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Comparison of Sums of Squares of Consecutive Primes Using Four Maximal Gap Conjectures

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Abstract: *We consider four Conjectures for $G(x)$. An attempt has been made to obtain the value of x for which the corresponding value of $G(x)$ is nearest to the actual gap while calculating the Sums of Squares of Consecutive Primes. In this paper we calculate sums of squares of consecutive primes using four conjectures and compare it with actual sums of squares of consecutive primes.*
Keywords: *Sums of Squares, Maximal gap.*

I. Introduction

Many number theorists and mathematical physicists are interested in understanding spacing statistics of various sequences of numbers occurring in nature.

Here we are interested in considering gaps between consecutive primes. Let $G(x)$ denote the largest gap between consecutive primes below x. More precisely, for $x \geq 2, G(x) \coloneqq \max_{p \leq x}(p' - p)$ where p', p are consecutive primes.

The twin prime conjecture says that the gap2 occurs infinitely. It was known only that there are infinitely many gaps which were about a quarter the size of the gap, Assuming certain conjectures on the distribution of primes in arithmetic progressions, Dan Goldston Janos Pintz and Cem Yildirim [10] are able to prove the existence of infinitely many prime pairs that differ by at most 16.The prime number theorem indicates that $\pi(x) = \frac{x}{\log x}$ denotes the number of primes $\leq x$.

Some results in number theory, including the Prime Number Theorem, can be obtained by assuming a random distribution of prime numbers. In addition, conjectural formulae, such as Cherwell's for the density of prime pairs $(p, p+2)$ obtained in this way, have been found to agree well with the available evidence. Recently, primes have been determined over ranges of 1,50,000 numbers with starting points upto 10^{15}.

Erdos [3] was the first to show that $\lim_{p\to\infty} \frac{P_{next} - P}{\log P} < 1$. Other landmark results in the area are the works of Bombieri and Davenport,Huxley, and Maier, who introduced several new ideas to this study and progressively reduced the liminf to ≤ 0.24.

In a series of papers from 1935 to 1963 Erdos Rankin and Schonhage showed that $G(x) \geq (c + o(1)) logx\, log_2 x log_4 x (log_3 x)^{-2}$ where $c = e^{\gamma}$ and γ is Euler's constant. Here, this result is shown with $c = c_o e^{\gamma}$ where $c_o = 1.31256 \ldots$ is the solution of the equation $\frac{4}{c_0} - e^{-\frac{4}{c_0}} = 3$,

Cramer conjectured that $G(x) \sim log^2 x$. Gauss Conjecture is$G(x) \sim logx[logx - 2loglogx + C]$. A.Granville argued [4] that the actual $G(x)$ can be larger than that given by $log^2 x$ namely he claims that there are infinitely many pairs of primes P_n, P_{n+1} for $P_{n+1} - P_n = G(P_n) > 2^{e^{-\gamma}} log^2(P_n) = 1.2292 \ldots\ldots log^2(P_n)$

The best estimate was obtained by Rankin who proved that there exist a positive constant C such that for infinitely many primes $P, P_{next} - P > Clogp \frac{log_2 P log_4 P}{(log_3 P)^2}$. Rankin results provides the largest known gap between primes.

In this paper we consider the four conjecture for $G(x)$ presented in [1,2] and denote them by $G_1(x), G_2(x), G_3(x)$ and $G_4(x)$. To start with, we choose the x value such that $G(x)$ gives the gap. It is observed that the Gauss Approximation Conjecture $G_1(x)$ gives the value nearest to the actual value while calculating the sums of squares of consecutive primes $P_{n+1}^2 + P_n^2$. However, after performing some Algebra and reducing all the four Conjectures $G_1(x)$ to $G_4(x)$ to a single approximate value $2[P_n + (logx)^2]^2$. It is seen that, for the value of x considered above, the Cramer's Conjecture $G_2(x)$ is nearest yo the actual value.

II. Method of Analysis

The four Conjectures considered are given below:

Gauss Conjecture: $G_1(x) \sim logx[logx - 2loglogx + C]$

Cramer's Conjecture: $G_2(x) \sim (logx)^2$
D.R.Heath Brown Conjecture: $G_3(x) \sim logx(logx + logloglogx)$
J.H.Cadwell Conjecture: $G_4(x) \sim logx(logx - loglogx)$

In all the four conjectures x has been chosen in such a way that $G(x)$ gives the gap and the numerical illustration are calculated upto 10^6 and to gap 72 which is presented below in table I.

Table I: Numerical Illustration

Gaps	$G_1(x)$	$G_2(x)$	$G_3(x)$	$G_4(x)$
2	10	5	6	6
8	91	17	17	35
14	288	43	38	98
18	526	70	59	171
20	690	88	73	220
26	1432	164	131	438
32	2712	287	222	802
38	4813	476	360	1384
40	5761	559	420	1644
48	11299	1021	748	3131
52	15461	1355	983	4230
58	24163	2030	1452	6497
60	27882	2313	1647	7457
66	42206	3375	2376	11117
72	62625	4843	3374	16267

The following table II represents the Sums of Squares of Consecutive Primes for the corresponding gap value presented in Tabel I.

Table II: Numerical Illustration

P_n	P_{n+1}	$P_{n+1} - P_n$	$P_n^2 + P_{n+1}^2$	$P_n^2 + (P_n + G_1(x))^2$	$P_n^2 + (P_n + G_2(N))^2$	$P_n^2 + (P_n + G_3(N))^2$	$P_n^2 + (P_n + G_4(N))^2$
821	823	2	1351370	1351536.02	1352341.97	1351772.07	1351642.36
37361	37363	2	2791838090	2791845626.58	2791882200.39	2791856340.72	2791850453.54
9883	9887	4	195426458	195426616.13	195432866.41	195432430.74	195429680.00
45823	45827	4	4199861258	4199861990.92	4199890961.07	4199888941.77	4199876192.13
947	953	6	1805018	1805107.34	1805351.13	1805829.27	1805144.33
9901	9907	6	196178450	196179378.69	196181912.75	196186881.88	196179763.20
449	457	8	410450	410459.30	410474.77	410579.86	410569.48
99809	99817	8	19925269970	19925272001.71	19925275379.65	19925298330.27	19925296063.7
8563	8573	10	146821298	146821340.21	146823013.06	146825581.44	146822604.11
37189	37199	10	2766787322	2766787505.17	2766794763.76	2766805908.02	2766792989.29
509	521	12	530522	530535.53	530533.80	530788.78	530600.30
67967	67979	12	9240657530	9240659294.76	9240659069.77	9240692330.18	9240667745.09
863	877	14	1513898	1513905.27	1514155.20	1514182.24	1513968.19
7283	7297	14	106288298	106288358.47	106290437.89	106290662.80	106288882.00
1831	1847	16	6763970	6764000.50	6764186.91	6764554.31	6764134.21
6841	6857	16	93817730	93817843.24	93818535.28	93819899.19	93818339.64
523	541	18	566210	566211.03	566263.79	566225.43	566229.33
80021	80039	18	12809601962	12809602113.91	12809609919.77	12809604244.34	12809604821.6
887	907	20	1609418	1609418.56	1609502.43	1609456.27	1609421.23
5717	5737	20	65597258	65597261.56	65597792.06	65597500.08	65597278.41
10039	10061	22	202005242	202005379.76	202005417.97	202006950.19	202005413.62

It is observed that the Gauss Approximation Conjecture $G_1(x)$ gives the value nearest to the actual value while calculating the sums of squares of consecutive primes $P_n^2 + P_{n+1}^2$.

III. Reduction to a single approximation

Now, we illustrate the process of reducing each of the four conjectures to a single approximation.

Gauss Conjecture [5]:

$G_1(x) \sim logx[logx - 2loglogx + C]$

$P_{n+1} - P_n = G_1(x)$

$P_{n+1}^2 = (P_n + G_1(x))^2$

$P_{n+1}^2 + P_n^2 = 2P_n^2 + 2P_n G_1(x) + G_1^2(x)$

$\sim 2P_n^2 + 2P_n[(logx)^2 - 2logxlog_2x + Clogx] + (logx)^2[logx - 2loglogx + C]^2$

$\sim 2P_n^2 + 2P_n[(logx)^2 - 2logxlog_2x] + [(logx)^2[logx - 2log_2(x)]^2]$

$P_{n+1}^2 + P_n^2 \sim 2[P_n + ((logx)^2 - 2logxlog_2x]^2$

$P_{n+1}^2 + P_n^2 < 2[P_n + (logx)^2]^2$

Cramer's Conjecture [5]:

$G_2(x) \sim (logx)^2$

$P_{n+1}^2 = (P_n + G_2(x))^2$

$P_{n+1}^2 + P_n^2 = 2P_n^2 + 2P_n G_2(x) + G_2^2(x)$

$P_{n+1}^2 + P_n^2 < 2[P_n + (logx)^2]^2$

D.R.Heath Brown Conjecture [5]:

$G_3(x) \sim logx(logx + logloglogx)$

$P_{n+1}^2 = (P_n + G_3(x))^2$

$P_{n+1}^2 + P_n^2 = 2P_n^2 + 2P_n G_3(x) + G_3^2(x)$

$P_{n+1}^2 + P_n^2 \sim 2P_n^2 + 2P_n logx(logx + log_3x) + (logx)^2[logx + log_3x]^2$

$\sim 2P_n^2 + 2P_n(logx)^2 + 2P_n logx\, log_3x + (logx)^4 + 2(logx)^3 log_3x + (logx)^2(log_3x)^2$

$\sim 2{P_n}^2 + (logx)^4 + 2(logx)^4 + (logx)^2[2P_n + (logx)^2] + 2P_n(logx)^2$

$\sim 2{P_n}^2 + 4(logx)^4 + 4P_n(logx)^2$

$P_{n+1}^2 + P_n^2 < 2[P_n + (logx)^2]^2$

J.H.Cadwell Conjecture [5]:

$G_4(x) \sim logx(logx - loglogx)$

$P_{n+1}^2 = (P_n + G_4(x))^2$

$P_{n+1}^2 + P_n^2 = 2P_n^2 + 2P_n G_4(x) + G_4^2(x)$

$P_{n+1}^2 + P_n^2 \sim 2P_n^2 + 2P_n logx(logx - log_2x) + (logx)^2[logx - log_2x]^2$

$\sim 2{P_n}^2 + 4(logx)^4 - 4P_n(logx)^2$

$P_{n+1}^2 + P_n^2 < 2[P_n + (logx)^2]^2$

The corresponding value of sums of squares of consecutive primes using reduced approximation is illustrated using numerical values in the table below.

Table III: Numerical Illustration

P_n	P_{n+1}	$P_{n+1} - P_n$	$P_n^2 + P_{n+1}^2$	$2(P_n + (logx)^2)^2$	$2(P_n + (logx)^2)^2$	$2(P_n + (logx)^2)^2$	$2(P_n + (logx)^2)^2$
5477	5479	2	60016970	60111268.20	60051819.50	60065412.10	60065412.10
99989	99991	2	19996000202	19997720824.18	19996636257.60	19996884282.15	19996884282.15
457	461	4	421370	438241.94	425639.81	426569.83	429810.91
99877	99881	4	19951629290	19955266476.37	19952557798.80	19952759047.68	19953458693.80
83471	83477	6	13935817370	13928826019.20	13925526828.41	13925661741.05	13926654930.35
99923	99929	6	19970410970	19975630915.38	19971679936.85	19971841504.33	19973030915.16
9311	9319	8	173538482	174148105.53	173688532.10	173688532.10	173860544.34
69653	69661	8	9704195330	9708750802.23	9705317392.66	9705317392.66	9706602932.51

After performing some Algebra and reducing all the four conjectures $G_1(x)$ to $G_4(x)$ to a single approximate value $2[P_n+(logx)^2]^2$. It is seen that, for the value of x considered above, the Cramer's Conjecture $G_2(x)$ is nearest yo the actual value.

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