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Properties of Right Strongly Prime Ternary Gamma Semirings

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Abstract: *In this paper we introduce the notion of right strongly prime gamma Semiring and study some properties of right strongly prime ternary gamma Semiring.*
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I. Introduction

The notion of ternary Γ-Semiring has been introduced by D. Madhusudhana Rao and M. Sajani Lavanya [5] in the year 2015. The notion of Strongly prime ring has been introduced by Handelman and Lawrence [3]. The notion of TernarySemiring was introduced by T. K. Dutta and S. Kar [1] in the year 2003 as a natural generalization of ternary ring which was introduced by W.G. Lister [4] in 1971. Some earlier works of Ternary Γ-Semiring may be found in [5, 6, 7, 8]. In 2007, T.K.Dutta and M.L. Das [2] introduced and studied right strongly prime Semiring.

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II. Preliminaries

Definition 2.1[5]: Let T and Γ be two additive commutative semigroups. T is said to be a ***Ternary Γ-Semiring*** if there exist a mapping from T ×Γ× T ×Γ× T to T which maps $(x_1, \alpha, x_2, \beta, x_3) \to [x_1\alpha x_2\beta x_3]$ satisfying the conditions:

i) $[[a\alpha b\beta c]\gamma d\delta e] = [a\alpha[b\beta c\gamma d]\delta e] = [a\alpha b\beta[c\gamma d\delta e]]$
ii)$[(a + b)\alpha c\beta d] = [a\alpha c\beta d] + [b\alpha c\beta d]$
iii) $[a\alpha (b + c)\beta d] = [a\alpha b\beta d] + [a\alpha c\beta d]$
iv) $[a\alpha b\beta(c + d)] = [a\alpha b\beta c] + [a\alpha b\beta d]$ for all $a, b, c, d \in$ T and $\alpha, \beta, \gamma, \delta \in \Gamma$.

Obviously, every ternary semiring T is a ternary Γ-semiring. Let T be a ternary semiring and Γ be a commutative ternary semigroup. Define a mapping T ×Γ× T ×Γ× T ⟶ T by $a\alpha b\beta c = abc$ for all $a, b, c \in$ T and $\alpha, \beta \in \Gamma$. Then T is a ternary Γ-semiring.

Definition 2.2[5]: An element 0of a ternary Γ-semiring T is said to be an ***absorbing zero*** of T provided $0 + x = x = x + 0$and $0\alpha a\beta b = a\alpha 0\beta b = a\alpha b\beta 0 = 0 \ \forall\ a, b, x \in$ T and $\alpha, \beta \in \Gamma$.

Note that a Ternary Γ-Semiring may not contain an identity but there are certain ternary Γ-semiring which generate identities in the sense defined below:

Definition 2.3[5]: An element a of a ternary Γ-semiring T is said to be an ***identity*** provided $a\alpha a\beta t = t\alpha a\beta a = a\alpha t\beta a = t \ \forall\ t \in$ T, $\alpha, \beta \in \Gamma$.

Note 2.4[5]: An identity element of a ternary Γ-semiring T is also called as ***unital element.***

Definition 2.5[5]: Let T be ternary Γ-semiring. A non empty subset 'S' is said to be a ***ternary subΓ-semiring*** of T if S is an additive sub-semigroup of T and $a\alpha b\beta c \in$ S for all $a, b, c \in$ S and $\alpha, \beta \in \Gamma$.

Note 2.6[5]: A non-empty subset S of a ternary Γ-semiring T is a ternary subΓ-semiring if and only if S + S ⊆ S and SΓSΓS ⊆ S.

Definition 2.7[5]: A nonempty subset A of a ternary Γ-semiring T is said to be ***left ternary Γ-ideal*** of T if (1) $a, b \in$ A implies $a + b \in$ A. (2) $b, c \in$ T, $a \in$ A, $\alpha, \beta \in \Gamma$ implies $b\alpha c\beta a \in$ A.

Note 2.8[5]: A non-empty subset A of a ternary Γ-semiring T is a left ternaryΓ-ideal of T if and only if A is additive sub-semigroup of T and TΓTΓA ⊆ A.

Definition II.9[5]: A nonempty subset of a ternary Γ-semiring T is said to be a ***lateral ternary Γ-ideal*** of T if (1) $a, b \in A \Rightarrow a + b \in A$. (2) $b, c \in T$, $a \in A$, $\alpha, \beta \in \Gamma \Rightarrow b\alpha a\beta c \in A$.

Note 2.10[5]: A nonempty subset of A of a ternary Γ-semiring T is a lateral ternary Γ-ideal of T if and only if A is additive sub-semigroup of T and TΓAΓT ⊆ A.

Definition 2.11[5]: A nonempty subset A of a ternary Γ-semiring T is a ***right ternary Γ-ideal*** of T if (1) $a, b \in A \Rightarrow a + b \in A$. (2) $b, c \in T$, $a \in A$, $\alpha, \beta \in \Gamma \Rightarrow a\alpha b\beta c \in A$.

Note 2.12[5]: A nonempty subset A of a ternary Γ-semiring T is a right ternary Γ-ideal of T if and only if A is additive sub-semigroup of T and AΓTΓT ⊆ A.

Definition 2.13[5]: A nonempty subset A of a ternary Γ-semiring T is said to be ***ternary Γ-ideal*** of T if
(1) $a, b \in A \Rightarrow a + b \in A$
(2) $b, c \in T$, $a \in A$, $\alpha, \beta \in \Gamma \Rightarrow b\alpha c\beta a \in A$, $b\alpha a\beta c \in A$, $a\alpha b\beta c \in A$.

Note 2.14[5]: A nonempty subset A of a ternary Γ-semiring T is a ternaryΓ-ideal of T if and only if it is left ternaryΓ-ideal, lateral ternaryΓ-ideal and right ternaryΓ-ideal of T.

Definition 2.15[6]: Let T be a ternary Γ-semiring and $a \in T$. Then
(i) principal left ternary **Γ**-ideal generated by *a* is given by

$$<a>_l = \left\{ \sum_{i=1}^{n} r_i\alpha_i t_i\beta_i a + na : r_i, t_i \in T, \alpha_i, \beta_i \in \Gamma \text{ and } n \in z_0^+ \right\}.$$

(*ii*)principal lateral ternary Γ-ideal generated by a is given by

$$< a >_m = \{\sum_{i=1}^{n} r_i\alpha_i a\beta_i t_i + \sum_{j=1}^{n} u_j\gamma_j v_j\delta_j a\varepsilon_j p_j\chi_j q_j + na : r_i, t_i, u_j v_j p_j q_j \in T,$$
$$\alpha_j, \beta_j, \chi_j, \delta_j, \gamma_j, \varepsilon_j \in \Gamma \text{ and } n \in z_0^+\}.$$

(iii) principal right ternary **Γ**-ideal generated by *a* is given by

$$<a>_r = \left\{ \sum_{i=1}^{n} a\alpha_i r_i\beta_i t_i + na : r_i, t_i \in T, \alpha_i, \beta_i \in \Gamma \text{ and } n \in z_0^+ \right\}$$

(iv) principal two sided ternary **Γ**-ideal generated by *a* is given by

$$<a>_t = \left\{ \begin{array}{l} \sum_{i=1}^{n} r_i\alpha_i s_i\beta_i a + \sum_{j=1}^{n} a\alpha_j t_j\beta_j u_j + \sum_{k=1}^{n} l_k\alpha_k m_k\beta_k a\gamma_k p_k\delta_k q_k + na : \\ r_i, s_i, t_j, u_j, l_k m_k, p_k, q_k \in T, \alpha_i, \beta_i, \alpha_j, \beta_j, \alpha_k, \beta_k, \gamma_k, \delta_k \in \Gamma \text{ and } n \in Z_0^+ \end{array} \right\}$$

(v) principal ternary **Γ**-ideal generated by a is given by

$$<a> = \{ \sum_{i=1}^{n} p_i\alpha_i q_i\beta_i a + \sum_{j=1}^{n} a\alpha_j r_j\beta_j s_j + \sum_{k=1}^{n} t_k\alpha_k a\beta_k u_k + \sum_{l=1}^{n} v_l\alpha_l w_l\beta_l a\gamma_l x_l\delta_l y_l + na$$
$$: p_i, q_i, r_j, s_j, t_k, u_k, v_l, w_l, x_l y_l \in T, \alpha_i, \beta_i, \alpha_j, \beta_j, \alpha_k, \beta_k, \alpha_l, \beta_l, \gamma_l, \delta_l \in \Gamma, n \in Z_0^+\}.$$

Where Σ denotes a finite sum and z_0^+ is the set of all positive integer with zero.

Definition 2.16: A ternary Γ-ideal I of a ternary Γ-semiring T is called a ***k-ternary Γ-ideal*** if $a + b \in I; a \in T, b \in I \Rightarrow a \in I$.

Definition 2.17: A proper ternary Γ-ideal P of a ternary Γ-semiring T is said to be a ***prime ternary Γ-ideal*** of T if for any three ternary Γ-ideal A, B, C of T, AΓBΓC ⊆P implies A ⊆ P or B ⊆ P or C ⊆ P.

III. Right Strongly Prime Ternary Γ-Semirings

Definition 3.1: A ternary Γ-semiring T is said to be ***right strongly prime ternary Γ-semiring*** provided for every $0 \neq x$ in T, there exist finite subsets S_1, S_2, S_3 of T such that $x\Gamma S_1\Gamma S_2\Gamma S_3 a = \{0\} \Rightarrow a = 0$ for all $a \in T$.

Example 3.2: Let $T = \{r\alpha i / r \in R, \alpha \in Q, i^2 = -1\}$ and Γ = Q, where R is the set of all real numbers and Q is the set of all rational numbers. Then together with usual binary addition and ternary multiplication, T forms a ternary Γ-semiring. Let $r\alpha i \neq 0 \in T$ and $S = \{r\alpha i\}$ then $r\Gamma i\Gamma S\Gamma S\Gamma a = 0$ implies that a = 0 for all $a \in T$. Hence T is a right strongly prime ternary Γ-semiring.

Theorem 3.3: A ternary Γ-semiring T is right strongly prime ternary Γ-semiring if and only if for every 0 ≠ *x* in T, there exist S of T such that $x\Gamma S\Gamma S\Gamma Sa = \{0\} \Rightarrow a = 0$ for all $a \in T$.

Proof: Suppose T is a right strongly prime ternary Γ-semiring. Let 0 ≠ *x* ∈ T. Then there exist finite subsets S_1, S_2, S_3 of T such that $x\Gamma S_1\Gamma S_2\Gamma S_3 a = \{0\} \Rightarrow a = 0$ for all $a \in T$. Let $S = S_1 \bigcap S_2 \bigcap S_3$. Then S ⊆ S_1, S ⊆ S_2, S ⊆ S_3 and S is finite. Suppose that $x\Gamma S\Gamma S\Gamma Sa = \{0\}$ for all $a \in T$. Then $x\Gamma S\Gamma S\Gamma Sa \subseteq x\Gamma S_1\Gamma S_2\Gamma S_3 a = \{0\}$ for all $a \in T$. Therefore $a = 0$ for all $a \in T$.

Converse part is obvious.

Definition 3.4: A ternary Γ-semiring T is said to be a ***prime ternary Γ-semiring*** provided the zero ternary Γ-ideal {0} is a prime ternary Γ-ideal of T.

Theorem 3.5: Every right strongly prime ternary Γ-semiring is a prime ternary Γ-semiring.

Proof: Suppose that T is a right strongly prime ternary Γ-semiring. Let X, Y, Z be three ternary Γ-ideals of T such that $X\Gamma Y\Gamma Z = \{0\}$. Suppose that $X \neq \{0\}$ and $Y \neq \{0\}$. Since $X \neq \{0\}$, there exists $x(\neq 0) \in X$. Since T is a right strongly prime ternary Γ-semiring, by theorem 3.3, there exists a finite subset S of T such that $x\Gamma S\Gamma S\Gamma S\Gamma y = \{0\} \Rightarrow y = 0$ for all $y \in T$.

Now $x\Gamma S\Gamma S\Gamma S\Gamma(Y\Gamma T\Gamma Z) = (x\Gamma S\Gamma S)\Gamma(S\Gamma Y\Gamma T)\Gamma Z \subseteq (X\Gamma T\Gamma T)\Gamma(T\Gamma Y\Gamma T)\Gamma Z \subseteq X\Gamma Y\Gamma Z = \{0\}$.

This implies that $Y\Gamma T\Gamma Z = \{0\}$. Again, since $Y \neq \{0\}$, there exists $p(\neq 0) \in Y$ and for this $p(\neq 0)$, there exists a finite subset U of T such that $p\Gamma U\Gamma U\Gamma U\Gamma z \subseteq Y\Gamma T\Gamma T\Gamma T\Gamma Z \subseteq Y\Gamma T\Gamma Z = \{0\}$ for $z \in Z$. This implies that *z* = 0. Since *z* is an arbitrary element of Z, we find that Z = {0}. This shows that {0} is a prime ternary Γ-ideal of T and hence T is a prime ternary Γ-semiring.

Theorem 3.6: Let T be a ternary Γ-semiring with identity element e'. Then the following are equivalent:

i) T is right strongly prime ternary Γ-semiring.

ii) if A is a non-zero ternary Γ-ideal of T, there exist finite subsets H of A and G of T such that $H\Gamma G\Gamma y = \{0\} \Rightarrow y = 0 \ \forall \ y \in T$.

iii) If $x(\neq 0) \in T$**, there exist** $t \in T$ **and finite subsets H, G of T such that**

$x\Gamma t\Gamma H\Gamma G\Gamma y = \{0\} \Rightarrow y = 0 \ \forall \ y \in T$

Proof: (i) ⇒(ii): Suppose that T is a right strongly prime ternary Γ-semiring and A be a non-zero ternary Γ-ideal of T. Since A is a non-zero ternary Γ-ideal of T, there exists $x(\neq 0) \in A$. Again since T is right strongly prime, there exists a finite subset G of T such that $x\Gamma G\Gamma G\Gamma G\Gamma y = 0 \Rightarrow y = 0 \ \forall \ y \in T$. Let H = *x*ΓGΓG. Then H = *x*ΓGΓG ⊆AΓGΓG⊆A i.e. H is a finite subset of A. Then there exist finite subsets H of A and G of T such that HΓGΓ*y* = {0} implies that *y* = 0 for all *y*∈T.

(ii) ⇒(iii): Suppose that A is a non-zero ternary Γ-ideal of T, there exist finite subsets H of A and G of T such that $H\Gamma G\Gamma y = \{0\} \Rightarrow y = 0 \ \forall \ y \in T$. Let $a(\neq 0) \in T$. Then <*a*> is a non-zero ternary Γ-ideal of T. Now by condition (ii), there exists finite subsets H of <*a*> and Gof T such that HΓGΓ*y* = {0} implies that *y* = 0 for all *y*∈T. If possible, let *a*ΓTΓT= {0}. Then <*a* >ΓTΓT= {0}. Since HΓGΓ*a*⊆<*a* >ΓTΓT, we have HΓGΓ*a* = {0}. This implies that *a* = 0, a contradiction. Therefore, *a*ΓTΓT ≠{0}. Thus there exist *r, x*∈ T and $\alpha, \beta \in \Gamma$ such that $a\alpha r\beta x \neq 0$. Then $A = < a\alpha r\beta x >$ is a non-zero ternary Γ-ideal of T. By condition (ii), there exists a finite subset I of A and a finite subset J of T such that IΓJΓ*y* = {0} implies that *y* = 0 for all *y*∈T. Since I is a finite subset of A, we find that

$I = \{n\Gamma a\Gamma r\Gamma x + \sum_{i=}^{m} a\Gamma r\Gamma x\Gamma s_i t_i + \sum_{j=1}^{l} p_j\Gamma q_j\Gamma a\Gamma r\Gamma x + \sum_{k=1}^{s} u_k\Gamma a\Gamma r\Gamma x\Gamma v_k + \sum_{w=1}^{t} c_p\Gamma d_p\Gamma a\Gamma r\Gamma x\Gamma e_p f_p\}$; where *n, m, l, s, t*∈ Z_0^+ ; s_i, t_i, p_j, q_j, u_k, v_k, c_p, d_p, e_p, f_p∈ T.

$$= \{n\Gamma a\Gamma r\Gamma x + \sum_{i=}^{m} a\Gamma r\Gamma x\Gamma s_i t_i + \sum_{j=1}^{l} p_j\Gamma q_j\Gamma a\Gamma r\Gamma x + \sum_{k=1}^{s} e\Gamma(u_k\Gamma a\Gamma r\Gamma x\Gamma v_k)\Gamma e + \sum_{w=1}^{t} c_p\Gamma d_p\Gamma a\Gamma r\Gamma x\Gamma e_p f_p\}$$

Let $H = \{x, x\alpha s_i\beta t_i, x\chi v_k\delta e, x\gamma u_p\sigma v_p : i = 1,2,3,.....m; k = 1,2,3,.....s; p = 1,2,3,.....t; m,s,t \in Z_0^+\}$

and let $a\Gamma r\Gamma H\Gamma J\Gamma y = \{0\}$. Then $J\Gamma I\Gamma y = \{0\}$. By condition (ii), we have *y* = 0.

(iii) ⇒(i): Suppose that If $x(\neq 0)\in T$, there exist $t\in T$ and finite subsets H, G of T such that $x\Gamma t\Gamma H\Gamma G\Gamma y=\{0\}\Rightarrow y=0\ \forall\ y\in T$. Let $a(\neq 0)\in T$. Now taking $G_1=\{t\},G_2=H$ and $G_3=G$ we find that there exists finite subset G_1,G_2,G_3 of T such that $a\Gamma G_1\Gamma G_2\Gamma G_3\Gamma y=\{0\}\Rightarrow y=0$. Hence T is right strongly prime ternary Γ-semiring.

Example 3.7: Let T and Γ be the set of all 2 × 2 matrices over Q, the set of rational numbers Define A + B = usual addition and A*α*B*β*C = usual matrix product of A, *α*, B, *β*, C; for all A, B, C ∈ T and for all *α*, *β*∈ Γ. Then T is a ternary Γ-semiring. Let I be a non-zero ternary Γ-ideal of T. Then I have a non-zero element, say $(a_{ij})_{2\times 2}$. Then $(a_{ij})_{2\times 2}$ has at least one non-zero element, say a_{rs}. Since I is a ternary Γ-ideal of T, $E_{11}\alpha_{11}E_{1r}\alpha_{1r}(a_{ij})_{2\times 2}(\beta_{ij})_{2\times 2}E_{s1}\alpha_{s1}E_{11}\in I$, where E_{rs},α_{rs} are the 2×2 matrices whose $(r,s)^{th}$ element is 1 and all others elements are zero. This shows that I has an element, say f_1 whose $(1,1)^{th}$ element is non-zero and all other elements are zero. Similarly, we can get an element, say f_2 in I whose $(2,2)^{th}$ element is non-zero and all others elements are zero. Let $f_1=\left\{\begin{pmatrix} a & 0\\ 0 & 0\end{pmatrix}/a\in Q\right\}$, $f_2=\left\{\begin{pmatrix} 0 & 0\\ 0 & b\end{pmatrix}/b\in Q\right\}$. Let F = $\{f_1,f_2\}$ and G = $\{g_1,g_2\}$ where $g_1=\left\{\begin{pmatrix} c & 0\\ 0 & 0\end{pmatrix}/c\in Q\right\}$, $g_2=\left\{\begin{pmatrix} 0 & 0\\ 0 & d\end{pmatrix}/d\in Q\right\}$. Suppose that $F\Gamma G\Gamma z=0$, where $z=\begin{pmatrix} a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix}\in T$. Then $f_1\Gamma g_1\Gamma z=f_2\Gamma g_2\Gamma z=f_2\Gamma g_1\Gamma z=0$. This implies that $a\alpha c\beta a_{11}=a\alpha c\beta a_{12}=b\gamma d\delta a_{21}=b\gamma d\delta a_{22}=0$. Since $a,b,c,d\in Q$ and $\alpha,\beta,\gamma,\delta\in\Gamma$, we must have $a_{11}=a_{12}=a_{21}=a_{22}=0$. Consequently, *z* = 0 and hence T is a right strongly prime ternary Γ-semiring.

Definition 3.8: Let X be a non-empty subset of a ternary Γ-semiring T. Then the right Γ-annihilator of X with respect to Y(⊆T) is T, denoted by $r_a(X,Y)$ and is denoted by $r_a(X,Y)=\{t\in T\ /\ X\Gamma Y\Gamma t=\{0\}\}$.

Theorem 3.9: The right annihilator of a subset X with respect to a subset Y of a ternary -semiringT is a right ternary -ideal of T.

Proof: We note that $0\in r_a(X,Y)$, Since $X\Gamma Y\Gamma 0=\{0\}$. So $r_a(X,Y)$ is non-empty. Let *s*, *t*∈ $r_a(X,Y)$. Then $X\Gamma Y\Gamma s=X\Gamma Y\Gamma t=\{0\}$. Now $X\Gamma Y\Gamma(s+t)=X\Gamma Y\Gamma s+X\Gamma Y\Gamma t=\{0\}+\{0\}=\{0\}$ implies that $s+t\in r_a(X,Y)$. Again, $X\Gamma Y\Gamma(s\Gamma x\Gamma y)=(X\Gamma Y\Gamma s)\Gamma x\Gamma y=0\Gamma x\Gamma y=0$ for all $x,y\in T$ implies that $s\Gamma x\Gamma y\subseteq r_a(X,Y)$. Hence $r_a(X,Y)$ is a right ternary Γ-ideal of T.

Theorem 3.10: The right annihilator of a subset X with respect to a right ternary Γ-ideal B of a ternary Γ-semiring T with identity element *e* is a ternary Γ-ideal of T.

Proof: from theorem 3.9, it follows that $r_a(X,Y)$ is a right ternary **Γ**-ideal of T. Therefore, it is enough to show that $r_a(X,Y)$ is a left ternary Γ-ideal as well as right ternary Γ-ideal of T. Let $s\in r_a(X,Y)$. Then $X\Gamma Y\Gamma s=\{0\}$.

Now since Y is a right ternary **Γ**-ideal of T, we find that

$X\Gamma Y\Gamma(x\Gamma y\Gamma s)=X\Gamma(Y\Gamma x\Gamma y)\Gamma s\subseteq X\Gamma(Y\Gamma T\Gamma T)\Gamma s\subseteq X\Gamma Y\Gamma s=\{0\}$ for all $x,y\in T$ implies that $x\Gamma y\Gamma s\subseteq r_a(X,Y)$. This implies that $r_a(X,Y)$ is a left ternary **Γ**-ideal of T.

Again, since Y is a right ternary **Γ**-ideal of T, we find that

$X\Gamma Y\Gamma(x\Gamma s\Gamma y)=X\Gamma Y\Gamma(e\Gamma x\Gamma e)\Gamma(e\Gamma s\Gamma e)\Gamma y=X\Gamma(Y\Gamma e\Gamma x)\Gamma(e\Gamma e\Gamma s\Gamma e\Gamma y)\subseteq X\Gamma(Y\Gamma T\Gamma T)\Gamma(e\Gamma e\Gamma s\Gamma y)\subseteq$
$X\Gamma Y(e\Gamma e\Gamma s\Gamma y)=X\Gamma(Y\Gamma e\Gamma e)\Gamma(s\Gamma e\Gamma y)\subseteq X\Gamma(Y\Gamma T\Gamma T)\Gamma(s\Gamma e\Gamma y)\subseteq X\Gamma Y\Gamma(s\Gamma e\Gamma y)=$
$(X\Gamma Y\Gamma s)\Gamma e\Gamma y=\{0\}\Gamma e\Gamma y=\{0\}$ for all $x,y\in T$ implies that $x\Gamma s\Gamma y\subseteq r_a(X,Y)$. This implies that $r_a(X,Y)$ is a lateral ternary **Γ**-ideal of T. Therefore, $r_a(X,Y)$ is a ternary **Γ**-ideal of T.

Definition 3.11: Let A be a proper ternary Γ-ideal of a ternary Γ-semiring T. Then the congruence of T, denoted by ρ_A and defined by $s\rho_A s'$ if and only if $s+a_1=s'+a_2$ for some $a_1,a_2\in A$, is called the ***Bourne congruence*** on T defined by the ternary Γ-ideal A.

We denote the Bourne congruence (ρ_A) class of an element *r* of T by *r*/ ρ_A or simply by *r*/A and denote the set of all such congruence classes of T by T/ ρ_A or simply by T/A.

Definition 3.12: For any proper ternary Γ-ideal A of a ternary Γ-semiring T if the Bourne congruence ρ_A, defined by A, is proper i.e. $0/A \neq T/A$, then we define the addition and ternary multiplication of T/A by *a/A* + *b/A* = (*a* + *b*)/A and (*a/A*)Γ(*b/A*)Γ(*c/A*) = (*a*Γ*b*Γ*c*)/A for all $a, b, c \in T$.

With reference these two operations T/A forms a ternary Γ-semiring and is called the Bourne factor ternary Γ-semiring or simply the factor ternary Γ-semiring.

Definition 3.13: A ternary Γ-ideal A of a ternary Γ-semiring T is called a ***right strongly prime ternary Γ-ideal*** if the factor ternary Γ-semiring T/A is right strongly prime.

Definition 3.14: A ternary Γ-ideal A of a ternary Γ-semiring T is said to be ***k-ternary* Γ*-ideal*** or ***subtractive*** provided for any two elements *a*∈ A and *x*∈ T such that *a* + *x*∈ A ⇒*x*∈ A.

Theorem 3.15: Let Q be a k-ternary Γ-ideal of a ternary Γ-semiring T. Then Q is a right strongly prime ternary Γ-ideal of T if and only if for every ternary Γ-ideal I of T not contained in Q, there exist finite subsets H of I and G of T such that $H\Gamma G\Gamma y \subseteq Q$ implies that $y \in Q$ for all $y \in T$.

Proof: Let Q be a right strongly prime ternary **Γ**-ideal of T**.** Then the factor ternary Γ-semiring T/Q is right strongly prime. Let I be a ternary Γ-ideal of T not contained in Q. Then (I+Q)/Q is a non-zero ternary Γ-ideal of the right strongly prime factor ternary Γ-semiring T/Q.

Thus there exist finite subsets $J = \{(i_1 + q_1)/Q, (\mathrm{i}_2 + q_2)/Q, ...(i_n + q_n)/Q\}$ of (I + Q)/Q and G/Q of T/Q such that JΓ(G/Q)Γ(*y*/Q)=0/Q implies that *y/Q* = *0/Q* for all $y/Q \in T/Q$. Let H = $\{i_1, i_2 ......i_n\}$. Then H is a finite subset of I. Let $i \in H$. Then $i/Q = ((i+q)/Q$, Since $iq_Q(i+q)$ as $i + q = (i+q) + 0$, where $q \in Q$. Let $H\Gamma G\Gamma y \subseteq Q$.

Then $(H/Q)\Gamma(G/Q)\Gamma(y/Q) = 0/Q$ i.e. $\Gamma(G/Q)\Gamma(y/Q) = 0/Q \Rightarrow y/Q = 0/Q \quad \forall \ y/Q \in T/Q$.

Since Q is a *k*-ternary Γ-ideal of T, $y \in Q$ for all $y \in T$.

Conversely, let I/Q be a non-zero ternary Γ-ideal of T/Q. Then I is a ternary Γ-ideal of T not contained in Q. Then by the statement there exist finite subsets H and G of I and T respectively such that $H\Gamma G\Gamma y \subseteq Q$ implies that $y \in Q$ for all $y \in T$. Since H is a finite subset of I, H/Q is a finite subset of I/Q. Let $(H/Q)\Gamma(G/Q)\Gamma(y/Q) = 0/Q$. Then $H\Gamma G\Gamma y \subseteq Q$ and hence *y*∈ Q i.e. $y/Q = 0/Q$. Thus T/Q is right strongly prime ternary Γ-semiring. Hence Q is a right strongly prime ternary Γ-ideal of T.

Corollary 3.16: A *k*-ternary □-ideal A of a ternary □-semiring T is a right strongly prime ternary □-idealif for $a \notin A$, there exist finite subsets H of <*a*> and G of T such that $H\Gamma G\Gamma b \subseteq A$ implies the $b \in A$.

Proof: Since $a \notin A$, <*a*> is not properly contained in A. Then by above theorem 3.15, there exist finite subsets H and G of <*a*> and T respectively such that $H\Gamma G\Gamma b \subseteq A$ implies that $b \in A$.

Definition 3.17: A nonempty subset A of a ternary Γ-semiring T is said to be an ***m*-system** provided for any *a*, *b*, *c*∈ A implies that $T'\Gamma T'\Gamma a\Gamma T \ \Gamma T'\Gamma b\Gamma T'\Gamma T'\Gamma c\Gamma T'\Gamma T' \cap A \neq \varnothing$.

We now prove a necessary and sufficient condition for a ternary Γ-ideal to be a prime ternary Γ-ideal in a ternary Γ-semiring.

Theorem 3.18: A ternary □-ideal A of a ternary □-semiring T is a prime ternary □-ideal of T if and only if T\A is an *m*-system of T or empty.

Proof: Suppose that A is a prime ternary Γ-ideal of a ternary Γ-semiring T and T\A $\neq \varnothing$.

Let *a*, *b*, *c*∈ T\A. Then *a*∉ A, *b*∉ A and *c*∉ A.

Suppose if possible $T'\Gamma T'\Gamma a\Gamma T \ \Gamma T'\Gamma b\Gamma T'\Gamma T'\Gamma c\Gamma T'\Gamma T' \cap$ T\A $= \varnothing$

$\Rightarrow T'\Gamma T'\Gamma a\Gamma T \ \Gamma T'\Gamma b\Gamma T'\Gamma T'\Gamma c\Gamma T'\Gamma T' \subseteq$ A. Since A is prime, either *a*∈ A or *b*∈ A or *c*∈ A.

It is a contradiction. Therefore, $T'\Gamma T'\Gamma a\Gamma T \ \Gamma T'\Gamma b\Gamma T'\Gamma T'\Gamma c\Gamma T'\Gamma T' \cap$ T\A $\neq \varnothing$.

Hence T\A is an *m*-system.

Conversely suppose that T\A is either an *m*-system of T or T\A = $\varnothing$.

If T\A = $\varnothing$, then T = A and hence A is a prime ternary Γ-ideal of T.

Assume that T\A is an *m*-system of T. Let *a*, *b*, *c*∈ T and <*a*>Γ< *b*>Γ<*c*>⊆ A.

Suppose if possible *a*∉ A, *b*∉ A and *c*∉ A. Then *a*, *b*, *c*∈ T\A. Sine T\A is an *m*-system,

$\Rightarrow T'\Gamma T'\Gamma a\Gamma T \ \Gamma T'\Gamma b\Gamma T'\Gamma T'\Gamma c\Gamma T'\Gamma T' \cap$ T\A $\neq \varnothing \Longrightarrow T'\Gamma T'\Gamma a\Gamma T \ \Gamma T'\Gamma b\Gamma T'\Gamma T'\Gamma c\Gamma T'\Gamma T' \nsubseteq$A

$\Rightarrow$ $<a>\Gamma<b>\Gamma<c>\not\subseteq$A. It is a contradiction.
Therefore,$a\in$ A or $b\in$ A or $c\in$ A. Hence A is a ternary Γ-ideal of T.

A similar type of result we obtain for right strongly prime ternary Γ-semiring. For this we introduce the following notion.

Definition 3.19: A non-empty subset G of a ternary Γ-semiring T is called an ***sp-system*** if for any $g\in G$ there is a finite subset $F_1\subseteq<g>$ and a finite subset F_2 of T such that $F_1\Gamma F_2\Gamma z\bigcap G\neq\emptyset$ for all $z\in G$.

Theorem 3.20: A proper ternary Γ-ideal I of a ternary Γ-semiring T is a right strongly prime if and only if T\I is an sp-system.
Proof: Suppose that I is a right strongly prime ternary **Γ**-ideal of a ternary Γ-semiring T.Let $g\in T\setminus I$. Then $g\notin I$. Therefore, there exist finite subsets H of <*g*> and G of T such that $H\Gamma G\Gamma b\subseteq I$ implies that $b\in I$, by using corollary 3.16, this implies that $H\Gamma G\Gamma z\bigcap(T/I)\neq\emptyset$ for all $z\in(T\setminus I)$. Hence $T\setminus I$ is an sp-system.

Conversely, suppose that $T\setminus I$ is an sp-system. Let $a\notin I$. Then $a\in T\setminus I$. Therefore, there exist a finite subset H of <*a*> and G of T such that $H\Gamma G\Gamma z\bigcap(T\setminus I)\neq\emptyset$ for all $z\in T\setminus I$. Let $H\Gamma G\Gamma b\subseteq I$. Then $H\Gamma G\Gamma b\bigcap(T\setminus I)=\emptyset$. If possible, let $b\notin I$. Then $b\in T\setminus I$ which implies that $H\Gamma G\Gamma b\bigcap(T\setminus I)\neq\emptyset$, a contradiction. Hence $b\in I$ and therefore I is a right strongly prime ternary Γ-ideal of T.

Definition 3.21: A pair of subsets (G, H), where H is a ternary Γ-ideal of a ternary Γ-semiring T and G is a non-empty subset of T is said to be a ***supper sp-system***of T provided $G\bigcap H$ contain no non-zero elements of T and for any $g\in G$, there exist a finite subset F of <*g*> and a finite subset I of T such that $F\Gamma I\Gamma z\bigcap G\neq\emptyset$ for all $z\notin H$.

Theorem 3.22: A ternary Γ-ideal I of a ternary Γ-semiring T is right strongly prime if and only if (T\I, I) is a supper sp-system of T.
Proof: Let I be a right strongly prime ternary Γ-ideal of a ternary Γ-semiring T. So T\I is an sp-system by theorem 3.20. Thus for any $g\in T\setminus I$, there exists a finite subset F of <*g*> and a finite subset F' of T such that $F\Gamma F'\Gamma z\bigcap(T\setminus I)\neq\emptyset$ for all $z\in I$. Also $T\setminus I\bigcap I$ contains no non-zero elements T. Thus the pair (T\I, I) is a upper sp-system of T. Converse follows from the definition.

Theorem 3.23: For any ternary Γ-semiring T, $SP(T)=\{x\in T$: whenever (G, H) is a super sp-system for some ternary Γ-ideal H of T and $x\in G$, then $0\in G\}$.
Proof: Let $x\in SP(T)$, if possible, let (G, H) be a super sp-system with $x\in G$ and $0\notin G$. Then $G\bigcap H=\emptyset$. By Zorn's lemma, choose a ternary Γ-ideal Q with $H\subseteq Q$ and Q is a maximal with respect to $G\bigcap Q=\emptyset$. We now prove that Q is a right strongly prime ternary Γ-ideal of T. Let $a\notin Q$. Then there exists $g\in G$ such that $<g>\subseteq Q+<a>$. Since (G, H) is a supper sp-system, there exists a finite subset $F=\{f_1,f_2,.....f_3\}\subseteq<g>$ and a finite subset F' of T such that $F'\Gamma z\bigcap G\neq\emptyset$ for all $z\notin H$.(1)

Since $F\subseteq<g>\subseteq Q+<a>$ each f_i is of the form $f_i=q_i+a$ for some $q_i\in Q$ and $a_i\in<a>$. Let $A=\{a_1,a_2,.....a_k\}$ then $F\subseteq<a>$. Let $z\in T$ be such that $F'\Gamma A\Gamma z\subseteq Q$. Now if $z\notin Q$, then $z\notin H$ so from (1) we have $F\Gamma F'\Gamma z\bigcap G\neq\emptyset$; but

$f_i\Gamma F'\Gamma z=(q_i+a_i)\Gamma F'\Gamma z=q_i\Gamma F'\Gamma z+a_i\Gamma F'\Gamma z\subseteq Q+A\Gamma F'z\subseteq Q+Q=Q$ for all $i\in\{1,2,.....k\}$. So $F\Gamma F'\Gamma z\subseteq Q$. Hence $G\bigcap Q=\emptyset$, a contradiction. Hence $z\in Q$. So Q is a right strongly prime ternary Γ-ideal of T. Now as SP(T) ⊆Q, so $x\in Q$. But by assumption $x\in G$, a contradiction. Hence $0\in G$.

Conversely, suppose that K= $\{x\in T$: whenever (G, H) is a super sp-system for some ternary Γ-ideal H of T and $x\in G$, then $0\in G\}$. Let $x\in K$. If possible let $x\notin SP(T)$. Then there exist a right strongly prime ternary Γ-ideal I of T such that $x\notin I$. Then (T\I, I) is a supper sp-system, where $x\in T\setminus I$ but $0\notin T\setminus I$, a contradiction. Hence the converse part is proved.

IV. Conclusion

In this paper mainly we studied about right strongly prime ternary Γ-semiring.

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