

Ordinary Differential Equation

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Abstract: The major purpose in this paper is to demonstrate on Differential equations, Types of differential equations, ordinary differential equations, partial differential equations, order and degree of a differential equation, Linear differential equation, Bernoulli's equation.

I. Introduction

Any student of Engineering and Technology, the general laws of medicine, social sciences, population dynamics and the rate of change of quantities needs a certain amount of Mathematics are usually modeled as differential equations. So, in this paper we study, methods of solution of ordinary differential equations of first order and degree.

II. Differential Equation

A Differential equation is an equation involving a relation between an unknown function and one or more of its derivatives.

Differential Equations are classified into Ordinary and Partial differential equations.

2.1 Ordinary Differential Equation:

A differential equation containing derivatives of a function of a single variable is called an ordinary differential equation.

Example: (1) $\frac{d^2 y}{dx^2} + m^2 y = 0$

(2) $\left(\frac{dy}{dx}\right)^3 - \left(\frac{dy}{dx}\right)^2 + 5y = \tan y$

The general solution of an ordinary differential equation is

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2 y}{dx^2}, \dots, \frac{d^n y}{dx^n}\right) = 0$$

2.2 Partial Differential Equation:

A Partial Differential Equation (or briefly PDE) is an mathematical equation that involves two or more independent variables, an unknown function (dependent on those variables), and partial derivatives of unknown function with respect to the independent variables.

Examples: (1) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(2) $\frac{\partial^2 z}{\partial x \partial y} = 0$

2.3 Order and Degree of a Differential equation:

- The order of a differential equation is the order of the highest derivative appearing in it.
- The degree of a differential equation is the degree or highest derivative, when the equation is freed from radicals and fractions in respect of the derivatives.

III. Equations

An equation of the form $\frac{dy}{dx} = f(x, y)$ is called a differential equation of first order and first degree.

→ In order to study the methods of solution, Linear equation and Bernoulli's equation are one of the classifications of first order and first degree equations.

3.1 Linear Differential Equations of First Order:

A differential equation is called linear if the dependent variable and its differential coefficients occur only in the first degree but not multiplied together. The standard form of a first order linear differential equation in 'y' can be written as

$$\frac{dy}{dx} + P(x)y = Q(x) \text{ and}$$

$$\frac{dx}{dy} + P(y)x = Q(y) \text{ is first order linear differential equation in 'x' where P and Q are constants.}$$

Working Principle. To solve the linear differential equation in $\frac{dy}{dx} + P(x)y = Q(x)$.

1. Get a differential equation $\frac{dy}{dx} + P(x)y = Q(x)$.
2. Find P and Q.
3. Compute an integrating factor(IF) as $e^{\int P(x)dx}$
4. The general solution is $y \times (\text{IF}) = \int Q(x) \times (\text{IF}) dx + c$.

Note: In the case of $\frac{dx}{dy} + P(y)x = Q(y)$, the IF = $e^{\int P(y)dy}$, and the general solution is $\times x (\text{IF}) = \int Q(y) \times (\text{IF}) dy + c$.

3.2 Bernoulli's Differential Equation:

A differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x) y^n$, where P(x) and Q(x) are the functions of x alone is known as Bernoulli's equation.

Working Principle

1. Multiply the above equation with y^{-n} , we get $\frac{dy}{dx} y^{-n} + py^{1-n} = Q \rightarrow (1)$
2. Let $y^{1-n} = u$ so that $(1-n)y^{-n} \frac{dy}{dx} = \frac{du}{dx}$ i.e., $y^{-n} \frac{dy}{dx} = \frac{1}{1-n} \frac{du}{dx} \rightarrow (2)$
3. From (1) and (2), we get $\frac{1}{1-n} \frac{du}{dx} + pu = Q$ i.e., $\frac{du}{dx} + (1-n)Pu = (1-n)Q$.

This is a linear equation of first order in u and this can be solved as a linear differential equation of first order model and we substitute for u and get the required solution.

IV. Conclusion

An ordinary differential equation gives a relationship between a function of one independent variable. The simplest case of which is shown above in Linear first order equations are important because they show up frequently in nature and physics, and can be solved by a fairly straight forward method.

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