IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 3 Ver. III (May. - Jun. 2016), PP 01-09
www.iosrjournals.org

Weakly Compatible Mapping in Intuitionistic Fuzzy Metric Space

Ankita Tiwari[1], Vandana Gupta[2], Sandeep K.Tiwari[3]
[1] *Faculty, Department of Mathematics, National Institute of Technology, Warangal.*
[2]*Professor and Head, Govt. Kalidas Girl's College, Ujjain (M.P.)*
[3]*Reader,School of Studies in Mathematics, Vikram University, Ujjain(M.P.)*

Abstract: *In 2006, Turkoglu et al. [18] extended the notion of compatible mappings to IFM spaces. Alaca [2] weakened the notion of compatibility by using the notion of weakly compatible maps in IFM spaces and showed that every pair of compatible mappings is weakly Compatible but reverse is not true. Many authors have proved a number of fixed point theorems for different contractions in IFM-spaces (see. 4,6,11].*

I. Introduction

In 2002, Aamri and El-Moutawakil [1] defined the defined the notion of property (E.A) for self mappings which contained the class of non-compatible mappings in metric spaces. It is observed that the property (E.A) requires the completeness (or closeness) of the underlying subspaces for the existence of common fixed point. In an interesting paper, Sintunavarat and Kumam [16] introduced the notion of 'common limit range property' (briefly,(CLRg) property with respect to mapping) in fuzzy metric spaces. They showed that the notion of (CLRg) property never requires the condition of the closeness of the subspace .

In 2008, Alaca et al. [3] proved a common fixed point theorem for continuous compatible mappings on complete IFM-space. Further, Kumar [10] and Manro [13] improved and generalized the results of Alaca et al.[3] and Kumar and Vats [12] under strict contractive conditions. Most recently Tanveer et al. [17] proved common fixed point theorems for weakly compatible mappings in modified IFM-spaces using common property (E.A).

The object of this paper is to prove a common fixed point theorem for a pair of weakly compatible mappings in IFM-space by using the notion of (CLRg) property. We also present a result for two finite families of self mappings by using pair wise commuting due to Imdad et al. [8].

Definition1.1. A binary operation $*$: $[0,1] \times [0,1] \rightarrow [0,1]$ is continuous t – norm if $*$ is satisfying the following conditions :

(i) $*$ is commutative and associative;

(ii) $*$ is continuous;

(iii) $a * 1 = a$ for all $a \in [0, 1]$;

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition1.2. A binary operation $\Diamond$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t – conorm if $\Diamond$ is satisfying the following conditions :

(i) $\Diamond$ is commutative and associative;

(ii) $\Diamond$ is continuous;

(iii) $a \Diamond 0 = a$ for all $a \in [0, 1]$;

(iv) $a \Diamond b \leq c \Diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition1.3. A 5–tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t – norm, $\Diamond$ is a continuous t – conorm and M, N are fuzzy sets on

$X^2 \times [0, \infty)$ satisfying the following conditions :
(i) M(x, y, t) + N(x, y, t) ≤ 1 for all x, y ∈ X and t > 0 ;
(ii) M(x, y, 0) = 0 for all x, y ∈X ;
(iii) M(x, y, t) = 1 for all x, y ∈ X and t > 0 if and only if
x = y;
(iv) M(x, y, t) = M(y, x, t) for all x, y ∈ X and t > 0 ;
(v) M(x, y, t) ∗ M(y, z, s) ≤ M(x, z, t + s) for all x, y, z ∈ X and s, t > 0 ;
(vi) for all x, y ∈X, M(x, y, .) : [0, ∞) → [0, 1] is left continuous;
(vii) $\lim_{t \to \infty}$M(x, y, t) = 1 for all x, y ∈X and t > 0 ;
(viii) N(x, y, 0) = 1 for all x, y ∈ X;
(ix) N(x, y, t) = 0 for all x, y ∈ X and t > 0 if and only if
x = y ;
(x) N(x, y, t) = N(y, x, t) for all x, y ∈ X and t > 0 ;
(xi) N(x, y, t) ◊ N(y, z, s) ≥ N(x, z, t + s) for all x, y, z ∈ X and s, t > 0 ;
(xii) for all x, y ∈X, N(x, y, .) : [0, ∞) → [0, 1] is right continuous;
(xiii) $\lim_{t \to \infty}$N(x, y, t) = 0 for all x, y in X :
The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non–nearness between x and y with respect to t, respectively.

Example1.4. Let $X = \{ \frac{1}{n} : n \in N\} \cup \{0\}$ with ∗ continuous t – norm and ◊ continuous t – conorm defined by a ∗ b = ab and
a ◊ b = min {1, a+b} respectively, for all a, b ∈ [0, 1].
For each t ∈ (0,∞) and x, y ∈ X, define M and N by Then, (X, M, N, ∗, ◊) is an intuitionistic fuzzy metric space.

Definition1.5. Let A and B be two self mappings of an intuitionistic fuzzy metric space (X, M, N, ∗, ◊). Then the maps A and B are said to be compatible if for all t > 0,

$$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1 \text{ and}$$

$$\lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$$

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some z ∈ X.

Definition 1.6 Self maps A and S of an intuitionistic fuzzy metric space (X, M, N, ∗, ◊)are said to be R-weakly commuting if there exists some real number R such that
M(ASx, SAx, t)≥M(Ax, Sx, t/R) and
N(ASx, SAx, t)≤ N(Ax, Sx, t/R) for each x ∈X and t > 0.

Definition 1.7. Self maps A and S of an intuitionistic fuzzy metric space (X, M, N, ∗, ◊) are said to be R-weakly commuting of type (A_g) if there exists some real number R such that
M(AAx, SAx, t) ≥ M(x, Sx, t/R) and
N(AAx, SAx, t) ≤ N(Ax, Sx, t/R) for each x ∈ X and t > 0.

Definition1.8. Let A and B be two self mappings of an intuitionistic fuzzy metric space (X, M, N, ∗, ◊). Then the maps A and B satisfy the property (E.A.) if there exists a sequence $\{x_n\}$ such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some z ∈X.

Remark 1.9. Two maps A and B on an intuitionistic fuzzy metric space (X, M, N, ∗, ◊) are non-compatible if and only if there exists at least one sequence $\{x_n\}$ in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some z ∈ X, but for some t > 0 either $\lim_{n\to\infty} M(ABx_n, BAx_n, t) \neq 1$ and

$\lim_{n\to\infty} N(ABx_n, BAx_n, t) \neq 0$ or the limit does not exist.

Remark 1.10 The concepts of t-norms and t-co norms are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions, respectively. These concepts were originally introduced by Menger [14] in his study of statistical metric spaces.

Atanassov [5], Kramosil and Michalek [9] and Alaca et al. [3] defined the following definition in framework of IFM-spaces:

Example 1.11. (Induced intuitionistic fuzzy metric)

Let (X, d) be a metric space .Denote a ◊ b = min { 1, a+b } , ∀ a , b ∈ [0,1] and let M_d and N_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows :

$$M_d(x, y, t) = \frac{ht^n}{ht^n + m\, d(x, y)}, \quad N_d = \frac{d(x,y)}{kt^n + m\, d(x, y)}$$

∀ h ,k ,m ,n ∈ N. Then (X ,M, N, *, ◊)is an IFM-Space.

Remark 1.12. Note the above example hold with t norm

a*b = min {a ,b} and the t-conorm a ◊ b = max { a,b} and hence

(M,N) is an intuitionistic fuzzy metric with respect to any continuous t norm and continuous t co-norms. In the above example by taking

h = k= m = n= 1, we get

$$M_d(x, y, t) = \frac{t}{t + m\, d(x, y)} \;;$$

$$N_d(x, y, t) = \frac{d(x,y)}{t + m\, d(x, y)}$$

Then (X ,M, N, *, ◊) is an IFM-Space induced by the metric d .It is obvious that

N (x, y, t) = 1 - M(x, y, t).

Remark 1.13. In IFM-Space (X ,M, N, *, ◊), M(x , y, .) is non decreasing and N(x , y ,.) is non decreasing ∀ x , y ∈ X.

The following definition is on the lines of Aamri and Moutawakil [1].

Definition 1.14 A pair (f ,g) self mappings defined on an IFM-space

(X ,M, N, *, ◊) is said to satisfy the property (E.A.) , if there exists a sequences

$\{x_n\}$in X such that $\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = z$, for some z ∈ X.

Definition 1.15 A pair (f, g) self mappings defined on a non-empty set X is said to be weakly compatible if they commute at their coincidence points, that is, if fx = gx for some x ∈ X. then f g x = g f x.

Remark 1.16 The notions of weak compatibility and property (E.A) are independent to each other .

Inspired by the work of Sintunavarat and Kumam [16], we defined the notion of (CLRg) property in framework of IFM-Space.

Definition 1.17 A pair (f ,g) self mappings defined on an IFM-space

(X,M, N, *, ◊) is said to satisfy the (CLRg) property, if there exists a sequences $\{x_n\}$in X such that

$\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = g u.$

For some u ∈ X.

Example 1.18. Let X = [0, ∞) with metric d defined by

d(x,y) = $|x-y|$ and define

$$M(x, y, t) = \begin{cases} \dfrac{t}{t+|x-y|} & ; \text{if } t > 0 \\ 0 & ; \text{if } t = 0 \end{cases}$$

$$N(x, y, t) = \begin{cases} \dfrac{|x-y|}{t+|x-y|} & ; \text{if } t > 0 \\ 1 & ; \text{if } t = 0 \end{cases}$$

For all x,y∈ X. clearly (X,M,N,*,◊) be an IFM-space where * and ◊ are continuous t-norm and continuous t co norm defined by

a*b = min { a ,b} and a ◊ b = Max { a ,b} ∀ a, b ∈ [0 ,1].

Define the self mappings f and g on X by f(x) = x + 3 and g(x) = 4x, ∀ x ∈ X.

Consider a sequence { x_n }= $\left\{1+\frac{1}{n}\right\}$, n ∈ N in X ,we have

$$\lim_{n\to\infty} f\left(1+\frac{1}{n}\right) = \lim_{n\to\infty}\left(4+\frac{1}{n}\right) = 4 = g(1) = \lim_{n\to\infty}\left\{4+\frac{4}{n}\right\} =$$

$$\lim_{n\to\infty} g\left(4+\frac{4}{n}\right) = \lim_{n\to\infty} g\left(1+\frac{1}{n}\right)$$

Which shows that pair (f , g) enjoys the (CLRg) property.

Example 1.19 The conclusion of Example 1.9 remains true if we replace the self mappings f and g by f(x) = x/4 and g(x) = 3x/4, ∀ x∈ X .

Consider a sequence {x_n} = { 1/n}, n ∈ N, in X , we have

$$\lim_{n\to\infty} f\left(\frac{1}{n}\right) = \lim_{n\to\infty}\left(\frac{1}{4n}\right) = 0 = g(0) = \lim_{n\to\infty}\left(\frac{3}{4n}\right) = \lim_{n\to\infty} g\left(\frac{1}{n}\right)$$

therefore, the mappings f and g satisfy the (CLRg) property.

Lemma 1.20. Let (X ,M, N, *, ◊) be an IFM-space and ∀ x ,y ∈ X , t > 0 and if for a constant k ∈ (0,1)

M(x, y ,k t) ≥ M (x, y , t) and N(x, y ,k t) ≤ N (x, y , t)

then x =y.

Sintuanavarat and Kuman [16]proved a fixed point theorem for weakly compatible map in Fuzzy metric space

Theorem 1.21 Let (X,M,*) be a KM-FUZZY METRIC SPACE satisfying the following property:

$\forall$ x,y $\in$ X, x$\neq$y, $\exists$ t > 0: 0< M(x,y,t) < 1,

and let f and g be weakly compatible self mappings of x such that for some $\varphi \in \Phi$,

$$M(fx,fy,t) \geq \varphi(\min\{M(gx,gy,t), M(fx,gx,t), M(fy,gy,t), M(fy,gx,t),M(fx,gy,t)\})$$

for all x, y $\in$ X, where t>0.If f and g satisfy the CLRg property, then f and g have a unique common fixed point. Our result improves the above result. we have proved the same theorem for Intuitionistic Fuzzy Metric Space.

II. Main Result

Theorem 2.1 Let (X ,M, N, *, ◊) be an IFM-space with t*t ≥ t and (1-t) ◊ (1-t) ≤ (1-t) , $\forall$ t $\in$ [0,1]. Further Let the pair (f ,g)of self mappings is weakly compatible satisfying

$$M(f x , f y , k t) \geq \{ M (g x ,g y ,t)* M (f x ,g x, t) * M(f y , g y, t)* M(f x, g y ,t) * M(f y ,g x, t) \} \quad (2.1)$$

And

$$N(f x , f y , k t) \leq \{ N (g x ,g y ,t) \lozenge N (f x ,g x, t) \lozenge N(f y , g y, t) \lozenge N(f x, g y ,t) \lozenge N(f y ,g x, t) \} \quad (2.2)$$

$\forall$ x,y $\in$ X , k $\in$ (0 ,1) and t > 0.If f and g enjoys (CLRg) property then f and g have a unique common fixed point in X.

Proof : Since the pair (f, g) satisfies the (CLRg) property ,there exists a sequence { x_n } in X such that

$$\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = gu, \text{ for some } u \in X.$$

Now we asserts that f u = g u.

On using inequalities (2.1) and (2.2) with x = x_n , y = u , we get

$$M(f x_n , fu , kt) \geq \{ M(g x_n , gu , t) * M(f x_n, g x_n ,t)* M(fu ,gu ,t) *M(f x_n ,gu, t)* M(fu ,g x_n ,t) \}$$

and

$$N(f x_n, fu , kt) \leq \{ N(g x_n , gu , t) \lozenge N(f x_n , g x_n ,t) \lozenge N(fu ,gu ,t) \lozenge N(f x_n ,gu, t) \lozenge N(fu ,g x_n ,t) \}$$

Taking limit as n→∞ we have

$$M(gu , fu , kt) \geq \{ M(gu, gu , t) * M(gu , gu ,t) * M(fu ,gu ,t) *M(gu ,gu, t) * M(fu ,g x_n ,t)\}$$

and

$$N(gu, fu , kt) \leq \{ N(gu, gu, t) \lozenge N(gu , gu ,t) \lozenge N(fu ,gu ,t) \lozenge N(gu ,gu, t) \lozenge N(fu ,g x_n ,t)\}.$$

It implies

$$M(fu , gu , kt) \geq \{ 1*1* M(fu ,gu ,t) *1* M(fu ,g x_n ,t) \} = M(fu ,gu ,t).$$

$$N(fu , gu , kt) \leq \{ 0 \lozenge 0 \lozenge N(fu ,gu ,t) \lozenge 0 \lozenge N(fu ,gu ,t) \} = N(fu ,gu ,t).$$

In view of Lemma 1.11, we have fu = gu.

Now we let z = fu = gu .Since the pair (f,g) is weakly compatible,

we get fz = fgu =gfu =gz..

Now we show that z is a common fixed point of the mappings f and g. To prove this, using inequalities (2.1) and (2.2) with x = z and y = u ,

we have

$$M(fz, fu ,kt) \geq \{M(gz,gu,t)*M(fz,gz,t)*M(fu ,gu,t) *M (fz, gu ,t) * M (fu ,gz ,t) \}$$ and

$N(fz,fu,kt) \leq \{N(gz,gu,t) \Diamond N(fz,gz,t) \Diamond N(fu,gu,t)$
$\Diamond N(fz, gu, t) \Diamond N(fu, gz, t)\}$

It implies

$M(fz, z, kt) \geq \{M(fz, z, t)*1*1* M(fz,z,t) * M(z, fz, t)\}$
$= M(fz, z, t)$ and

$N(fz, z, kt) \leq \{N(fz, z, t) \Diamond 0 \Diamond 0 \Diamond N(fz, z, t) \Diamond N(z, fz, t)\}$
$= N(fz, z, t)$

On employing Lemma 1.11, we get z = fz = gz which shows that z is a common fixed point of the mappings f and g.

Uniqueness: Let w be another common fixed point of the mappings f and g. On using inequalities (2.1) and (2.2) with x = z,y = w, we have

$M(fz, fw, kt) \geq \{M(gz,gw,t)*M(fz, gz, t) * M(fu, gw, t)$
$* M(fz, gw, t) * M(fw, gz, t)\}$

and

$N(fz, fw, kt) \leq \{N(gz, gw, t) \Diamond N(fz, gz, t) \Diamond N(fw, gw, t)$
$\Diamond N(fz, gw, t) \Diamond N(fw, gz, t)\}$

Or equivalently,

$M(z, w, kt) \geq \{M(z, w, t) * M(z, z, t) * M(w, w, t)$
$* M(z, w, t) * M(w, z, t)\}$
$= M(z, w, t)$

And

$N(z, w, kt) \leq \{N(z, w, t) \Diamond N(z, z, t) \Diamond N(w, w, t)$
$\Diamond N(z, w, t) \Diamond N(w, z, t)\}$
$= N(z, w, t)$.

Appealing to Lemma 1.11, we have z= w. Therefore the mappings f and g have a unique common fixed point in X.

Remark 2.2. From the proof of Theorem 2.1, it is asserted that (CLRg) property never requires the completeness (or closeness) of the underlying subspace and containment of ranges amongst involved mappings.

Example 2.3 Let X = [1,1.5) with metric d defined by

$d(x,y) = |x-y|$ and define

$$M(x,y,t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0 \\ 0, & \text{if } t = 0 \end{cases} \quad \text{and}$$

$$N(x,y,t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0 \\ 1, & \text{if } t = 0 \end{cases}$$

$\forall$ x,y $\in$ X .Then (X ,M, N, *, $\Diamond$) is an IFM-space where * is a Continuous t norm, $\Diamond$ is a continuous t-co norm defined by

a*b = min { a ,b} and a $\Diamond$ b = Max { a ,b}, for all a , b ∈ [0 ,1].

Now we define the self mappings f and g on X by

$$f(x) = \begin{cases} 1, \text{if } x \in \{1\} \cup (8,15) \\ 8, \quad \text{if } x \in (1,3] \end{cases} \quad \text{and}$$

$$g(x) = \begin{cases} 1, \text{ if } x = 1 \\ 7, \text{ if } x \in (1,3] \\ \dfrac{x+1}{4}, \text{ if } x \in (3,15) \end{cases}$$

Consider a sequence $\{ x_n \} = \left\{3+\frac{1}{n}\right\}$ n∈ N or $\{ x_n\} = 1$.

Then we have

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = g(1) \in X.$$

Hence the pair (f ,g) enjoys the (CLRg) Property .It is noticed that f (X) = $\{1,8\} \not\subset [1,4) \cup \{7\} = g(X)$.

Here g(X) is not a closed subspace of X. Thus all the conditions of theorem 2.1 are satisfied for some k ∈(0,1) and 1 is a unique common fixed point of the mappings f and g . Also all the involved mappings are discontinuous at their unique common fixed point.

Our next theorem is proved for a pair of weakly compatible mappings by using the notion of property (E.A) under additional assumption of closeness of the underlying subspace.

Theorem 2.4. Let (X ,M, N, *, ◊) be an IFM-space with $t * t \geq t$ and $(1-t) ◊ (1-t) \leq (1-t)$ ∀ t∈ [0,1]. Further let the pair (f ,g) of self mappings is weakly compatible satisfying inequalities (2.1) and (2.2) of Theorem 2.1. If f and g enjoy the property (E.A) and g (X) is a closed subspace of X then and f and g have a unique common fixed point in X .

Proof. If the pairs (f,g) satisfies the property (E.A), then there exists a sequence $\{ x_n \}$ such that

$$\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = z \text{ for some } z \in X.$$

Since g (X) is a closed subspace of X, there exists a point u ∈ X such that gu = z which shows that the pair (f,g) satisfies the (CLRg) property. The rest of the proof can be completed on the lines of the proof of Theorem 2.1. Therefore the mappings f and g have a unique common fixed point in X.

Example 2.5 In the setting of Example **2.3** replace the mapping g by the following: besides retaining the rest

$$g(x) = \begin{cases} 1, & \text{if } x = 1 \\ 4, & \text{if } x \in (1,3] \\ \dfrac{x+1}{4}, & \text{if } x \in (3,15) \end{cases}$$

Choose a sequence $\{x_n\} = \left\{3+\frac{1}{n}\right\}$, n∈ N or $\{x_n\} = 1$.

Then we have

$$\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = 1 \in X.$$

It implies the pair(f ,g) satisfies the property (E.A).

Also f(X)= $\{1,8\} \not\subset \{1,4\} = g(X)$. Here is closed subspace of X. Thus all the conditions of Theorem 2.4 are satisfied for some

k∈ (0,1) and 1 is a unique common fixed point of the mappings f and g .

Since the pair of non-compatible mappings implies to the pair satisfying the property (E.A), we get the following results:

Corollary 2.6 Let (X ,M, N, *, ◊) be an IFM-space with $t * t \geq t$ and $(1-t) \lozenge (1-t) \leq (1-t)$ $\forall$ $t \in [0,1]$. Further let the pair (f ,g) of self mappings is weakly compatible satisfying inequalities (2.1) and (2.2) of Theorem 2.1. If f and g are non- compatible and g (X) is a closed subspace of X then and f and g have a unique common fixed point in X.

Now we utilize Definition 1.7 (which is a natural extension of commutativity condition to two finite families) to define a new result in IFM-space.

Theorem 2.7. Let $\{f_i\}$(i= 1,2,…,m) and $\{g_j\}$(j= 1,2,…,n) be two finite families of self mappings in IFM-space (X ,M, N, *, ◊) with $t*t \geq t$ and $(1-t) \lozenge (1-t) \leq (1-t)$ $\forall$ $t \in [0,1]$ such that $f=f_1f_2\ldots f_m$ and $g=g_1g_2\ldots g_n$ which satisfy the inequalities (2.1) and (2.2). Suppose that the pair(f ,g) enjoys the (CLRg) property. Moreover { f_i}(i= 1,2,…,m) and $\{g_j\}$(j= 1,2,…,n) have a unique common fixed point provided the pair of families (f_i,g_i) commutes pairwise where(i= 1,2,…,m) and(j= 1,2,…,n)

Corollary 2.8 Let f and g be two self mappings in IFM-Space (X ,M, N, *, ◊) with $t * t \geq t$ and $(1-t) \lozenge (1-t) \leq (1-t)$ $\forall$ $t \in [0,1]$.Further , Let the pair (f^m,g^n) of self mappings enjoys the CLRg^n property. If there exists a constant $k \in [0,1]$ such that

$$M(fx, fy, kt) \geq \{ M(gx, gy, t) * M(fx, gx, t) * M(fy, gy, t) * M(fx, gy, t) * M(fy, gx, t) \} \quad (2.3)$$

and

$$N(fx, fy, kt) \leq \{ N(gx, gy, t) \lozenge N(fx, gx, t) \; N(fy, gy, t) \lozenge N(fx, gy, t) \lozenge N(fy, gx, t) \} \quad (2.4)$$

$\forall$ $x,y \in X$, $t > 0$ and m ,n are fixed positive integer then f and g have a unique common fixed point in X provided the pair (f^m, g^n) commutes pairwise.

Remark 2.9 The results similar to Theorem 2.7 and Corollary 2.8 can be outlined in view of Theorem .2.4 and Corollary 2.6 The details of possible corollaries are not included here.

III. Conclusion

Theorem 7.2.1 is proved for a pair of weakly compatible mappings using the (CLRg) property in IFM-space without any requirement on containment of ranges amongst the involved mappings and completeness of the whole space (or closedness of any subspace). Theorem 2.1 improves the results of Kumar [10,Theorem 3.3] and Tanveer et al. [17, Corollary 3.2]. Theorem 2.7 and Corollary 2.8 also generalize the main results of Alaca et al. [3, theorem 18] and Huang et al.[7] as the conditions of completeness (or closedness) of the underlying subspace, continuity of one or more mappings and containment of ranges amongst involved mappings are completely relaxed.

References

[1] Aamri, M. and Mountawakli, D.El.,Some new common fixed point theorem under strict contractive conditions,J.Math.Anal.Appl.270(2000),181-188.

[2] Alaca, C., “A common fixed point theorem for weak compatible mappings in intuitionistic fuzzy metric spaces”, Int. J. Pure Appl. Math., 32(4):537-548(2006).

[3] Alaca, C., Turkoglu D., Yildiz,C., “Common fixed points of compatible maps in intuitionistic fuzzy metric spaces”, Southeast Asian Bull. Math., 32(1):21-33(2008).

[4] Alaca,C., “On fixed point theorems intuitionistic fuzzy metric spaces”, Commun Korean Math. Soc., 24(4):565-579(2009).

[5] Atanassov, K., “Intuitionistic fuzzy sets”, Fuzzy Sets Syst., 20(1):87-96(1986).

[6] Ćiŕic,LjB Ješić ,S.N., Ume, J.S., “The existence theorems for fixed and periodic points of non expansive mappings in intuitionistic fuzzy metric spaces”, Chaos Solit. Fract., 37(3):781 791(2008).

[7] Huang,X., Zhu,C., & Wen, Xi, “Common fixed point theorems for families of compatible mappings in intuitionistic fuzzy metric spaces”, Ann. Univ. Ferrara Sez. VII Sci. Mat 56(2):305-326(2010).

[8] Imdad, M., Ali,J.& Tanveer,M., "Coincidence and common fixed point theorems for nonlinear contractions in Menger PM spaces", Chaos Solit. Fract., 42(5):3121-3129(2009).
[9] Kramosil and J. Machalek, Fuzzy metric and statistical metric spaces, Kybernetika 1975), 336-344.
[10] Kumar, S., "Common fixed point theorems in intuitionistic fuzzy metric spaces using property (E.A)", J. Indian Math. Soc. (N.S.), 76(1- 4):93-104(2009).
[11] Kumar,S., Kutukcu, S., "Fixed points of expansion maps in intuitionistic fuzzy metric spaces", Sci. Stud. Res. Ser. Math. Inform., 20(1):119-132(2010).
[12] Kumar,S.,Vats, R.K., "Common fixed points for weakly compatible maps in intuitionistic fuzzy metric spaces", Adv. Fuzzy Math., 4(1):9- 22(2009).
[13] Manro,S ,Bhatia,S.S., Kumar, S., "Common fixed point theorem for weakly compatible maps satisfying (E.A.) property in intuitionistic fuzzy metric spaces", Punjab Univ. J. Math. (Lahore), 42:51-56(2010).
[14] Menger,K., "Statistical metrics", Proc. Nat Acad. Sci. U.S.A., 28:535-537(1942).
[15] Sintunavarat W., Kumam, P., "Common fixed points for -weakly commuting in fuzzy metric spaces", Ann. Univ. Ferrara Sez. VII Sci. Mat., 58(2):389-406(2012).
[16] Sintunavarat,W., Kumam,P., "Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces", J. Appl. Math., Vol. 2011, Article ID 637958, 14 pages, (2011).
[17] Tanveer,M., Imdad M., Gopal D., Kumar, D., "Common fixed point theorems in modified intuitionistic fuzzy metric spaces with common property (E.A)", Fixed Point Theory Appl., 2012, 2012:36, 17 pages.
[18] Turkoglu,D.,Alaca, C., Cho, Y.J.,Yildiz,C., "Common fixed point theorems in intuitionistic fuzzy metric spaces", J. Appl. Math. Comput., 22(1-2):411-424(2006).