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Numerical Solution of Convection Diffusion Problem Using Non-Standard Finite Difference Method and Comparison With Standard Finite Difference Methods

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Abstract: *In this article we found the numerical solution of singularly perturbed one dimensional convection diffusion equation using Non-Standard finite difference method by following the Mickens Rules. To compare the results with the known methods we also found solution of one dimensional convection diffusion equation using standard backward and central finite difference schemes. The work has been illustrated through the examples for different values of small parameter ϵ, with different step lengths. The approximate solution is compared with the solution obtained by standard finite difference methods and exact solution. It has been observed that the approximate solution is an excellent agreement with exact solution. Low absolute error indicates that our numerical method is effective for solving perturbation problems.*

Keywords:*Convection diffusion problem; Non-standard finite difference method; Perturbation problem; Absolute error.*

I. Introduction

The non-standard finite difference approach was initiated almost three decades ago byMickens [1]. An important observation fromthis pioneer researcher [2] was that the traditional procedures in the design of finite difference schemes have to be suitably changed by nonstandard procedures to avoid instabilityand chaotic behavior. Subsequently, a remarkable effort was made to designnonstandard finite difference approach for a variety of ordinary and partial differentialequations of interest in applications [3]. One of the culminating points of this effortwas fromthe author's point of view, the identification byMickens's five rules for the constructionof non-standard finite difference schemes as more reliable numerical methods.Since the publication ofMickens's book, the nonstandard finite difference approach wasextensively been applied to differential models originating problems from Engineering, Physics, Biology, Chemistry, etc. In all these contributions of different areas ofapplication, the non-standard finite difference scheme have shown a great potential inreplicating the essential physical properties of the exact solutions of the involved differentialmodels.Despite the success of the new approach, Mickens's himself acknowledgethat the general rules for constructing the nonstandard finite difference scheme are notprecisely known at present time. Consequently, there exists a certain level of ambiguityin the practical implementation of non-standard procedures to the formulation offinite difference schemes for differential equations.

Singularly perturbed differential equations is one of the area of increasing interest in theapplied mathematics and engineering since recent years.In this type of problems, thereare regions where the solution varies very rapidly known as boundary layers and the regionwhere the solution varies uniformly known as the outer region. Standard finite difference or finite element methods are applied on the singularly perturbeddifferential equation on uniform mesh give unsatisfactory result as $\epsilon \to 0$ [4]. Since for most application problems, finding the analytical solutionof singularly perturbed one dimensional convection diffusion problems is difficult even impossible, so we are applying the efficient numerical technique, the non-standard finite difference scheme to singularly perturbed one dimensional convection diffusion problem for numerical simulations.

Kadalbajooand Vikasgupta [5] presented a survey on numerical methods for solving singularlyperturbed problems. Spline approximation method for solving self-adjoint singular perturbationproblems on non-uniform grids have been investigated by Kadalbajoo andK.C. Patidar [6]. Reddy and Chakravarthy [7] constructed an exponentially fitted finitedifference method for solving singularly perturbed two-point boundary value problems. Ravikanth [8] has given numerical treatment of singular boundary valueproblems.Chawla and Katti [9] employed finite difference method for a class of singulartwo-point BVPs. A class of BVPs has been solved by Rama Chandra Rao [10] usingnumerical integration.ParchaKalyani [11] has employed numerical integration method to solve perturbation problems, by reducing it to a differential equation of first order with a small deviating argument.Ravikanthand Reddy [12] dealt with cubic spine for a class of singulartwo-point boundary valueproblems.Adomian et al. [13] solved a generalizationof Airy's equation by decomposition method. For the numerical solution of singularlyperturbed two-point boundary value problems a numerical algorithm based on optimalmonitor function for mesh selection has been developed by Capper and Cash [14]. Rashidinia.et.al.[15] have developed quintic non polynomial spine functions to obtainapproximate solutions of BVPs with singular perturbation. Linand Cheng [16] consideredspline scaling functions and wavelets for singularly perturbed problems arising inbiology and discussed their convergence. A conventional approach for the solution of fifth order boundary value problems using sixth degree spline functions has been given by ParchaKalyani et al. [17].

In this study we applied the non-standard finite difference scheme by applying Mickens Rules on singularlyperturbed one dimensional convection diffusion problems.
The governing equation of the problem is given by

$$Lu = -\epsilon u'' + a(x)u' = r(x)\,,\ 0 < x < 1 \qquad (1)$$
$$u(0) = \alpha\,, u(1) = \beta\,, a(x) > a_o > 0$$

Where$\epsilon$is a small parameter $0 < \epsilon << 1$, is used to measure the relative amount of diffusionto convection.We also solved one dimensional convection diffusion problems with standard finite difference schemes and compared the solutions with exact solution.We simulated the solution of the standard and nonstandard finite difference and theexact solution with the same window.The errors obtained from the standard and non-standard schemes are plotted on the same windowand shown that the non-standard finite difference scheme is more powerfulthan the standard finite difference scheme in solving the one dimensional convectiondiffusion problems.

II. Approximation Of Convection Diffusion Problem With Non - Standard Finite Difference Scheme

In this section we apply the non-standard modeling rulesof Mickens to find the solution of the one dimensional convection diffusion equation by constructing the appropriate denominator function.
Consider the one dimensional convection diffusion equation (1) i.e.

$$-\epsilon u'' + a(x)u' = r(x)$$

assume a(x) = 1, then the equation (1) becomes

$$-\epsilon u'' + u' = r(x)(2)$$

The discretization is as follows

$$-\epsilon\frac{u_{i+1}-2u_i+u_{i-1}}{h^2} + a\frac{u_{i+1}-u_i}{h} = r(x_i)\ (3)$$

As per the rules of Mickens the denominator of the highest derivative (h^2) of the discretized equation (3) must be replaced by the functionϕ (h), where

$$\phi(h) = \frac{\left[\exp\left(\frac{-h}{\epsilon}\right) - 1\right]h}{\frac{-1}{\epsilon}} \quad , \quad 0<(h)<1.$$

Equation (3) becomes

$$-\epsilon\frac{u_{i+1}-2u_i+u_{i-1}}{\Phi(h)} + a\frac{u_{i+1}-u_i}{h} = r(x_i) \qquad (4)$$

$$-\epsilon\frac{u_{i+1}-2u_i+u_{i-1}}{\frac{\left[\exp\left(\frac{-h}{\epsilon}\right)-1\right]h}{\frac{-1}{\epsilon}}} + a\frac{u_{i+1}-u_i}{h} = r(x_i)\ (5)$$

For a = 1 we have

$$\frac{u_{i+1}-2u_i+u_{i-1}}{\left[\exp\left(\frac{-h}{\epsilon}\right)-1\right]h} + \frac{u_{i+1}-u_i}{h} = r(x_i) \qquad (6)$$

Simplifying, we get

$$u_{i+1} - 2u_i + u_{i-1} + \left(\exp\left(\frac{-h}{\epsilon}\right) - 1\right)(u_{i+1} - u_i) = r(x)\left(\left[\exp\left(\frac{-h}{\epsilon}\right) - 1\right]h\right)(7)$$

Arranging the coefficients of the same indices, we get

$$\left(\exp\left(\frac{-h}{\epsilon}\right)\right)u_{i+1} - \left(1 + \exp\left(\frac{-h}{\epsilon}\right)\right)u_i + u_{i-1} = 0(8)$$

Now we find the roots of equation (8) by considering homogeneous case $u^i = r^i$

$$\left(\exp\left(\frac{-h}{\epsilon}\right)\right)r^{i+1} - \left(1 + \exp\left(\frac{-h}{\epsilon}\right)\right)r^i + r^{i-1} = 0 \qquad (9)$$

$\left(\exp\left(\frac{-h}{\epsilon}\right)\right) r^2 - \left(1 + \exp\left(\frac{-h}{\epsilon}\right)\right) r + 1 = 0$(10)

$$r_{1,2} = \frac{\left(1+\exp\left(\frac{-h}{\epsilon}\right)\right) \pm \sqrt{\left(1+\exp\left(\frac{-h}{\epsilon}\right)\right)^2 - 4\exp\left(\frac{-h}{\epsilon}\right)}}{2\exp\left(\frac{-h}{\epsilon}\right)} \qquad (11)$$

$$\Rightarrow r_1 = 1 \; and r_2 = \frac{1}{\exp\left(\frac{-h}{\epsilon}\right)}$$

This indicates that for all values of h and ϵ, $r_2$is always positive so that it is stable and we also observed that for all values of ϵthere will not be any oscillations.

III. Approximation of convection diffusion problem with standard finite difference schemes

In this section, we present and analyze central-difference and back ward-difference approximations for convection diffusion problem. We simulate some numerical results for different values of small parameter ϵ and discuss the behavior of the numerical solution.

3.1Approximationofthe Convection Term by Central Difference Scheme

We study one dimensional convection diffusion problem (1 and 2) with central difference method. i.e.,

$$-\epsilon u'' + a(x)u' = r(x)$$

We approximate the diffusion term with second order central difference operator and convective term by central-difference operator as described below

$$-\epsilon \frac{u_{i+1}-2u_i+u_i}{h^2} + a\frac{u_{i+1}-u_i}{2h} = r(x_i)(12)$$

Rearranging the coefficients of like terms gives

$$\left(\frac{\epsilon}{h^2}+\frac{a}{h}\right)u_{i+1} + \left(\frac{2\epsilon}{h^2}\right)u_i + \left(\frac{-\epsilon}{h^2}-\frac{a}{h^2}\right)u_{i-1} = r(x_i)(13)$$

$$\left(\frac{-\epsilon+ah}{h^2}\right)u_{i+1} + \left(\frac{2\epsilon}{h^2}\right)u_i + \left(\frac{-\epsilon-ah}{h^2}\right)u_{i-1} = r(x_i) \qquad (14)$$

$$\text{Let } a_1 = \frac{-\epsilon}{h^2}+\frac{a}{2h}, \; b_1 = \left(\frac{-\epsilon}{h^2}\right), \; c_1 = \frac{-\epsilon}{h^2}-\frac{a}{2h} \qquad (15)$$

Now let us see the solution of equation (14), by considering homogeneous case $u^i = r^i$

$a_1 r^{i+1} - 2b_1 r^i + c_1 r^{i-1} = 0$ (16)

$$a_1 r^2 - 2b_1 r + 1 = 0(17)$$

The characteristic roots of equation (17) can be obtained as

$$r_{1,2} = \frac{2b_1 \pm \sqrt{4{b_1}^2 - 4a_1c_1}}{2a_1} \quad \rightarrow r_{1,2} = \frac{b_1 \pm \sqrt{{b_1}^2 - a_1c_1}}{a_1}$$

From equation (15) we have

$${b_1}^2 - a_1c_1 = \left(\frac{a_1+c_1}{2}\right)^2 - a_1c_1 = \frac{{a_1}^2 + 2a_1c_1 + {c_1}^2 - 4a_1c_1}{4} = \left(\frac{a_1-c_1}{2}\right)^2$$

$$r_{1,2} = b_1 \pm \left(\frac{a_1-c_1}{a_1}\right) \Rightarrow r_1 = 1 \; and r_2 = \frac{c_1}{a_1}$$

$$r_2 = \frac{c_1}{a_1} = \frac{\frac{-2\epsilon - ah}{2h^2}}{\frac{-2\epsilon + ah}{2h^2}} = \frac{-2\epsilon - ah}{-2\epsilon + ah} = \frac{-2\epsilon - \frac{2\epsilon ah}{2\epsilon}}{-2\epsilon + \frac{2\epsilon ah}{2\epsilon}} \qquad \text{(From (15))}$$

Let $\alpha = \frac{ah}{2\epsilon}$then we have $r_2 = \frac{-2\epsilon - 2\epsilon\alpha}{-2\epsilon + 2\epsilon\alpha} = \frac{1+\alpha}{1-\alpha}$

This result shows that if **α**< 1 the approximate solution to be consistent but if **α**>1 the numerical solution oscillates this is because when we take **α**>1,r_2 will be negative.

3.2 Approximation of the convective term by back ward difference scheme

We study one dimensional convection diffusion problem (1 and 2) with backward difference method. i.e.

$$-\epsilon u'' + a(x)u' = r(x)$$

In this case the diffusive term is discretized with second order central difference whereas the convective term of the equation discretized using first order back ward difference.

$$-\epsilon \frac{u_{i+1} - 2u_i + u_i}{h^2} + a\frac{u_{i-}u_i}{h} = r(x_i)$$

$$\left(\frac{-\epsilon}{h^2}\right)u_{i+1} + \left(\frac{2\epsilon}{h^2} + \frac{a}{h}\right)u_i + \left(\frac{-\epsilon}{h^2} - \frac{a}{h}\right)u_{i-1} = r(x_i)(18)$$

Let $a_2 = \frac{\epsilon}{h^2}, b_2 = \frac{2\epsilon}{h^2} + \frac{a}{h}, c_2 = \frac{-\epsilon}{h^2} - \frac{a}{h}(19)$

Consider the homogeneous case of equation (18)$u^i = r^i$, then we have the following equation.

$$-a_2 r^{i+1} + b_2 r^i + c_2 r^{i-1} = 0 \qquad (20)$$

$-a_2 r^2 + b_2 r + 1 = 0(21)$

the characteristic roots of this equation are

$$r_{1,2} = \frac{-b_2 \pm \sqrt{{b_2}^2 - 4a_2c_2}}{-2a_2}$$

From equation (19) we have

$${b_2}^2 + 4a_2c_2 = (a_2 - c_2)^2 + 4a_2c_2 = {a_2}^2 + 2a_2c_2 + {c_2}^2 = (a_{2+}c_2)^2$$

then we have$r_{1,2} = \frac{-b_2 \pm (a_2 + c_2)}{-2a_2}$

So the roots of the homogeneous case of the equation are

$$r_1 = 1\ and r_2 = \frac{-c_2}{a_2} = \frac{\epsilon + ah}{\epsilon} = \frac{\epsilon + 2\alpha\epsilon}{\epsilon} = 1 + \boldsymbol{\alpha}$$

From this we have that if **α**>1 or **α**< 1, r_2 will always have positive results and did not observeoscillations. Therefore the back ward approximation of the convective term of thegiven convection diffusion equation is more stable than the central difference approximation ofthe convective term of the one dimensional convection diffusion problem.

IV. Numerical illustrations

In this section we consider two examples of one dimensional singularly perturbed convection diffusion problems. Their numerical solution and absolute errors are given for different values of small parameter ϵ. The approximate solution obtained by non-standard finite difference, standard finite difference, exact solutions and absolute errors at the grid points are summarized in tabular form. The approximate solution and exact solution have been shown graphically. Further the comparison of numerical solutions obtained by SFDM, NSFD with exact solutionand also absolute errors at different step lengthshas been shown graphically.

4.1Numerical solution of convection diffusion problem with non-standard finite difference scheme

Example 1.

Consider the singular perturbed convection diffusion problem

$$-\epsilon u'' + au' = 1 \ on[0,1], u(0) = 0, u(1) = 0 \qquad (22)$$

The exact solution of (22) is

$u(x) = x - \frac{\exp\left(-\frac{1-x}{\epsilon}\right) - \exp(\frac{-1}{\epsilon})}{1 - \exp(\frac{-1}{\epsilon})}(23)$

Approximating the derivatives with finite differences

$$-\epsilon \frac{u_{i+1} - 2u_i + u_{i-1}}{\frac{\left[\exp\left(\frac{-h}{\epsilon}\right) - 1\right]h}{\frac{-1}{\epsilon}}} + a\frac{u_{i+1} - u_i}{h} = 1 \qquad (24)$$

$$\frac{u_{i+1} - 2u_i + u_{i-1}}{\left[\exp\left(\frac{-h}{\epsilon}\right) - 1\right]h} + a\frac{u_{i+1} - u_i}{h} = 1(25)$$

$$u_{i+1} - 2u_i + u_{i-1} + a\left(\exp\left(\frac{-h}{\epsilon}\right) - 1\right)(u_{i+1} - u_i) = \left[\exp\left(\frac{-h}{\epsilon}\right) - 1\right]h \quad (26)$$

Arranging the coefficients of the same indices gives

$$(1 + a\exp\left(\frac{-h}{\epsilon}\right) - 1))u_{i+1} + (-2 - a(\exp\left(\frac{-h}{\epsilon}\right) - 1))u_i + u_{i-1} = [\exp\left(\frac{-h}{\epsilon}\right) - 1]h$$
(27)

In this article, in all experiments of MATLAB coding the value of ais considered as 1.
The comparison of numerical solution of the discretized equation (27) obtained by non-standard finite difference method for several values of ϵ with exact solution has been shown graphically.

Example 2:

$$\epsilon u'' + u' = 2x on [0,1] \ , u(0) = 0 \ , u(1) = 0 \quad (28)$$

The exact solution is

$$u(x) = 2\epsilon x + x^2 + \frac{((2\epsilon+1)(\exp\left(\frac{-1}{\epsilon}\right) - \exp\left(\left(\frac{x-1}{\epsilon}\right)\right))}{(1 - \exp\left(\frac{-1}{\epsilon}\right))} \quad (29)$$

$$-\epsilon \frac{u_{i+1} - 2u_i + u_{i-1}}{\frac{\left[\exp\left\{\frac{-h}{\epsilon}\right) - 1\right]h}{\frac{-1}{\epsilon}}} + \frac{u_{i+1} - u_i}{h} = 2x_i \quad (30)$$

$$\frac{u_{i+1} - 2u_i + u_{i-1}}{\left[\exp\left(\frac{-h}{\epsilon}\right) - 1\right]h} + \frac{u_{i+1} - u_i}{h} = 2x_i \quad (31)$$

Arranging the coefficients of the same indices gives

$$(\exp\left(\frac{-h}{\epsilon}\right) - 1))u_{i+1} + (-1 - \exp(\left(\frac{-h}{\epsilon}\right))u_i + u_{i-1} = 2.i.h.h[\exp\left(\frac{-h}{\epsilon}\right) - 1] \quad (32)$$

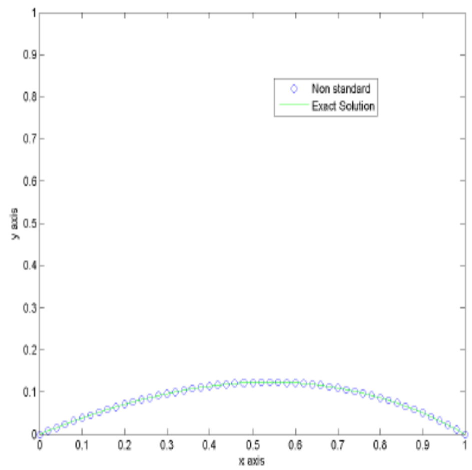

Figure 1: Numerical solution obtained by using non standard finite difference method for example 1 with $n = 50$ and $\epsilon = 1$.

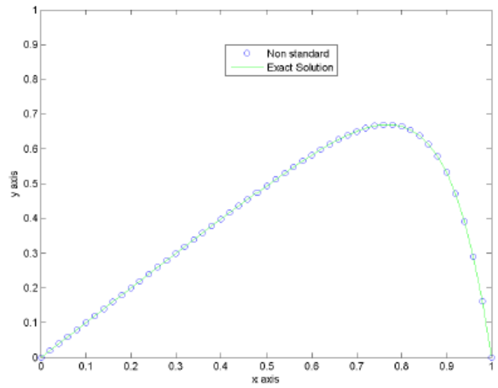

Figure 2: Numerical solution obtained by using non standard finite difference method for example 1 with $n = 50$ and $\epsilon = 0.1$.

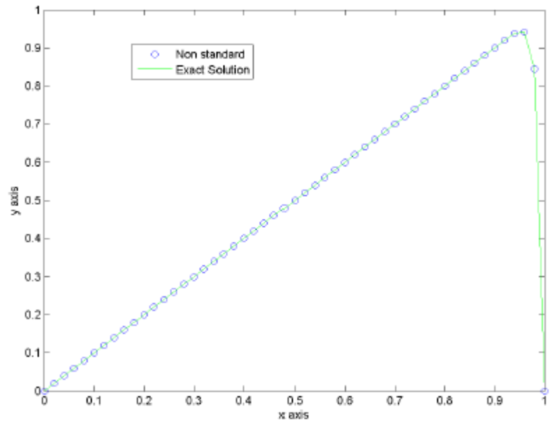

Figure 3: Numerical solution obtained by using non standard finite difference method for example 1 with $n = 50$ and $\epsilon = 0.01$.

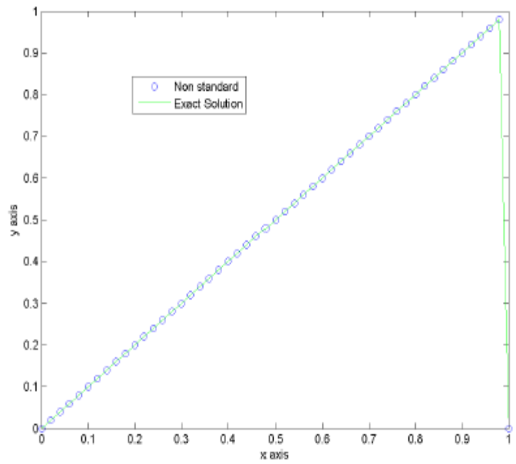

Figure 4: Numerical solution obtained by using non standard finite difference method for example 1 with $n = 50$ and $\epsilon = 0.001$.

The comparison of numerical solution of the discretized equation (32) obtained by non-standard finite difference method for several values of ϵ with exact solution has been shown graphically.

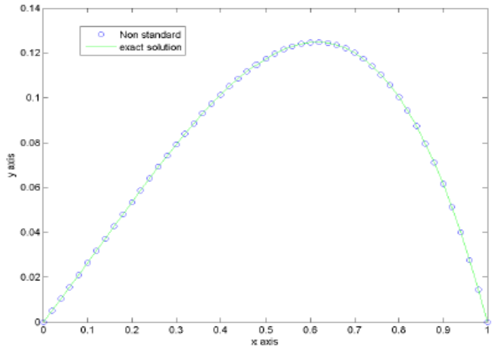

Figure 5: Numerical solution obtained by using non standard finite difference method for example 2 with $n = 50$ and $\epsilon = 1$.

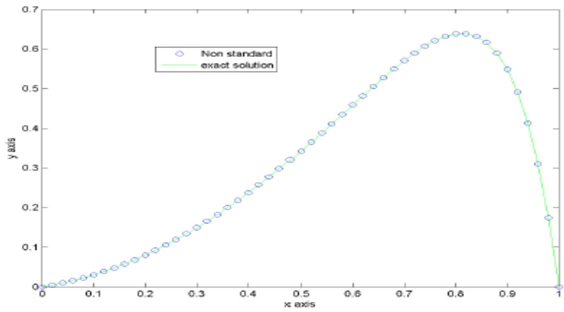

Figure 6: Numerical solution obtained by using non standard finite difference method for example 2 with $n = 50$ an $\epsilon = 0.1$.

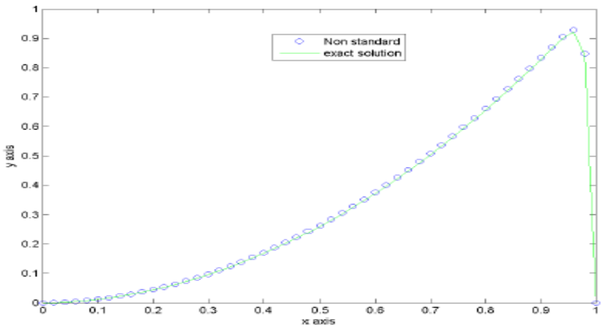

Figure 7: Numerical solution obtained by using non standard finite difference method for example 2 with $n = 50$ and $\epsilon = 0.01$.

4.2 NumericalSolution of Convection Diffusion Problem with Standard Finite Difference Method

In this section we found the numerical solution of convection diffusion problem using central and backward difference schemes. We have chosen the same problem for the sake of comparison.
i.e,$-\epsilon u'' + au' = 1$

4.2.1 The Solution with Central Difference Scheme

$$-\epsilon \frac{u_{i+1} - 2u_i + u_i}{h^2} + a\frac{u_{i+1} - u_i}{2h} = 1$$

$$\left(\frac{-\epsilon + ah}{h^2}\right) u_{i+1} + \left(\frac{2\epsilon}{h^2}\right) u_i + \left(\frac{-\epsilon - ah}{h^2}\right) u_{i-1} = 1 \quad (33)$$

Now let us consider the numerical solution of equation (33) for different values of ϵandcompare with the exact solution as follows.

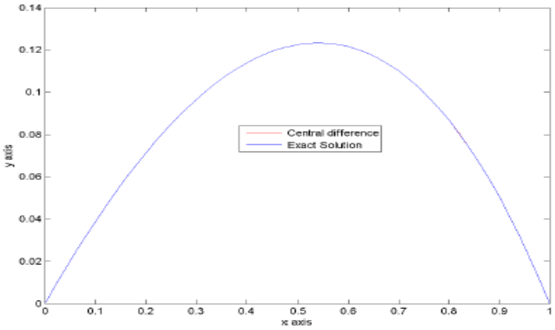

Figure 8: Numerical solution obtained by using central difference for example 1 with $n = 50$ and $\epsilon = 1$

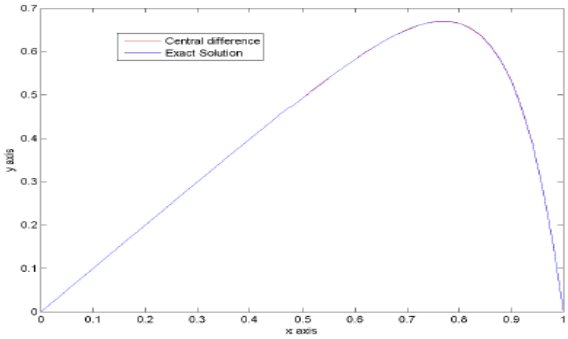

Figure 9: Numerical solution obtained by using central difference for example 1 with $n = 50$ and $\epsilon = 0.1$.

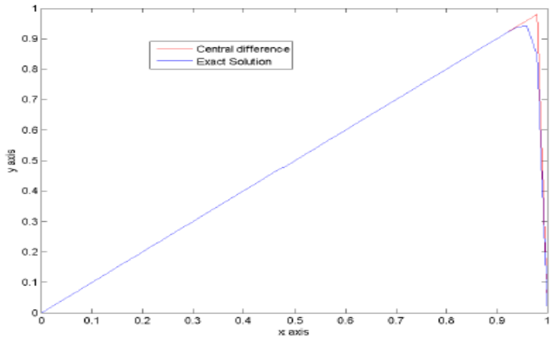

Figure 10: Numerical solution obtained by using central difference for example 1 with $n = 50$ and $\epsilon = 0.01$.

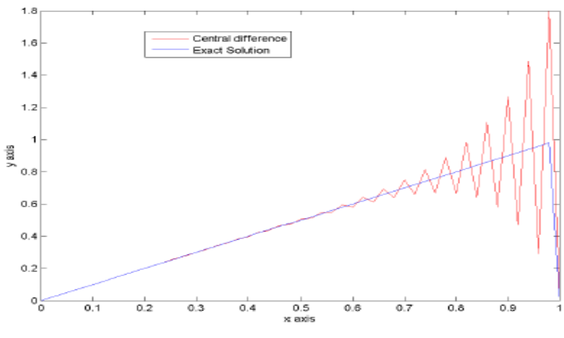

Figure 11: Numerical solution obtained by using central difference for example 1 with $n = 50$ and $\epsilon = 0.001$.

4.2.2 .Solution With Back Ward Difference Scheme

$$-\epsilon \frac{u_{i+1} - 2u_i + u_i}{h^2} + a\frac{u_{i-}u_i}{h} = 1$$

$$\left(\frac{-\epsilon}{h^2}\right)u_{i+1} + \left(\frac{2\epsilon}{h^2} + \frac{a}{h}\right)u_i + \left(\frac{-\epsilon}{h^2} - \frac{a}{h}\right)u_{i-1} = 1$$

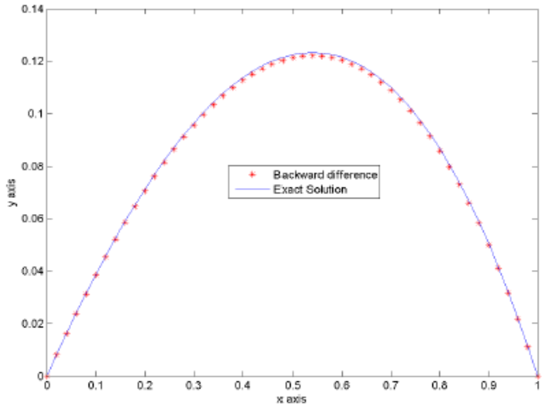

Figure 12: Numerical solution obtained by using back ward difference for example 1 with $n = 50$ and $\epsilon = 1$.

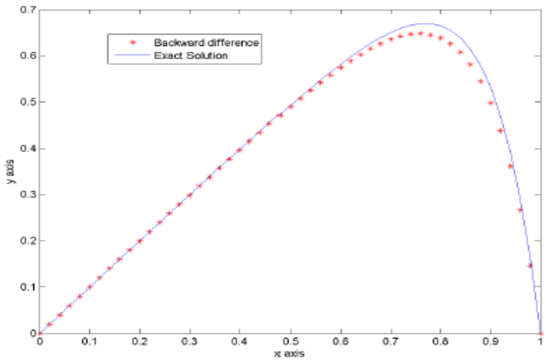

Figure 13: Numerical solution obtained by using back ward difference for example 1 with $n = 50$ and $\epsilon = 0.1$.

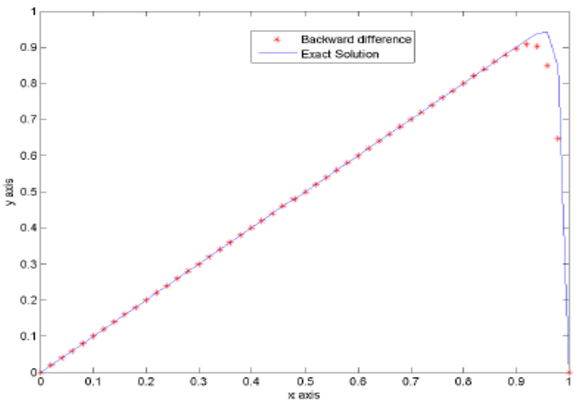

Figure 14: Numerical solution obtained by using back ward difference for example 1 with $n = 50$ and $\epsilon = 0.01$.

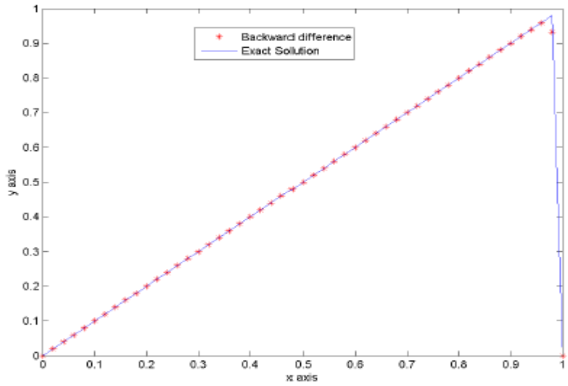

Figure 15: Numerical solution obtained by using back ward difference for example 1 with $n = 50$ and $\epsilon = 0.001$.

From the figures (8 - 15), we observed that the back ward discretization of the convective term of the one dimensional convection diffusion problem is more stable than the central difference approximation of the convective term of the one dimensional convection diffusion problem.

V. Comparative Study of Non-Standard and Standard Finite Difference Methods.

In this section the performance of standard and non-standard finite difference schemes are compared. The performance of the scheme was evaluated by comparing the result with exact solution. As discussed earlier the back ward discretization of the convective term of the one dimensional convection diffusion problem is more stable than the central difference approximation of the convective term of the one dimensional convection diffusion problem. So the performance of the non-standard finite difference scheme is compared with back ward difference approximation.

The comparison of numerical solution obtained by non-standard finite difference method for several values of ϵ and the solution obtained by back ward difference approximation, with exact solutions is given in tabular form and has been shown graphically. We also plotted the graph of exact solution for different values of ϵ.

Table 1: Comparison of numerical solutions obtained by using standard and non standard finite difference methods for example 1 with $n = 50$ and $\epsilon = 1$.

x	standard FDM	Non standard FDM	Exact solution
0	0.000000000000000	0.000000000000000	0.000000000000000
0.02	0.008176790296303	0.0082433290656881	0.0082433290656881
0.04	0.016117116398533	0.0.0162490800030726	0.016249080030726
0.1	0.038471541881861	0.038792975439910	0.038792975439911
0.9	0.049997410369012	0.050544988032654	0.050544988032655
0.92	0.041174148872695	0.041628149155351	0.041628149155352
0.94	0.031774422146453	0.032127151383389	0.032127151383390
0.96	0.021786700885685	0.022030193924307	0.022030193924307
0.98	0.011199225199702	0.011325237593823	0.011325237593823
1	0.000000000000000	0.000000000000000	0.000000000000000

Table 2: Comparison of absolute errors by standard and non standard finite difference methods for example 1 with $n = 50$ and $\epsilon = 1$.

x	Err1	Err2
0	0.000000000000000	0.000000000000000
0.02	0.000066538000000	0.000000000000000
0.04	0.000131963000000	0.000000000000000
0.1	0.000321433000000	0.000000000000001
0.9	0.000547577000000	0.000000000000001
0.92	0.000454000000000	0.000000000000001
0.94	0.000352729000000	0.000000000000001
0.96	0.000243493038622	0.000000000000000
0.98	0.000126012394121	0.000000000000000
1	0.000000000000000	0.000000000000000

$*Err1 = |exact - SFDM|$
$*Err2 = |exact - NSFDM|$

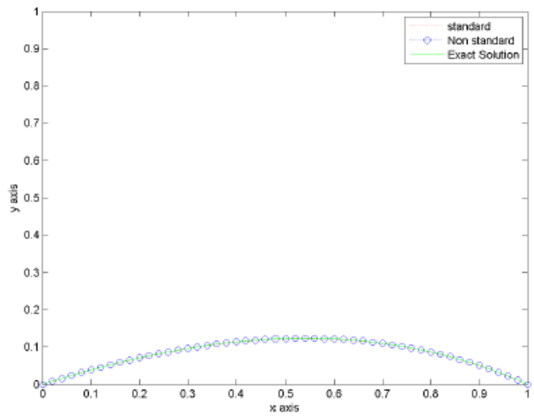

Figure 16: Comparison of numerical solutions obtained by using standard and non standard finite difference methods for example 1 with $n = 50$ and $\epsilon = 1$.

Table 3: Comparison of numerical solutions obtained by using standard and non standard finite difference methods for example 1 with $n = 50$ and $\epsilon = 0.1$.

x	standard FDM	Non standard FDM	Exact solution
0	0.000000000000000	0.000000000000000	0.000000000000000
0.02	0.019978020620976	0.01998947873965	0.019989947873964
0.04	0.039951645366148	0.039977670179499	0.039977670179499
0.06	0.05991995060354	0.059962674169616	0.059962674169616
0.6	0.574022977034060	0.581728931535809	0.581728931535803
0.68	0.626016059871227	0.639281347327432	0.639281347327425
0.9	0.498188159781275	0.532149258360493	0.532149258360487
0.94	0.3613425954451184	0.391208848756020	0.391208848756015
0.98	0.146684982815853	0.161277476906739	0.161277476906737
1	0.000000000000000	0.000000000000000	0.000000000000000

Table 4: Comparison of absolute errors by standard and non standard finite difference methods for example 1 with $n = 50$ and $\epsilon = 0.1$.

x	Err1	Err2
0	0.000000000000000	0.000000000000000
0.02	0.000011927000000	0.000000000000001
0.04	0.000026024000000	0.000000000000000
0.06	0.000042723000000	0.000000000000000
0.6	0.007705954000000	0.000000000000006
0.68	0.013265287456198	0.000000000000007
0.9	0.033961098579212	0.000000000000006
0.94	0.0298662533108966	0.000000000000005
0.98	0.014592494090884	0.000000000000006
1	0.000000000000000	0.000000000000000

$*Err1 = |exact - SFDM|$
$*Err2 = |exact - NSFDM|$

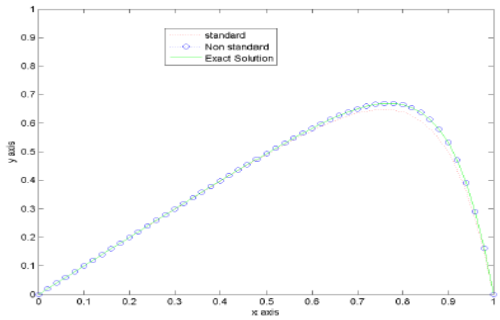

Figure 17: Comparison of numerical solutions obtained by using standard and non standard finite difference methods for example 1 with $n = 50$ and $\epsilon = 0.1$.

Table 5: Comparison of numerical solutions obtained by using standard and non standard finite difference methods for example 1 with $n = 50$ and $\epsilon = 0.01$.

x	standard FDM	Non standard FDM	Exact solution
0	0.000000000000000	0.000000000000000	0.000000000000000
0.02	0.020000000000000	0.020000000000000	0.020000000000000
0.04	0.040000000000000	0.040000000000000	0.040000000000000
0.06	0.060000000000000	0.060000000000000	0.060000000000000
0.6	0.59999999713203	0.59999999999998	0.600000000000000
0.68	0.679999976769427	0.67999999999985	0.67999999999987
0.9	0.895884773662552	0.899954600070233	0.89954600070238
0.94	0.902962962962963	0.937521247823328	0.937521247823334
0.98	0.646666666666667	0.844664716763382	0.844664716763388
1	0.000000000000000	0.000000000000000	0.000000000000000

Table 6: Comparison of absolute errors of standard and non standard finite difference methods for example 1 with $n = 50$ and $\epsilon = 0.01$.

x	Err1	Err2
0	0.000000000000000	0.000000000000000
0.02	0.000000000000000	0.000000000000000
0.04	0.000000000000000	0.000000000000000
0.06	0.000000000000000	0.000000000000000
0.6	0.000000002867920	0.000000000000002
0.68	0.000000023230563	0.000000000000002
0.9	0.003661227039828	0.000000000000005
0.94	0.034558284860371	0.000000000000006
0.98	0.197998050096721	0.000000000000006
1	0.000000000000000	0.000000000000000

$*Err1 = |exact - SFDM|$
$*Err2 = |exact - NSFDM|$

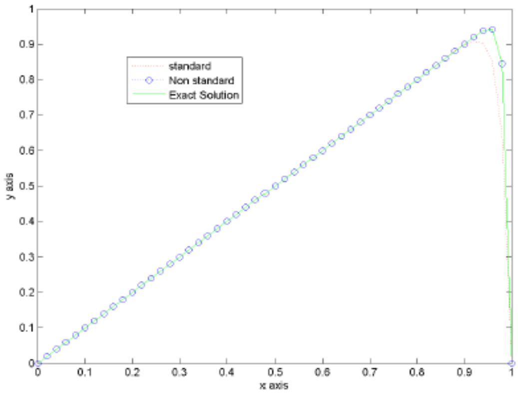

Figure 18: Comparison of numerical solutions obtained by using standard and non standard finite difference methods for example 1 with $n = 50$ and $\epsilon = 0.01$.

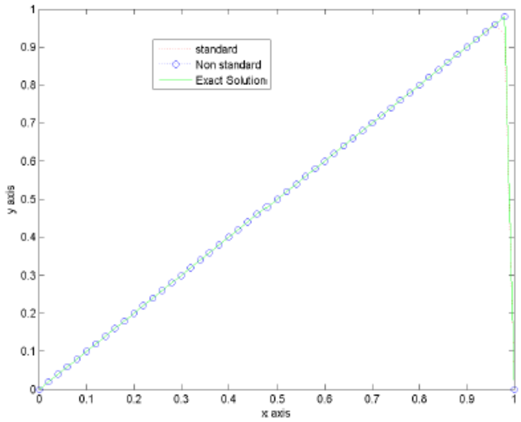

Figure 19: Comparison of numerical solutions obtained by using standard and non standard finite difference methods for example 1 with $n = 50$ and $\epsilon = 0.001$.

We have also shownthat the comparison of the errors of standard and non-standard finite difference schemes of the numerical solution of example 1 for several values of ϵ and different step lengths graphically.

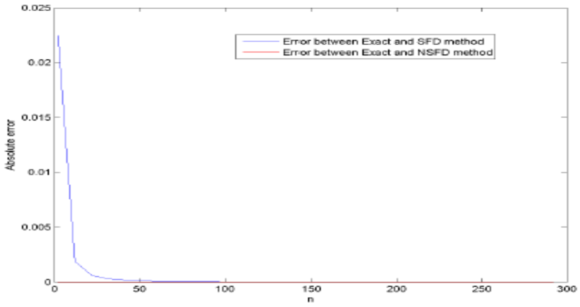

Figure 20: Comparison of absolute errors of standard and non standard finite difference methods for example 1 with $\epsilon = 1$, $L = 1$ and $n = 2:10:300$.

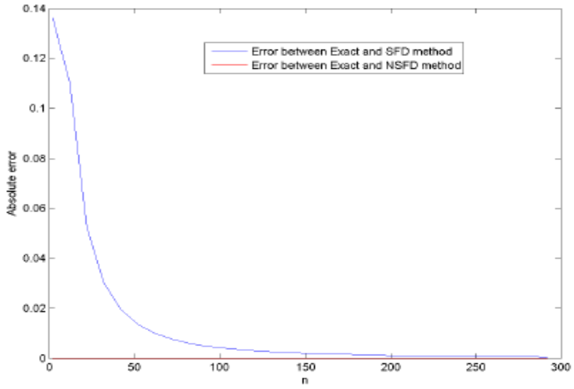

Figure 21: Comparison of absolute errors of standard and non standard finite difference methods for example 1 with $\epsilon = 0.1$, $L = 1$ and $n = 2:10:300$.

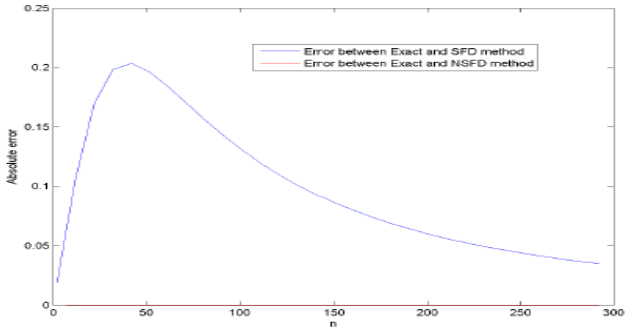

Figure 22: Comparison of absolute errors of standard and non standard finite difference methods for example 1 with $\epsilon = 0.01$, $L = 1$ and $n = 2:10:300$.

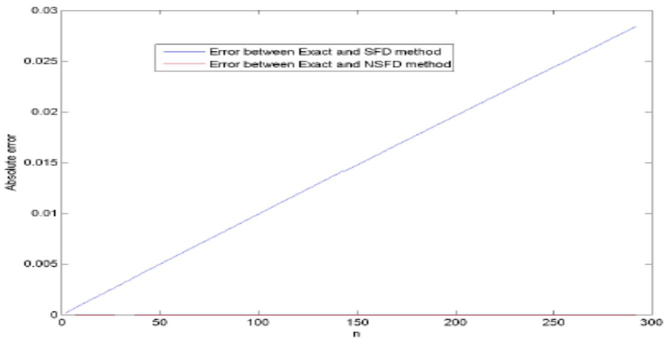

Figure 23: Comparison of absolute errors of standard and non standard finite difference methods for example 1 with $\epsilon = 0.001$, $L = 1$ and $n = 2:10:300$.

The comparison of numerical solution obtained by non-standard finite difference method for several values of ϵ and the solution obtained by back ward difference approximation, of example 2 with exact solutions has been shown graphically.

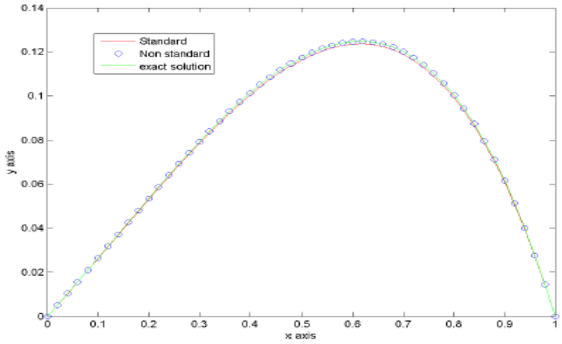

Figure 24: Comparison of numerical solutions obtained by using standard and non standard finite difference methods for example 2 with $n = 50$ and $\epsilon = 1$.

The following figures shows the comparison of absolute errors of example 2 for different step lengths and ϵ.

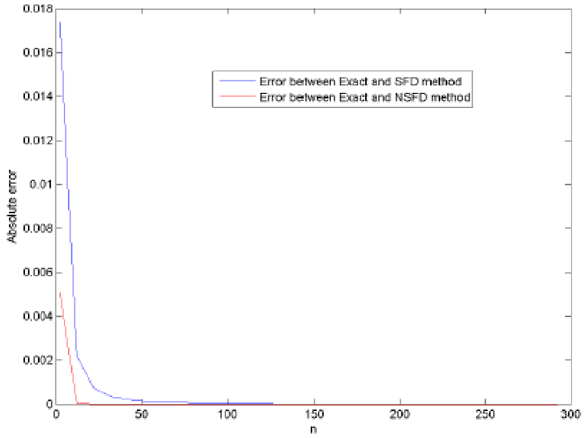

Figure 25: Comparison of absolute errors of standard and non standard finite difference methods for example 2 with $\epsilon = 1$, $L = 1$ and $n = 2:10:300$.

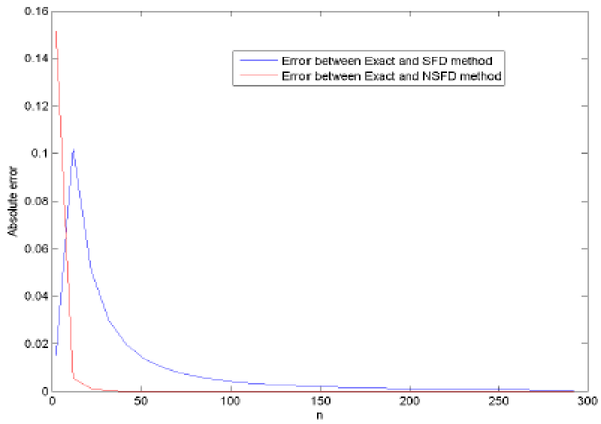

Figure 26: Comparison of absolute errors of standard and non standard finite difference methods for example 2 with $\epsilon = 0.1$, $L = 1$ and $n = 2:10:300$.

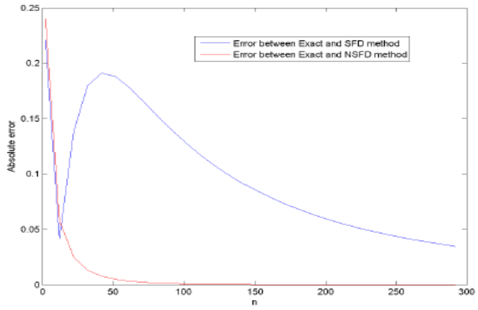

Figure 27: Comparison of absolute errors of standard and non standard finite difference methods for example 2 with $\epsilon = 0.01$, $L = 1$ and $n = 2:10:300$.

VI. Conclusion

Non-standard and standard finite difference schemes are applied to find the numerical solution of example 1 and 2at differentstep lengths for different values of small parameter ϵ. Numerical solutions are summarized in the tables and the comparison has been shown in figures.From the figures 12, 13, 14 and 15, we observed that the back ward discretization of the convectiveterm of the one dimensional convection diffusion problem is more stable than thecentralapproximation of the convective termof the one dimensional convection diffusionproblem. Therefore we compared the backward scheme with NSFD.

From the figures 1-7, we observed that even if the small parameter ϵgets smallerand smaller, the nonstandard finite difference scheme performed well and there is no oscillations observed so that itis stable on the given domain.It is also observed fromthe tables,eventhough the standard finite difference methodyield good result when the small parameter ϵ large enough, the non-standard finitedifference scheme perform better than the standard finite difference method. The graphs(figure 23 and 27) of the errors shows that the error of the standard finite differencescheme increases as the value of the small parameter ϵdecreases and the error plots shows that instability of the numerical scheme for different values of n. The error plots (figures 20-27) ofnon-standard finite scheme shows that the error decreases as the value of *n* increasesthis shows that the scheme is dynamically consistent and it is stable for all values of ϵ. From all the tables and graphs we conclude that the non-standard finite difference scheme is more powerfulthan the standard finite difference method.

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