We have presented the simulation results using MATLAB program. A simulation results are conducted to examine and compare the performance of the estimates for shape parameter respecting to their MSE. The best estimator has the smallest value of MSE. When we have as well as singly type II censored sample are summarized tables (3) and (4) in table (5).

Table (5)the best performances for loss function and prior dist.

Test	prior	(Doubly); MSE					
		10	20	30	50	75	100
1	Jeffery	BL	BL	BL	BL	BQ	BQ
	Chi-Sq.	BQ	BQ	BQ	BQ	BQ	BQ
	Exp.	BL	BL	BL	BL	BL	BL, BQ
	Gamma	BQ	BQ	BQ	BQ	BQ	BQ
	Best prior	Exp.	Exp.	Exp.	Exp.	Exp.	Exp.
2	Jeffery	BL	BL	BL	BL	BQ	BQ
	Chi-Sq.	BQ	BQ	BQ	BQ	BQ	BQ
	Exp.	BL	BL	BL	BL	BL	BL
	Gamma	BQ	BQ	BQ	BQ	BQ	BQ
	Best prior	**Exp.**	**Exp.**	**Exp.**	**Exp.**	**Exp.**	**Exp.**
3	Jeffery	BL	BL	BL	BL	BL	BL
	Chi-Sq.	BL	BL	BL	BL	BL	BL
	Exp.	BNL	BNL	BL	BL	BL	BL
	Gamma	BL	BL	BL	BL	BL	BL
	Best prior	Gamma	Gamma	Gamma	Gamma	Gamma	Jeffery

4	Jeffery	BL	BL	BL	BL	BL	BL
	Chi-Sq.	BL	BL	BL	BL	BL	BL
	Exp.	BD	BD	BD	BD	BP	BP
	Gamma	BL	BL	BL	BL	BL	BL
	Best prior	**Gamma**	**Gamma**	**Gamma**	**Gamma**	**Gamma**	**Gamma**

From the table (5) it can be assessed that the rate of converges of the estimates towards the true value of the shape parameter increases with increase of sample size. However, the rate of convergence is random with respect to loss function and prior distribution. The estimates with $(\lambda = 1.5)$under Exponential prior $(b = 1.2$with linear Exponential loss function the Best than those under Jeffery, Chi-Square and Gamma prior using other loss function. And the estimates with $(\lambda = 3)$under Gamma prior$(a = 4, b = 1.2)$ with linear Exponential loss function the Best than those under Jeffery, Chi-Square and Exponentialprior using other loss function for the singly type II censoring Similarly, the estimates based on linear Exponential loss function are associated with the minimum MSE.

V. Conclusions

The above study suggests that in order to estimate the parameter of Burr type X distribution under a Bayesian framework, the use of Exponential prior$(b = 1.2)$with$(\lambda = 1.5)$anduse of Gamma prior$(a = 4, b = 1.2$ withλ=3 along with linear Exponential loss function can be preferred for the singly type II censored sample.

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