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Queueing Analysis of Mainte-Nance Float System with Reneging

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Abstract: *The present paper deals with a Markovian queueing model for maintenance float system, which has following elements: workstations, repair stations and a set of standby components. In case of long queue of the failed machines, the reneging of failed machine takes place. Particularly we consider exponential repair and failure. Determine the four optimal parameters; population size (M), size of repair crew (S), the mean repair rate μ and reneges exponentially with parameter v such that the total cost is minimized.*

Keywords: *Repair, Failure, Optimal Cost, Reneging*
Mathematical Subject Classification (AMS 2000): *90B25*

I. Introduction

Manufacturing or production units are concerned for successful and smooth running in industrial environments. The system designers and cost analysts may be interested to know the number of machines and size of repair crew to be facilited for effective running of a system. The cost is also associated with replacement and repair. The analysis of maintenance cost is an important factor for optimal maintenance policy. The cost effective maintenance schedules are required for many complex equipments for the smooth operation of the machining system. The machine repair problems have been discussed by several research workers in different frame-works. Shanthi kumar and Yao [1] obtained the allocation of optimal server in a system of multi server stations. Madu [2] considered a closed queuing network with two types of repairs by assuming exponential failure and repair times. Albright and Soni [3] considered the continuous Markov process to study the repairable model. Georgantzas and Madu [4] considered the maintenance float system with imperfect information. Madu [5] obtained the economic design for optimal maintenance float system. Montoro-cazorla, and Perez-Ocon [6] a system with n components one online and the rest in standby subject to repair is considered. Khalil et al. [7] developed the survival function of a repairable standby series system using maximum Likelihood and Monte Carlo simulation. Lin et al. [8] considered the flexible manufacturing system with a single repair station of a single channel and another with two separate and independent repair stations each having a single repair channel. Lieckens et al. [9] established comprehensive service contracts and sustainable supply chain are two recent trends that create the opportunity to develop maintenance contracts with an uptime guarantee for the customer and a remanufacturing process for removed parts. Zeng and Zhang [10] developed the model for designing an optimal maintenance float system with three parameters. Ke and Wang [11] considered the M/M/R machine repair problem with balking, reneging and unreliable servers. Jain et al. [12] established machine repair problem with reneging and additional repairman. Shawky [13] established hyper-exponential machine interference system with reneging. Lam et al. [14] Geometric process model for M/M/1 is evaluated by supplementary variable technique, some queueing characteristics of the system and reliability indices of the service station are derived. Ke et al. [15] studied the machine repair problem consisting of M operating machines with two types of spare machines S and R servers, who leave for a vacation of random length when there are no failed machines queueing up for repair in the repair facility. Jain et al. [16] developed queueing model for the performance prediction of flexible manufacturing systems with a multiple discrete material handling devices. Jeyakumar et al. [17] completing a batch of service, if the server is breakdown with probability π then the renovation of service station will be considered. Wang et al. [18] derived the explicit expressions for reliability function and mean time to system failure (MTTF).

In this paper, the four decision variables define as: the finite population size of the standby units (M), the size of the repair crew (S), mean repair rate (μ) and reneging with (v). The present paper is organized as follows. In section 2, describe the model of maintenance float system. The assumption along with notations for mathematical formulation is given in section 3. For optimization, the total cost function and several results in the form of lemma, corollary and theorem are established in section 4. Algorithm to determine the optimal parameters is given in section 5. Future scope and economic justification are discussed in final section 6.

II. Model Description

Consider a maintenance float system with multiple repairmen and standby. In order to maintain a high level of availability of the workstation units, the identical key components are kept in the operation line in a buffer but the total number of key components is fixed. Also assume that the failed machine is replaced by the available standby machine and failed machine is immediately sent for repair. When the repair is finished, the repaired machine is transferred back to the buffer and serves as a standby. To study this problem assume the following :

- The life time of all the units in operation is an independent and identically distributed (iid) exponential variable with mean $(1/\lambda)$ where λ is constant.
- The repair time is independent and identically distributed (iid) and follows an exponential distribution with mean $1/\mu$.
- When a failed machine waits for a long time in the queue for repair, it may renege exponentially with parameter ν, The total number of key components M, is greater than or equal to the number of repair servers in the repair station S.
- If all M units are in repair station then workstation is idle or unavailable.

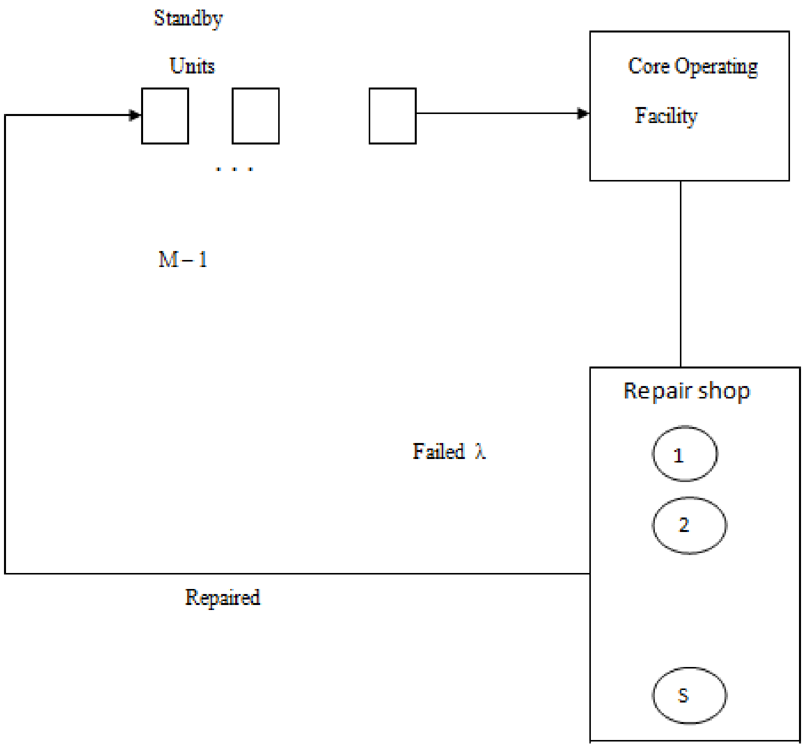

Fig: 2. A maintenance float system with multiple repair servers

III. Model Formulation

Consider the M/M/S/M/M queueing system with reneging for finite float system .The transition states follow the following flow equations:

$$(M-n+1)\lambda P_{n-1} = n\mu P_n , \qquad n = 1,2,------,S$$

$$(M-n+1)\lambda P_{n-1} = \left[s\mu + (n-s)\nu\right] P_n \qquad n = s+1, s+2, ---, M$$

where P_n denotes the probability that there are n units in the workstation.
Thus the steady-state probability P_n is obtained as.

$$P_n = \begin{cases} \dfrac{(S\rho)^n M!}{n!(M-n)!} P_0 & ; n \le S-1 \\ \dfrac{M! S^S \rho^n}{S!} \dfrac{P_0}{\prod_{n=s+1}^{M}\left[1+\dfrac{(n-s)\nu}{S\mu}\right]} & ; S \le n \le M \end{cases} \qquad \ldots(1)$$

Now by using the normalizing condition $\sum_{n=0}^{M} P_n = 1$, we have

$$P_0 = \left[\sum_{n=0}^{S-1} \frac{(S\rho)^n M!}{n!(M-n)!} + \frac{S^S M!}{S!} \sum_{n=S}^{M} \frac{\rho^n}{\prod_{n=S+1}^{M} \left[1 + \frac{(n-s)\nu}{s\mu} \right]} \right]^{-1} \qquad \ldots(2)$$

where $\rho = \frac{\lambda}{S\mu} > 0$.

The idle probability involves the parameters M, S, μ and ν is defined as

$$IP(M,S,\mu,\nu) = P_M = \frac{S^S \rho^M}{S!} P_0$$

$$= \frac{S^S \rho^M}{S!} \left[\sum_{n=0}^{S-1} \frac{(S\rho)^n M!}{n!(M-n)!} + \frac{S^S M!}{S!} \sum_{n=S}^{M} \frac{\rho^n}{\prod_{n=S+1}^{M} \left[1 + \frac{(n-s)\nu}{s\mu} \right]} \right]^{-1} \qquad \ldots(3)$$

IV. Cost Optimization

Address the issue of the operational availability of the repair station and the cost of investment on server's maintenance (i.e. training and new technology) and standbys. The combination of optimal parameters M, S, μ and ν are obtained through cost optimization. The objective is to minimize the total cost so as to determine the optimal values of the parameters M, S, μ and ν. Consider the following assumptions for developing the total cost of the system:

(i) $f(IP(M,S,\mu,\nu))$ is increasing and convex loss function of the idle probability.

(ii) $h_1(M)$ is increasing and convex cost function of holding M units in the float system.

(iii) $h_2(S)$ is increasing and convex cost function of employing servers in the float system.

(iv) $h_3(\mu)$ is increasing and convex cost function of improving the mean repair rate.

(v) $h_4(\nu)$ is increasing and convex function corresponding to reneging parameter.

Mathematically, It can write as

$$Y(M,S,\mu,\nu) = f(IP(M,S,\mu,\nu)) + h_1(M) + h_2(s) + h_3(\mu) + h_4(\nu) \qquad \ldots(4)$$

Now formulate the optimization problem (OP) as

$$\text{Min } Y(M,S,\mu,\nu) = f(IP(M,S,\mu,\nu)) + h_1(M) + h_2(s) + h_3(\mu) + h_4(\nu)$$

subject to constraints**:** $M \geq 1;\ M \geq S \geq 1.$ …(5)

Lemma 1: IP(M, S, μ , ν) is a decreasing function in M for given S, μ and ν.

Proof: In order to prove this lemma, suppose that $\frac{1}{IP(M,\, S,\, \mu,\, \nu)}$ is an increasing function of M.

Now by equation (3), we have

$$\frac{1}{IP(M,S,\mu,\nu)} = \frac{S!}{S^S \rho^M}\left[\sum_{n=0}^{S-1}\frac{(S\rho)^n M!}{n!(M-n)!} + \frac{S^S M!}{S!}\sum_{n=S}^{M}\frac{\rho^n}{\prod_{n=S+1}^{M}\left[1+\frac{(n-S)\nu}{S\mu}\right]}\right]$$

$$= \left[\frac{S!}{S^S}\left[\sum_{n=0}^{S-1}\frac{(S\rho)^n M!}{n!(M-n)!}\right] + M!\sum_{n=S}^{M}\frac{\rho^n}{\prod_{n=S+1}^{M}\left[1+\frac{(n-S)\nu}{S\mu}\right]}\right]\rho^{-M} \qquad \ldots(6)$$

$= A\rho^{-M}, when\, \rho \neq 1$

where

$$A = \left[\frac{S!}{S^S}\left[\sum_{n=0}^{S-1}\frac{(S\rho)^n M!}{n!(M-n)!}\right] + M!\sum_{n=S}^{M}\frac{\rho^n}{\prod_{n=S+1}^{M}\left[1+\frac{(n-S)\nu}{S\mu}\right]}\right] \qquad \ldots(7)$$

It has been observed that A is independent of M. When $\rho \neq 1$, to prove IP(M, S, μ, ν) as decreasing function, Now by induction that for $\rho < 1,\ \rho^{-M} < \rho^{-(M+1)}$.

Now taking S=1

$$A = \left[1 + M!\sum_{n=1}^{M}\frac{\rho^n}{\prod_{n=2}^{M}\left[1+\frac{(n-1)\nu}{\mu}\right]}\right] > 0 \qquad \ldots(8)$$

For S=S+1

$$A = \left[\frac{(S+1)!}{(S+1)^{S+1}}\sum_{n=0}^{S}\frac{((S+1)\rho)^n M!}{n!\,(M-n)!} + M!\sum_{n=S+1}^{M}\frac{\rho^n}{\prod_{n=S+2}^{M}\left[1+\frac{(n-S-1)\nu}{(S+1)\mu}\right]}\right]$$

…(9)

Thus A > 0. In other words it can say that for $\rho > 1,\ \rho^{-M} > \rho^{-(M+1)}$ evidently A>0 therefore $A\rho^{-M}$ is increasing in M when $\rho \neq 1$.

Hence $IP(M,\ S,\ \mu,\ \nu) > IP(M+1,\ S,\ \mu,\ \nu)$.

Lemma 2: Prove that idle probability function of M is a convex function for given S, μ and ν.

Proof: Let us denote $q(M) = \dfrac{1}{IP(M,S,\mu,\nu)}$ for a given S, μ and ν.

Now,

IP(M +1, S, μ, ν) + IP(M-1, S, μ, ν) – 2 IP(M, S, μ, ν)

$$= \frac{1}{q(M+1)} + \frac{1}{q(M-1)} - \frac{2}{q(M)}$$

$$= \frac{[q(M+1)+q(M-1)]q(M) - 2q(M+1)q(M-1)}{q(M+1)q(M-1)q(M)} \qquad \ldots(10)$$

Since q(M)>0, let q(M) = $A\rho^{-M}$ when $\rho \neq 1$ then, we get

$$= \left(\frac{A}{\rho^{M+1}} + \frac{A}{\rho^{M-1}} \right)\left(\frac{A}{\rho^{M}} \right) - 2\left(\frac{A}{\rho^{M+1}} \right)\left(\frac{A}{\rho^{M-1}} \right)$$

$$= \frac{A^2(1-\rho)^2}{\rho^{2M+1}} > 0 \qquad \ldots(11)$$

We have seen that, when ρ <1, A>0 and we already discussed when ρ >1, A<0 in lemma 1. Thus A>0 when $\rho \neq 1$.

Hence IP(M+1, S, μ, ν) + IP(M-1, S, μ, ν) > 2 IP(M, S, μ, ν) …(12)

Lemma 3: Y(M, S, μ, ν) is convex function of M for given S, μ and ν.

Proof: In order to prove that $f(IP(M,S,\mu,\nu))$ is a convex function of M for S, μ and ν. Now by lemma 2, we have

IP(M+1,S,μ,ν) + IP(M-1,S,μ,ν) > 2 IP(M,S,μ,ν)

Therefore,

$$f(IP(M,S,\mu,\nu)) \leq f\left(\frac{1}{2} IP(M+1,S,\mu,\nu) + \frac{1}{2} IP(M-1,S,\mu,\nu) \right)$$

(because $f(IP(M,S,\mu,\nu))$ is increasing function)

$$\leq \frac{1}{2} f(IP(M+1,S,\mu,\nu)) + \frac{1}{2} f(IP(M-1,S,\mu,\nu))$$

...(13)

Hence total cost function $Y(M,S,\mu,\nu)$ is a convex function of M.

Lemma 4: The idle probability function IP is decreasing in μ, for given M, S and ν.

Proof: Suppose that $g(\mu)_{(M,S,\nu)} = \dfrac{1}{IP(M,S,\mu,\nu)}$ for fixed M, S and ν,

we have

$$g(\mu)_{(M,S,\nu)} = \frac{S!}{S^S \rho^M}\left[\sum_{n=0}^{S-1}\frac{(S\rho)^n M!}{n!(M-n)!} + \frac{S^S M!}{S!}\sum_{n=S}^{M}\frac{\rho^n}{\prod_{n=S+1}^{M}\left[1+\frac{(n-S)\nu}{S\mu}\right]}\right]$$

…(14)

$$= \frac{S!M!}{S^{S-M}}\sum_{n=0}^{S-1}\frac{\mu^{M-n}}{n!(M-n)!\lambda^{M-n}} + \frac{S^{S+M}M!}{S!}\sum_{n=S}^{M}\frac{\mu^{M-n}}{S^n\lambda^{M-n}\prod_{n=S+1}^{M}\left[1+\frac{(n-S)\nu}{S\mu}\right]}$$

…(15)

Differenating equation (15) with respect to μ, we have.

$$g'(\mu)_{(M,S,\nu)} = \frac{S!M!}{S^{S-M}}\sum_{n=0}^{S-1}\frac{\mu^{M-n-1}}{n!(M-n-1)!\lambda^{M-n}} + \frac{M!}{S!}$$

$$\times\sum_{n=S}^{M}\frac{S^{M+S-n}}{\lambda^{M-n}}\left[\frac{(M-n)\mu^{M-n-1}\psi - \mu^{M-n}\psi\left[\frac{\psi_1'}{\psi_1}+\frac{\psi_2'}{\psi_2}+.............+\frac{\psi_M'}{\psi_M}\right]}{\psi^2}\right]$$

…(16)

where $\psi = \prod_{n=S+1}^{M}\left[1+\frac{(n-S)\nu}{S\mu}\right]$

Thus $g'(\mu)_{(M,S,\nu)} \geq 0$. Therefore $g(\mu)_{(M,S,\nu)}$ is increasing in μ and $IP(M,S,\mu,\nu)$ is decreasing in μ .

Lemma 5: The idle probability function $IP(M,S,\mu,\nu)$ is a convex function of μ for fixed M, S and ν.
Proof: The proof can be done in the similar manner as described in Zeng and Zhang's (1997).

Lemma 6: The function $f(IP(M,S,\mu,\nu))$ is a convex function of μ for given M, S and ν.
Proof: Let us consider that $f(IP(M,S,\mu,\nu))$ is convex in μ for given M, S and ν. Now by chain rule we have

$$\frac{\partial f(IP(M,S,\mu,\nu))}{\partial\mu} = f'\times\frac{\partial IP(M,S,\mu,\nu)}{\partial\mu} \quad \text{…(17)}$$

$$\frac{\partial^2 f(IP(M,S,\mu,\nu))}{\partial\mu^2} = f''\times\left[\frac{\partial IP(M,S,\mu,\nu)}{\partial\mu}\right]^2 + f'\times\frac{\partial^2 IP(M,S,\mu,\nu)}{\partial\mu^2} > 0.$$

…(18)

From the equation (4) we have seen that the cost function $f' > 0$, i.e., it is increasing function and convex $(f'' \geq 0)$.

Corollary 1: There exit one finite μ_M that minimizes the total cost function $Y(M,S,\mu,\nu)$ for given M, S, and ν.

Proof: Consider that $K(\mu) = f(IP(M,S,\mu,\nu)) + h_3(\mu)$ for given M, S, and ν, then the total cost function.

$$Y(M,S,\mu,\nu) = K(\mu) + h_1(M) + h_2(S) + h_4(\nu). \quad \dots(19)$$

Now show that $K(\mu)$ is minimum at $\mu_{f.}$ It has already proved in lemma 6 that $K(\mu)$ is a convex function; so that it must have one minimum. Which show that this minimum value can not be infinity. Since $h_3(\mu)$ is increasing in μ, $\lim_{\mu\to\infty} h_3(\mu) = \infty$.

On the other hand the function IP satisfies $\lim_{\mu\to\infty} IP(M,S,\mu,\nu) = 0$, which gives $\lim_{\mu\to\infty} f(IP(M,S,\mu,\nu)) = f(0) = a$ constant. Thus, Min $K(\mu)$ can be obtained at a finite $\mu_{f.}$ Let us suppose that

$$H(M,S,\mu,\nu) = f(IP(M,S,\mu,\nu)) + h_2(S) + h_3(\mu) + h_4(\nu) \quad \dots(20)$$

Now by lemma 3, we note that $H(M,S,\mu,\nu)$ is convex in M, for fixed S, μ *and* $\nu.$ Also assume that $H(M,S,\mu,\nu) = H(M,M,\mu,\nu)$ *if* $S \geq M$.

Then

$$H(M,S_M,\mu_M,\nu) = \underset{S,\mu,\nu}{Min}\, H(M,S,\mu,\nu) \quad \dots(21)$$

The optimal solution is obtained as given below

$$\underset{M,S,\mu,\nu}{Min}\; Y(M,S,\mu,\nu) = \underset{M}{Min}\, \{H(M,S_M,\mu_M) + h_1(M)\}. \quad \dots(22)$$

It is clear that when the number of floats M, is zero, the function IP is one, i.e. IP(0,0,0) = 1.0. Thus we have H (0,0,0) = f (1.0)

Lemma 7: $\phi(n)$ is convex (concave) if and only if

$$(j-i)[\phi(k)-\phi(j)] \geq (k-j)[\phi(j)-\phi(i)] \forall\, i \leq j \leq k.$$

Proof: The proof of lemma can be seen by equation (22).
(**cf. Shanthikumar and Yao 1987**)

Theorem 1: The optimal solution M^* to equation (22) is given by

Min. $H(M,S_M,\mu_M,\nu_M) + h_1(M)$, *for* $1 \leq M \leq M_0$.

where F_0 is smallest integer which satisfies the following condition

$$\left[\, f(1.0) - H(M_0,S_{M_0},\mu_{M_0},\nu_{M_0}) \,\right] / M_0 \leq h_1(M_0) - h_1(M_0 - 1).$$

Proof: For proof see Zeng and Zhang's (1997).

V. Algorithm

In this section, algorithm is developed for obtaining the optimal values of the number of floats (M^*), the number of servers (S^*), the mean repair rate (μ^*) and reneges exponentially with parameter (ν^*). The minimum total cost of the system $Y(M^*,S^*,\mu^*,\nu^*)$.

Step 1: Initialization

Set $M^* = 1,\quad S^* = 1,\quad \mu^* = 0,\quad \nu^* = 0,\quad M = 1,$

$S = 1,\qquad H(M^*, S^*, \mu^*, \nu^*) = \infty$

Step 2: Set $S_M = 1, \mu_M = 0, \nu_M = 0, H(M, S_M, \mu_M, \nu_M) = \infty$

Step 3: Find μ_M by solving $\frac{\partial H(M,S,\mu,\nu)}{\partial \mu} = 0$

Step 4: Find ν_M by solving $\frac{\partial H(M,S,\mu,\nu)}{\partial \nu} = 0$

Step 5: Compute $H(M,S,\mu_M,\nu_M)$ using equation (22)

Step 6: If $H(M,S,\mu_M,\nu_M) < H(M,S_M,\mu_M,\nu_M)$

Step 6.1 $H(M,S,\mu_M,\nu_M) = H(M,S,\mu_M,\nu_M)$

Step 6.2 $S_M = S$

end if

Step 7: If S < M

Step 7.1 S = S+1; goto Step 2

else Step 7.2 goto Step 8

end if

Step 8: If M satisfies equation (16)

Step 8.1 stop

else Step 8.2 M = M+1; goto Step 2

End If

VI. Discussion

This investigation proposed the maintenances float system consisting of three elements workstations, repair stations and a set of standby components. An algorithm is developed to obtain the optimal values of system parameters. The scope of this paper is in production units, manufacturing system and safety-sustaining systems.

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