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Two Warehouse Inventory Model with Ramp Type Demand, Shortages under Inflationary Environment

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Abstract: *In the literature of inventory after the development of classical economic order quantity (EOQ) model researchers extensively studied several aspects of inventory modeling by assuming constant demand rate. But in a real market demand of a product is always dynamic state due to the variability of time, price or even of the instantaneous level of inventory displayed in retail shop. This impressed researchers and marketing practitioners to think about the variability of demand rate.*
Keywords: *Ramp Type demand, Present worth Ordering Cost, Present worth Deterioration Cost*

I. Introduction

The ramp type demand is very commonly seen in real life situations when some fresh products come to the market. In case of ramp type demand rate, the demand increases linearly at the beginning and then the market grows into a stable stage such that the demand becomes a constant until the end of the inventory cycle. **Hill (1995)** first proposed a time dependent demand pattern by considering it as the combination of two different types of demand such as increasing demand followed by a constant demand in two successive time periods over the entire time horizon and termed it as ramp-type time dependent demand pattern. **Mandal** and **Pal (1998)** extended the inventory model with ramp type demand for deteriorating items and allowing shortage. **Wu** and **Ouyang (2000)** extended the inventory model to include two different replenishment policies: (a) models starting with no shortage and (b) models starting with shortage. **Wu (2001)** further investigated the inventory model with ramp type demand rate such that the deterioration followed the Weibull distribution deterioration and partial backlogging. **Giri** *et al.* **(2003)** extended the ramp type demand inventory model with Weibull deterioration distribution. **Manna** and **Chaudhuri (2006)** have developed a production inventory model with ramp-type two time periods classified demand pattern where the finite production rate depends on the demand. **Deng** *et al.* **(2007)** point out some questionable results of **Mandal** and **Pal (1998)** and **Wu** and **Ouyang (2000)**, and then resolved the similar problem by offering a rigorous ad efficient method to derive the optimal solution

Trapezoidal type demand pattern is more realistic in comparison to ramp type demand in many cases like fad or seasonal goods coming to market. The demand rate for such items increases with the time up to certain time and then ultimately stabilizes and becomes constant, and finally the demand rate approximately decreases to a constant or zero, and the begins the next replenishment cycle.

The effect of deterioration of physical goods in stock is very realistic feature of inventory control. For more details about deteriorating items one can see the review paper of **Goyal** and **Giri (2001).**

So far it seems none has tried to investigate this important issue with the assumptions of two warehouse, deterioration, and shortages with the effect of learning. The whole combination is very unique and very much practical. We think that our work will provide a solid foundation for the further study of this kind of important models with trapezoidal type demand rate.

In this paper, we proposed the work as follows:

(i) We introduce a trapezoidal type demand rate, with its variable part being linear function of time.
(ii) Three parameter Weibull distribution deterioration.
(iii) Shortages are allowed.
(iv) Infinite planning horizon consider in this model.

II. Assumptions And Notations

2.1 Assumptions

The mathematical model of the economic production quantity problem here is based on the following assumptions:

- The trapezoidal type demand rate, D(t), which is positive and consecutive, is assumed to be a trapezoidal type function of time, that is,

$$D(t)=\begin{cases} a_1+b_1t, & t\le\mu_1 \\ D_{1,} & \mu_1\le t\le t_0 \\ a_2 - b_2t, & t_0\le t\le t_s \end{cases} \quad \text{and} \quad D(t)=\begin{cases} a_3-b_3t, & t\le\mu_2 \\ D_{2,} & t_s^{'}\le t\le t_r^{'} \\ a_4+b_4t, & t_r^{'}\le t\le t_0^{'} \end{cases}$$

Where μ_1 is the time at which demand is changing from the linearly increasing demand to constant demand and t_0 is time at which demand is changing from the constant demand to the linearly deceasing demand for Model 1. (See Fig 4.1 and 4.2) and $a_{1,}a_2 > b_1, b_2$ and μ_2 is the time at which demand is changing from the linearly decreasing demand to constant demand, and $t_r^{'}$ is time at which demand is changing from the constant demand to the linearly increasing demand for Model 2. (See Fig 4.3 and 4.4) and $a_{3,} a_4 > b_3, b_4$.

- Shortages are allowed and fully backlogged.
- There is no repair or replenishment of deteriorated units during the period.
- In this model deterioration rate at any time is assumed to follow three parameter Weibull distribution function

$$\eta_1 = \alpha_1\, \beta_1\, (t-\gamma_1)^{\beta_1 -1} \quad \text{and} \quad \eta_2 = \alpha_2\, \beta_2\, (t-\gamma_2)^{\beta_2 -1}$$

where $\gamma_{1,}\ \gamma_2$ is the location parameter, $t \ge \gamma_{1,}\ \gamma_2$; where $\alpha_{1,}\ \alpha_2$ and β_1, β_2 are scale parameter and shape parameter respectively and $\alpha_{1,}\ \alpha_2 \ge \beta_1, \beta_2$.

- A single item inventory is considered over the prescribed period.
- Planning horizon is infinite and lead-time is zero.

2.2 Notations

The following notations are used in the proposed study:

W: Capacity of the OW

C_{h1}: The holding cost per unit per unit time in OW

C_{h2}: The holding cost per unit per unit time in RW

C_D: The purchasing cost per unit

C_S: Shortage Cost per unit per unit time

C_0: Fix amount of the replenishment costs per \$ per order

C_1: Amount of the set up cost on which learning effect is applied.

t_r: The time at which the inventory level reaches zero in RW for Model.1

t_0: The time at which the inventory level reaches zero in OW for Model.1

t_s: The time at which the shortage reaches the lowest point in the replenishment cycle for Model.1

$t_S^{'}$: The time at which the shortage occur for Model.2

$t_r^{'}$: The time at which the inventory level reaches zero in RW for Model.2

$t_0^{'}$: The time at which the inventory level reaches zero in OW for Model.2

$I_1(t)$: The level of positive inventory in RW at time t for Model.1

$I_2(t)$, $I_3(t)$, $I_4(t)$: The level of positive inventory in OW at time t for model.1

$I_5(t)$: The level of negative inventory at time t for Model.1

$I_1(t)$: The level of negative inventory at time t for Model.2

$I_2(t)$: The level of positive inventory in RW at time t for Model.2

$I_3(t)$: The level of positive inventory in OW at time t for Model.2

$I_4(t)$: The level of positive inventory in OW at time t for Model.2

TC_i: The present value of the total relevant cost per unit time for Model i, i = 1, 2

III. Formulations And Solution Of The Model

There are two storage shortage models under the assumptions described in previous section i.e., one is the traditional model and another is staring with shortage model so far can be found in literature. The traditional model is depicted graphically in Figure 4. 1. for case 1 and 4.2 for case 2. It starts with an instant replenishment and ends with shortages. It has been studied in several papers. In contrast, we propose the other shortage model in which the demand will be met at the end of cycle. In fact, the inventory level in our proposed model starts with shortages and ends without shortages. The proposed shortage model is depicted graphically in Figure 3 for case 1 and 4 for case 2.

IV. 4. Shortages Occur At The End Of The Cycle (Model 1):

4.1 Case 1: When $\eta_1 < \mu_1$

In this section, we discussed the deterministic inventory model for deteriorating items with the traditional two warehouse model where shortage occur at the end of the cycle at time t = 0, a lot size of certain units enters the system from which a portion is backlogged towards previous shortages, W units are kept in OW and the rest is stored in RW. The goods of OW are consumed only after consuming the goods kept in RW. During the interval $(0, t_r)$, the inventory in RW gradually decreases due to linearly increasing demand and it vanishes at $t = t_r$. At OW, the inventory W remains same in the interval $(0, t_r)$, but during (t_r, μ_1) the inventory is depleted due to linearly increasing demand and deterioration. During the interval (μ_1, t_0) the inventory level is depleted due to constant demand and deterioration. By the time t_0, both warehouses are empty and thereafter the shortages are allowed to occur with linearly decreasing function of time. The shortage quantity is supplied to customers at the beginning of the next cycle. By the time t_s, the replenishment cycle restarts. The objective of the traditional model is to determine the timings of t_r , t_s and t_0 so that the total relevant cost (including holding, deterioration, shortage, and ordering costs) per unit time of the inventory system is minimized.

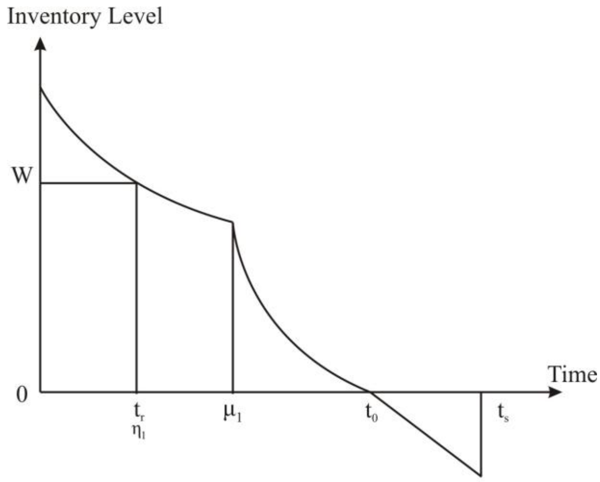

Fig. 4.1 Graphical representation of a two warehouse inventory system for model 1 (Case 1. $\eta_1 < \mu_1$)

Mathematically, the system can be represented by the following system of differential equations:

$$I_1^{'}(t) = -(a_1 + b_1 t), \quad 0 \le t \le t_r \text{ or } 0 \le t \le \eta_1 \quad \ldots(1)$$

$$I_2(t) = W, \quad 0 \le t \le t_r \text{ or } 0 \le t \le \eta_1 \quad \ldots(2)$$

$$I_3^{'}(t) + \alpha_1 \beta_1 \left(t - \gamma_1\right)^{\beta_1 - 1} I_3(t) = -(a_1 + b_1 t), \quad t_r \le t \le \mu_1 \text{ or } \eta_1 \le t \le \mu_1 \quad \ldots(3)$$

$$I_4^{'}(t) + \alpha_1 \beta_1 \left(t - \gamma_1\right)^{\beta_1 - 1} I_4(t) = -D_1, \quad \mu_1 \le t \le t_0 \quad \ldots(4)$$

$$I_5^{'}(t) = -(a_2 - b_2 t), \quad t_0 \le t \le t_s \quad \ldots(5)$$

With the boundary conditions $I_1(t_r)=0$, $I_3(t_r)=W$, $I_4(t_0)=0$, $I_5(t_0)=0$, one can arrive the following equations

$$I_1(t) = a_1\left(t_r - t\right) + \frac{b_1}{2}\left(t_r^{\,2} - t^2\right), \quad 0 \le t \le t_r \text{ or } 0 \le t \le \eta_1 \quad \ldots(6)$$

$$I_2(t) = W, \quad 0 \le t \le t_r \text{ or } 0 \le t \le \eta_1 \quad \ldots(7)$$

$$I_3(t) = We^{\alpha_1\left[\left(t_r - \gamma_1\right)^{\beta_1} - \left(t - \gamma_1\right)^{\beta_1}\right]} + \left[a_1\left(t_r - t\right) + \frac{b_1}{2}\left(t_r^{\,2} - t^2\right) + \alpha_1 a_1 \left\{\frac{1}{\beta_1 + 1}\left(\left(t_r - \gamma_1\right)^{\beta_1 + 1} - \left(t - \gamma_1\right)^{\beta_1 + 1}\right)\right.\right.$$

$$\left. - \left(t - \gamma_1\right)^{\beta_1}\left(t_r - t\right)\right\} + \alpha_1 b_1 \left\{\frac{1}{\beta_1 + 1}\left(t_r\left(t_r - \gamma_1\right)^{\beta_1 + 1} - t\left(t - \gamma_1\right)^{\beta_1 + 1}\right)\right.$$

$$-\frac{1}{(\beta_1+1)(\beta_1+2)}\left(\left(t_r-\gamma_1\right)^{\beta_1+2}-\left(t-\gamma_1\right)^{\beta_1+2}\right)-\frac{1}{2}\left(t-\gamma_1\right)^{\beta_1}\left(t_r^{\ 2}-t^2\right)\Big\}\Big]$$

$t_r \le t \le \mu_1$ or $\eta_1 \le t \le \mu_1$ … (8)

$$I_4(t)=D_1\left[\left(t_0-t\right)+\frac{\alpha_1}{(\beta_1+1)}\left\{\left(t_0-\gamma_1\right)^{\beta_1+1}-\left(t-\gamma_1\right)^{\beta_1+1}\right\}-\alpha_1\left(t-\gamma_1\right)^{\beta_1}\left(t_0-t\right)\right],$$

$\mu_1 \le t \le t_0$ …(9)

$$I_5(t)=a_2\left(t_0-t\right)-\frac{b_2}{2}\left(t_0^{\ 2}-t^2\right),\qquad t_0 \le t \le t_s \qquad \ldots(10)$$

Present worth Ordering Cost

Since replenishment is done at the start of the cycle, the present value of the ordering cost per cycle is given by

$$C(N) = C_0 \qquad \ldots(11)$$

Present worth Holding Cost for RW

The present value of inventory holding cost in RW per cycle can be derived as

$$HC_{RW}=C_{h2}\left[\int_0^{t_r} I_1(t)dt\right]$$

$$=C_{h2}\left(\frac{a_1}{2}t_r^{\ 2}+\frac{b_1}{3}t_r^{\ 3}\right) \qquad \ldots(12)$$

Present worth Holding Cost for OW

The present value of inventory holding cost in OW per cycle can be derived as

$$HC_{OW}=C_{h1}\left[\int_0^{t_r} I_2(t)dt+\int_{t_r}^{\mu_1} I_3(t)dt+\int_{\mu_1}^{t_0} I_4(t)dt\right]$$

$$=C_{h1}\Bigg[Wt_r+W\left(\mu_1-t_r\right)+W\alpha_1\left(t_r-\gamma_1\right)^{\beta_1}\left(\mu_1-t_r\right)-\frac{W\alpha_1}{(\beta_1+1)}\left\{\left(\mu_1-\gamma_1\right)^{\beta_1+1}-\left(t_r-\gamma_1\right)^{\beta_1+1}\right\}$$

$$+a_1\left\{\mu_1 t_r-\frac{\mu_1^{\ 2}}{2}-\frac{t_r^{\ 2}}{2}\right\}+\frac{b_1}{2}\left\{\mu_1 t_r^{\ 2}-\frac{\mu_1^{\ 3}}{3}-\frac{2t_r^{\ 3}}{3}\right\}+\frac{\alpha_1 a_1}{(\beta_1+1)}\left(t_r-\gamma_1\right)^{\beta_1+1}\left(\mu_1-t_r\right)$$

$$-\frac{\alpha_1 a_1}{(\beta_1+1)(\beta_1+2)}\left\{\left(\mu_1-\gamma_1\right)^{\beta_1+2}-\left(t_r-\gamma_1\right)^{\beta_1+2}\right\}-\alpha_1 a_1\Bigg\{\left(t_r-\mu_1\right)\frac{\left(\mu_1-\gamma_1\right)^{\beta_1+1}}{(\beta_1+1)}$$

$$+\frac{\left(\mu_1-\gamma_1\right)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}-\frac{\left(t_r-\gamma_1\right)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}\Bigg\}+\frac{\alpha_1 b_1}{(\beta_1+1)}t_r\left(t_r-\gamma_1\right)^{\beta_1+1}\left(\mu_1-t_r\right)$$

$$-\frac{\alpha_1 b_1}{(\beta_1+1)}\left\{\frac{\mu_1\left(\mu_1-\gamma_1\right)^{\beta_1+2}}{(\beta_1+2)}-\frac{\left(\mu_1-\gamma_1\right)^{\beta_1+3}}{(\beta_1+2)(\beta_1+3)}-t_r\frac{\left(t_r-\gamma_1\right)^{\beta_1+2}}{(\beta_1+2)}+\frac{\left(t_r-\gamma_1\right)^{\beta_1+3}}{(\beta_1+2)(\beta_1+3)}\right\}$$

$$-\frac{\alpha_1 b_1}{(\beta_1+1)(\beta_1+2)}\left(t_r-\gamma_1\right)^{\beta_1+2}\left(\mu_1-t_r\right)+\frac{\alpha_1 b_1}{(\beta_1+1)(\beta_1+2)(\beta_1+3)}\left\{\left(\mu_1-t_r\right)^{\beta_1+3}-\left(t_r-\gamma_1\right)^{\beta_1+3}\right\}$$

$$-\frac{\alpha_1 b_1}{2}\Bigg\{\frac{\left(t_r^{\ 2}-\mu_1^{\ 2}\right)\left(\mu_1-\gamma_1\right)^{\beta_1+1}}{(\beta_1+1)}+\frac{2\mu_1\left(\mu_1-\gamma_1\right)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}-\frac{2\left(\mu_1-\gamma_1\right)^{\beta_1+3}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)}$$

$$-\frac{2t_r\left(t_r-\gamma_1\right)^{\beta_1+2}}{\left(\beta_1+1\right)\left(\beta_1+2\right)}+\frac{2\left(t_r-\gamma_1\right)^{\beta_1+3}}{\left(\beta_1+1\right)\left(\beta_1+2\right)\left(\beta_1+3\right)}\Bigg\}+D_1\Bigg\{\left(\frac{t_0^{\ 2}}{2}-t_0\mu_1+\frac{\mu_1^{\ 2}}{2}\right)$$

$$+\frac{\alpha_1}{\left(\beta_1+1\right)}\left(t_0-\gamma_1\right)^{\beta_1+1}\left(t_0-\mu_1\right)-\frac{\alpha_1}{\left(\beta_1+1\right)\left(\beta_1+2\right)}\left\{\left(t_0-\gamma_1\right)^{\beta_1+2}-\left(\mu_1-\gamma_1\right)^{\beta_1+2}\right\}$$

$$+\frac{\alpha_1}{\left(\beta_1+1\right)}\left(t_0-\mu_1\right)\left(\mu_1-t_r\right)^{\beta_1+1}-\frac{\alpha_1}{\left(\beta_1+1\right)\left(\beta_1+2\right)}\left\{\left(t_0-\gamma_1\right)^{\beta_1+2}-\left(\mu_1-\gamma_1\right)^{\beta_1+2}\right\}\Bigg\}\Bigg]$$

…(13)

Present worth Deterioration Cost

The present value of the deterioration cost per cycle can be derived as

$$DC=C_D\left[\int_{t_r}^{\mu_1}\alpha_1\beta_1\left(t\text{-}\gamma_1\right)^{\beta_1\text{-}1}I_3(t)dt+\int_{\mu_1}^{t_0}\alpha_1\beta_1\left(t\text{-}\gamma_1\right)^{\beta_1\text{-}1}I_4(t)dt\right]$$

$$=\alpha_1\beta_1C_D\Bigg[\Bigg\{\frac{W}{\beta_1}\left(\left(\mu_1-\gamma_1\right)^{\beta_1}-\left(t_r-\gamma_1\right)^{\beta_1}\right)+\frac{W\alpha_1}{\beta_1}\left(t_r-\gamma_1\right)^{\beta_1}\left(\left(\mu_1-\gamma_1\right)^{\beta_1}-\left(t_r-\gamma_1\right)^{\beta_1}\right)$$

$$-\frac{W\alpha_1}{2\beta_1}\left(\left(\mu_1-\gamma_1\right)^{2\beta_1}-\left(t_r-\gamma_1\right)^{2\beta_1}\right)+\frac{a_1}{\beta_1}\left(t_r-\mu_1\right)\left(\mu_1-\gamma_1\right)^{\beta_1}$$

$$+\frac{a_1}{\beta_1\left(\beta_1+1\right)}\left(\left(\mu_1-\gamma_1\right)^{\beta_1+1}-\left(t_r-\gamma_1\right)^{\beta_1+1}\right)+\frac{b_1}{2\beta_1}\left(t_r^{\ 2}-\mu_1^{\ 2}\right)\left(\mu_1-\gamma_1\right)^{\beta_1}$$

$$+\frac{b_1}{\beta_1\left(\beta_1+1\right)}\left(\mu_1\left(\mu_1-\gamma_1\right)^{\beta_1+1}-t_r\left(t_r-\gamma_1\right)^{\beta_1+1}\right)-\frac{b_1}{\beta_1\left(\beta_1+1\right)\left(\beta_1+2\right)}$$

$$\left(\left(\mu_1-\gamma_1\right)^{\beta_1+2}-\left(t_r-\gamma_1\right)^{\beta_1+2}\right)+\frac{\alpha_1a_1}{\beta_1\left(\beta_1+1\right)}\left(t_r-\gamma_1\right)^{\beta_1+1}\left(\left(\mu_1-\gamma_1\right)^{\beta_1}-\left(t_r-\gamma_1\right)^{\beta_1}\right)$$

$$-\frac{\alpha_1a_1}{\left(\beta_1+1\right)\left(2\beta_1+1\right)}\left(\left(\mu_1-\gamma_1\right)^{2\beta_1+1}-\left(t_r-\gamma_1\right)^{2\beta_1+1}\right)-\frac{\alpha_1a_1}{2\beta_1}\left(\left(t_r-\mu_1\right)\left(\mu_1-\gamma_1\right)^{2\beta_1}\right)$$

$$-\frac{\alpha_1a_1}{2\beta_1\left(2\beta_1+1\right)}\left(\left(\mu_1-\gamma_1\right)^{2\beta_1+1}-\left(t_r-\gamma_1\right)^{2\beta_1+1}\right)+\frac{\alpha_1b_1}{\beta_1\left(\beta_1+1\right)}t_r\left(t_r-\gamma_1\right)^{\beta_1+1}$$

$$\left(\left(\mu_1-\gamma_1\right)^{\beta_1}-\left(t_r-\gamma_1\right)^{\beta_1}\right)-\frac{\alpha_1b_1}{\left(\beta_1+1\right)\left(2\beta_1+1\right)}\left(\mu_1\left(\mu_1-\gamma_1\right)^{2\beta_1+1}-t_r\left(t_r-\gamma_1\right)^{2\beta_1+1}\right)$$

$$+\frac{\alpha_1b_1}{\left(\beta_1+1\right)\left(2\beta_1+1\right)\left(2\beta_1+2\right)}\left(\left(\mu_1-\gamma_1\right)^{2\beta_1+2}-\left(t_r-\gamma_1\right)^{2\beta_1+2}\right)-\frac{\alpha_1b_1}{\beta_1\left(\beta_1+1\right)\left(\beta_1+2\right)}$$

$$\left(t_r-\gamma_1\right)^{\beta_1+2}\left(\left(\mu_1-\gamma_1\right)^{\beta_1}-\left(t_r-\gamma_1\right)^{\beta_1}\right)+\frac{\alpha_1b_1}{\left(\beta_1+1\right)\left(\beta_1+2\right)\left(2\beta_1+2\right)}$$

$$\left(\left(\mu_1-\gamma_1\right)^{2\beta_1+2}-\left(t_r-\gamma_1\right)^{2\beta_1+2}\right)-\frac{\alpha_1b_1}{4\beta_1}\left(t_r^{\ 2}-\mu_1^{\ 2}\right)\left(\mu_1-\gamma_1\right)^{2\beta_1}-\frac{\alpha_1b_1}{2\beta_1\left(2\beta_1+1\right)}$$

$$\left(\mu_1\left(\mu_1-\gamma_1\right)^{2\beta_1+1}-t_r\left(t_r-\gamma_1\right)^{2\beta_1+1}\right)+\frac{\alpha_1b_1}{2\beta_1\left(2\beta_1+1\right)\left(2\beta_1+2\right)}\left(\left(\mu_1-\gamma_1\right)^{2\beta_1+2}-\left(t_r-\gamma_1\right)^{2\beta_1+2}\right)\Bigg\}$$

$$+D_1\Bigg\{\left(\mu_1-t_0\right)\frac{\left(\mu_1-\gamma_1\right)^{\beta_1}}{\beta_1}+\frac{\left(t_0-\gamma_1\right)^{\beta_1+1}}{\beta_1\left(\beta_1+1\right)}-\frac{\left(\mu_1-\gamma_1\right)^{\beta_1+1}}{\beta_1+1}+\frac{\alpha_1}{\beta_1\left(\beta_1+1\right)}\left(t_0-\gamma_1\right)^{\beta_1+1}$$

$$\left(\left(t_0-\gamma_1\right)^{\beta_1}-\left(\mu_1-\gamma_1\right)^{\beta_1}\right)+\frac{\alpha_1}{2\beta_1}\left(t_0-\mu_1\right)\left(\mu_1-\gamma_1\right)^{2\beta_1}-\frac{\alpha_1}{\left(\beta_1+1\right)\left(2\beta_1+1\right)}$$

$$\left.\left.\left(\left(t_0-\gamma_1\right)^{2\beta_1+1}-\left(\mu_1-\gamma_1\right)^{2\beta_1+1}\right)-\frac{\alpha_1}{2\beta_1\left(2\beta_1+1\right)}\left(\left(t_0-\gamma_1\right)^{2\beta_1+1}-\left(\mu_1-\gamma_1\right)^{2\beta_1+1}\right)\right\}\right] \quad \ldots(14)$$

Present worth Shortage Cost

The present value of the shortage cost per cycle can be derived as

$$SC=C_S\left[\int_{t_0}^{t_s}\left[-I_5(t)\right]dt\right]$$

$$=C_S\left[a_2\left(\frac{t_0^{\,2}}{2}-t_0t_s+\frac{t_s^{\,2}}{2}\right)-\frac{b_2}{2}\left(\frac{2t_0^{\,3}}{3}-t_0^{\,2}t_s+\frac{t_s^{\,3}}{3}\right)\right] \qquad \ldots(15)$$

Present worth Total Cost

Consequently, the present value of the total relevant cost per unit time for Model 1. (Case 1.$\eta_1 < \mu_1$) during the cycle $(0, t_s)$ is

$$TC_1=\frac{1}{t_s}\left[C(N)+HC_{OW}+HC_{RW}+DC+SC\right] \qquad \ldots(16)$$

4.2 Case 2: When $\eta_1 = \mu_1$

This case is different from the previous one by considering same time to deteriorate the items and changing the demand trend because of the ramp parameter. During the interval $(0, t_r)$, the inventory in RW gradually decreases due to linearly increasing demand and it vanishes at $t = t_r$. In OW, the inventory W remains same in the interval $(0, t_r)$. Afterwards (t_r, μ_1) the inventory is depleted due to linearly increasing demand. At the time interval (μ_1, t_0) the inventory level is depleted due to constant demand and deterioration. By the time t_0, both warehouses are empty and thereafter the shortages are allowed to occur with linearly decreasing function of time. The shortage quantity is supplied to customers at the beginning of the next cycle. By the time t_s, the replenishment cycle restarts.

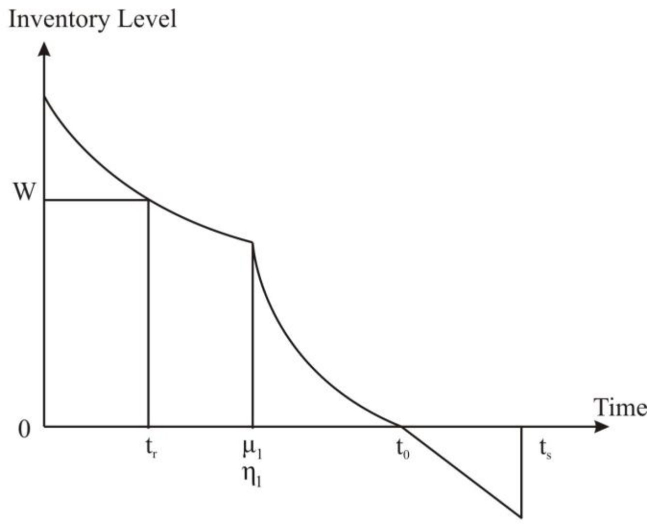

Fig. 4.2 Graphical representation of a two warehouse inventory system for model 1 (Case 2. $\eta_1 = \mu_1$)

The model can be described by the following system of equations:

$I_1'(t)=-(a_1+b_1t)$, $\quad 0 \le t \le t_r \quad \ldots(1)$

$I_2(t)= W$, $\quad 0 \le t \le t_r \quad \ldots(2)$

$I_3'(t)=-(a_1+b_1t)$, $\quad t_r \le t \le \mu_1$ or $t_r \le t \le \eta_1 \quad \ldots(3)$

$$I_4^{'}(t)+\alpha_1\beta_1\left(t-\gamma_1\right)^{\beta_1-1}I_4(t)=-D_1, \quad \mu_1 \le t \le t_0 \text{ or } \eta_1 \le t \le t_0 \qquad \ldots(4)$$

$$I_5^{'}(t)=-(a_2-b_2t), \quad t_0 \le t \le t_s \qquad \ldots(5)$$

With the boundary conditions $I_1(t_r)=0$, $I_3(t_r)=W$, $I_4(t_0)=0$, $I_5(t_0)=0$,one can arrived the following equations

$$I_1(t)=a_1\left(t_r-t\right)+\frac{b_1}{2}\left(t_r^{\,2}-t^2\right), \quad 0 \le t \le t_r \qquad \ldots(6)$$

$$I_2(t)=W, \quad 0 \le t \le t_r \qquad \ldots(7)$$

$$I_3(t)=W+a_1\left(t_r-t\right)+\frac{b_1}{2}\left(t_r^{\,2}-t^2\right) \quad t_r \le t \le \mu_1 \text{ or } t_r \le t \le \eta_1 \qquad \ldots(8)$$

$$I_4(t)=D_1\left[\left(t_0-t\right)+\frac{\alpha_1}{(\beta_1+1)}\left\{\left(t_0-\gamma_1\right)^{\beta_1+1}-\left(t-\gamma_1\right)^{\beta_1+1}\right\}-\alpha_1\left(t-\gamma_1\right)^{\beta_1}\left(t_0-t\right)\right],$$

$$\mu_1 \le t \le t_0 \text{ or } \eta_1 \le t \le t_0 \qquad \ldots(9)$$

$$I_5(t)=a_2\left(t_0-t\right)-\frac{b_2}{2}\left(t_0^{\,2}-t^2\right), \quad t_0 \le t \le t_s \qquad \ldots(10)$$

Present worth Ordering Cost

Since replenishment is done at the start of the cycle, the present value of the ordering cost per cycle is given by

$$C(N) = C_0 \qquad \ldots(11)$$

Present worth Holding Cost for RW

The present value of inventory holding cost in RW per cycle can be derived as

$$HC_{RW}=C_{h2}\left[\int_0^{t_r} I_1(t)dt\right]$$

$$=C_{h2}\left[\frac{a_1}{2}t_r^{\,2}+\frac{b_1}{3}t_r^{\,3}\right] \qquad \ldots(12)$$

Present worth Holding Cost for OW

The present value of inventory holding cost in OW per cycle can be derived as

$$HC_{OW}=C_{h1}\left[\int_0^{t_r} I_2(t)dt+\int_{t_r}^{\mu_1} I_3(t)dt+\int_{\mu_1}^{t_0} I_4(t)dt\right]$$

$$=C_{h1}\left[Wt_r+W\left(\mu_1-t_r\right)+a_1\left\{\mu_1 t_r-\frac{\mu_1^{\,2}}{2}-\frac{t_r^{\,2}}{2}\right\}+\frac{b_1}{2}\left\{\mu_1 t_r^{\,2}-\frac{\mu_1^{\,3}}{3}-\frac{2t_r^{\,3}}{3}\right\}\right.$$

$$+D_1\left\{\left(\frac{t_0^{\,2}}{2}-t_0\mu_1+\frac{\mu_1^{\,2}}{2}\right)+\frac{\alpha_1}{(\beta_1+1)}\left(t_0-\gamma_1\right)^{\beta_1+1}\left(t_0-\mu_1\right)\right.$$

$$-\frac{\alpha_1}{(\beta_1+1)(\beta_1+2)}\left\{\left(t_0-\gamma_1\right)^{\beta_1+2}-\left(\mu_1-\gamma_1\right)^{\beta_1+2}\right\}+\frac{\alpha_1}{(\beta_1+1)}\left(t_0-\mu_1\right)\left(\mu_1-\gamma_1\right)^{\beta_1+1}$$

$$\left.\left.-\frac{\alpha_1}{(\beta_1+1)(\beta_1+2)}\left\{\left(t_0-\gamma_1\right)^{\beta_1+2}-\left(\mu_1-\gamma_1\right)^{\beta_1+2}\right\}\right\}\right] \qquad \ldots(13)$$

Present worth Deterioration Cost

The present value of the deterioration cost per cycle can be derived as

$$DC=C_D\left[\int_{\mu_1}^{t_0}\alpha_1\beta_1\left(t-\gamma_1\right)^{\beta_1-1}I_4(t)dt\right]$$

$$= C_D\alpha_1\beta_1 D_1\Bigg[(\mu_1 - t_0)\frac{(\mu_1-\gamma_1)^{\beta_1}}{\beta_1} + \frac{1}{(\beta_1+1)}\left\{(t_0-\gamma_1)^{\beta_1+1} - (\mu_1-\gamma_1)^{\beta_1+1}\right\}$$

$$-\frac{\alpha_1}{\beta_1(\beta_1+1)}(t_0-\gamma_1)^{\beta_1+1}\left\{(t_0-\gamma_1)^{\beta_1} - (\mu_1-\gamma_1)^{\beta_1}\right\} - \frac{\alpha_1}{(\beta_1+1)(2\beta_1+1)}$$

$$\left\{(t_0-\gamma_1)^{2\beta_1+1} - (\mu_1-\gamma_1)^{2\beta_1+1}\right\} + \frac{\alpha_1}{2\beta_1}(t_0-\mu_1)(\mu_1-\gamma_1)^{2\beta_1}$$

$$-\frac{\alpha_1}{2\beta_1(2\beta_1+1)}\left\{(t_0-\gamma_1)^{2\beta_1+1} - (\mu_1-\gamma_1)^{2\beta_1+1}\right\}\Bigg]\Bigg] \qquad \ldots(14)$$

Present worth Shortage Cost

The present value of the shortage cost per cycle can be derived as

$$SC = C_S\left[\int_{t_0}^{t_s}\left[-I_5(t)\right]dt\right]$$

$$= C_S\left[a_2\left(\frac{t_0^{\,2}}{2} - t_0 t_s + \frac{t_s^{\,2}}{2}\right) - \frac{b_2}{2}\left(\frac{2t_0^{\,3}}{3} - t_0^{\,2}t_s + \frac{t_s^{\,3}}{3}\right)\right] \qquad \ldots(15)$$

Present worth Total Cost

The present value of the total relevant cost per unit time for Model 1. (Case 2. $\mu_1 = \eta_1$) during the cycle $(0, t_s)$ is

$$TC_2 = \frac{1}{t_s}\left[C(N) + HC_{OW} + HC_{RW} + DC + SC\right] \qquad \ldots(16)$$

V. Appendix

To minimize total average cost per unit time (TC_i), i = 1, 2 for both cases of Model 1 and Model 2. The optimal values of t_r , t_0 and t_s for Model 1 and $t_s^{'}$, $t_r^{'}$ and $t_0^{'}$ for Model 2 can be obtained by solving the following equations simultaneously from the equations

$$\frac{\partial TC_i}{\partial t_r} = 0, \qquad \frac{\partial TC_i}{\partial t_0} = 0, \qquad \frac{\partial TC_i}{\partial t_S} = 0, \qquad \ldots(5.1)$$

$$\frac{\partial TC_i}{\partial t_S^{'}} = 0, \qquad \frac{\partial TC_i}{\partial t_r^{'}} = 0, \qquad \frac{\partial TC_i}{\partial t_0^{'}} = 0, \qquad \ldots(5.2)$$

Provided, they satisfy the following conditions

$$\frac{\partial^2 TC_i}{\partial t_r^{\,2}} > 0, \frac{\partial^2 TC_i}{\partial t_0^{\,2}} > 0 \text{and} \frac{\partial^2 TC_i}{\partial t_s^{\,2}} > 0 \qquad \ldots(5.3)$$

$$\frac{\partial^2 TC_i}{\partial t_S^{'2}} > 0, \frac{\partial^2 TC_i}{\partial t_r^{'2}} > 0 \text{and} \frac{\partial^2 TC_i}{\partial t_0^{'2}} > 0 \qquad \ldots(5.4)$$

VI. Solution Procedure

We use the classical optimization techniques for finding the minimum value of the total cost. The equation (5.1) consists of different equations for both cases of Model 1 and equation (5.2) consists of different equations for both cases of Model 2 are highly non-linear in the continuous variable t_r, t_0, t_s for both cases of Model 1 and $t_s^{'}$, $t_r^{'}$, $t_0^{'}$ for both cases of Model 2. We have used the mathematical software ***MATLAB 7.0.1.*** to arrive at the solution of our system. We obtained the optimal values. With the use of these optimal values equations provides minimum total average cost per unit time of the system in consideration. Here numerical illustration is to be given from 1st to 10th replenishment.

VII. Numerical Illustrations And Analysis

To elucidate, by the preceding theory the following numerical data is given by:

Example 1:

W = 500 units , C_0 = \$100 per setup, C_{h1} =\$ 0.3, C_{h2} =\$ 0.6, C_D = \$0.5 per unit, C_S = \$3/ unit/unit time, a_1 = 200, b_1 = 5, α_1 = 0.002, β_1 = 2, γ_1 = 0.5, α_2 = 0.002, β_2 = 2, γ_2 = 0.5, a_2 = 220, b_2 = 10, D_1 = 500 units, μ_1 = 10, a_3 = 220, b_3 = 10, a_4 = 200, b_4 = 5, μ_2 = 10, D_2 = 500 units.

Table 1. Optimal Solution for Model 1 (Case 1: $\square_1 < \square_1$)

N	t_r	t_0	t_s	Total Cost (TC)
1	9.3595	10.3638	15.7584	723.334
2	9.3595	10.3637	15.7579	723.290
3	9.3595	10.3637	15.7575	723.264
4	9.3595	10.3637	15.7573	723.246
5	9.3595	10.3637	15.7572	723.232
6	9.3595	10.3637	15.7570	723.220
7	9.3595	10.3637	15.7569	723.211
8	9.3595	10.3637	15.7568	723.202
9	9.3595	10.3637	15.7567	723.195
10	9.3595	10.3636	15.7566	723.188

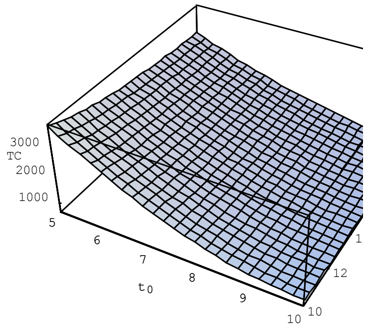

Fig.3. Convexity of the Total Cost Function for Model .1 (Case 1) when t_r fixed

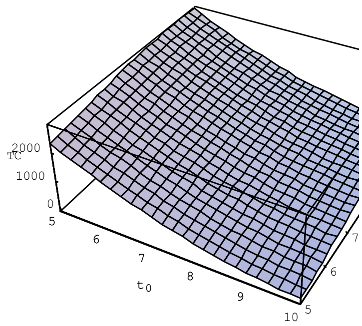

Fig. 4. Convexity of the Total Cost Function for Model .1 (Case 1) when t_s fixed

Table 2. Optimal Solution for Model 2 (Case 1: $\square_2 = \square_2$)

N	t_s	t_r	t_0	Total Cost (TC)
1	0.2212	0.3731	3.8076	155.168
2	0.2209	0.3723	3.8034	154.987
3	0.2208	0.3719	3.8008	154.881
4	0.2207	0.3716	3.7991	154.805
5	0.2206	0.3713	3.7978	154.747
6	0.2206	0.3711	3.7966	154.699
7	0.2205	0.3709	3.7957	154.659
8	0.2205	0.3708	3.7949	154.624
9	0.2204	0.3707	3.7942	154.594
10	0.2204	0.3705	3.7935	154.566

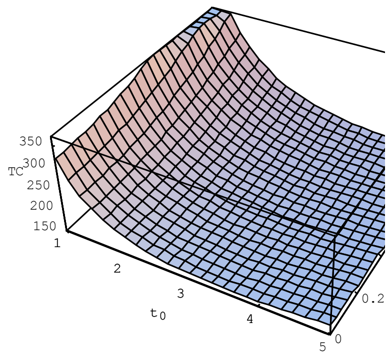

Fig.5. Convexity of the Total Cost Function for Model .2 (Case 1) when t_r fixed

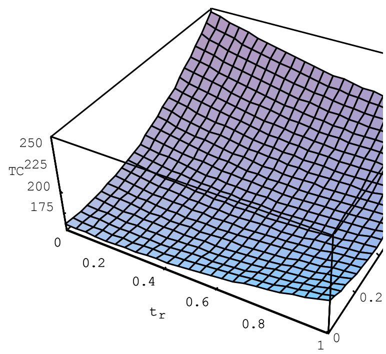

Fig.6. Convexity of the Total Cost Function for Model .2 (Case 1) when t_s fixed

VIII. Sensitivity Analysis

In Tables 1 and 2, some sensitivity analysis of the model is performed by changing the parameter values -50%, -25%, 25% and 50%, taking one at a time and keeping the remaining unchanged. The analysis is performed based on the adjusted results obtained in Tables 4.1, 4.2, 4.3 and 4.4. From Table 3 and 4, some simple characteristics of parameter's impact on total cost.

8.1 Sensitivity Analysis for Model 1:

Table 3: Sensitivity Analysis of Optimal Solution for Model 1: Case 1 (When $\square_1 < \square_1$)

Parameter	-50% Changed			-25%Changed			+25%Changed			+50%Changed		
	N	TC	PCV	N	TC	PCV	N	TC	PCV	N	TC	PCV
α_1	1	723.679	0.05	1	723.484	0.02	1	723.095	-0.03	1	722.900	-0.05
γ_1	1	725.680	0.32	1	724.484	0.16	1	722.096	-0.17	1	720.903	-0.33
W	1	629.634	-12.95	1	676.462	-6.48	1	770.117	6.47	1	816.945	12.94
a_1	1	562.969	-22.17	1	643.129	-11.08	1	803.450	11.08	1	883.610	22.16
a_2	1	570.954	-21.07	1	647.122	-10.54	1	799.457	10.52	1	875.625	21.05
b_1	1	698.630	-3.42	1	710.960	-1.71	1	735.620	1.69	1	747.949	3.40
b_2	1	807.502	11.64	1	765.396	5.82	1	681.184	-5.82	1	639.078	-11.64
D_1	1	717.901	-0.75	1	720.595	-0.37	1	725.984	0.37	1	728.678	0.74

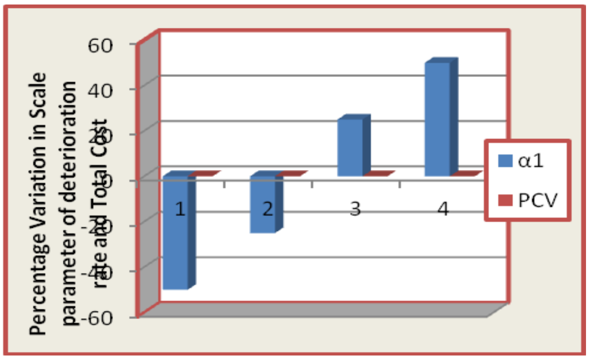

Fig 7. % Variation of total cost with the parameter 'α_1'

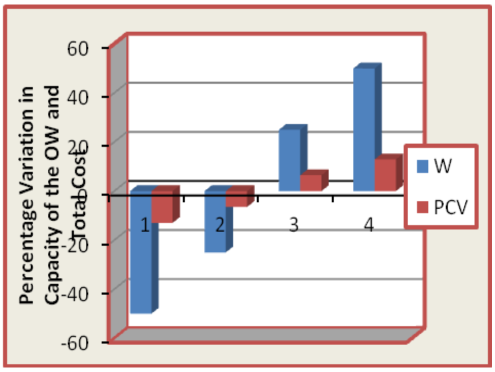

Fig 8. % Variation of total cost with the parameter 'W'

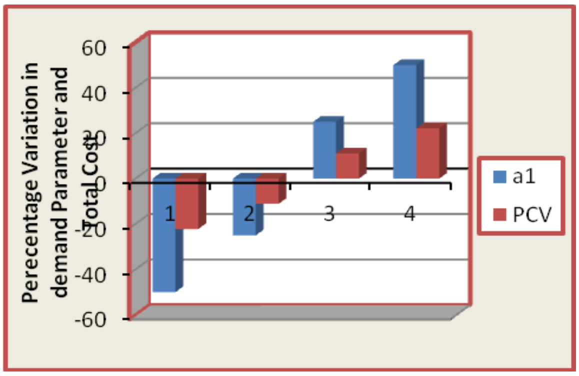

Fig 9.% Variation of total cost with the parameter 'a_1'

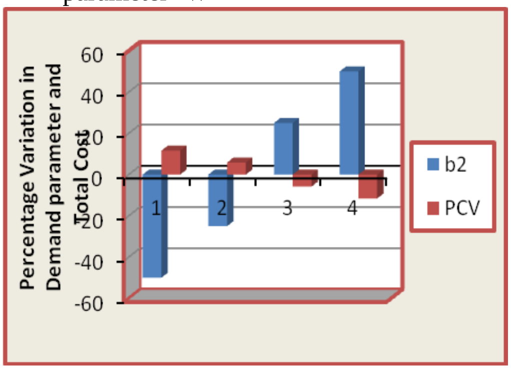

Fig 10. % Variation of total cost with the parameter 'b_2'

8.1.1 Observations for Model 1 (Case 1: $\eta_1 < \mu_1$): The following observations are made based on the above findings:

The values of percentage variation in total costs are the highly sensitive to the following parameters:

- When 'W' decreases by 50%, the value of percentage variation in total cost decreases by over 12%.
- When 'a_1' and 'a_2' increases by 50%, the value of percentage variation in total cost increases by over 22% and 21% respectively.
- When 'b_2' decreases by 50%, the value of percentage variation in total cost increases by over 11%.

  The values of percentage variation in total costs are quite sensitive to the following parameters.
- When 'b_1' decreases by 50%, the value of percentage variation in total cost decreases by over 3%.

  The values of percentage variation in total costs are not so sensitive to the following parameters.
- When 'α_1' decreases by 50%, the value of percentage variation in total cost increases by over 0.005%.
- When 'γ_1' increases by 50%, the value of percentage variation in total cost decreases by over 0.30%.
- When 'D_1' decreases by 50%, the value of percentage variation in total cost decreases by over 0.70%.

Table 4. : Sensitivity Analysis of Optimal Solution for Model 1: Case 2 (When $\eta_1 = \mu_1$)

Parameter	-50% Changed			-25%Changed			+25%Changed			+50%Changed		
	N	TC	PCV	N	TC	PCV	N	TC	PCV	N	TC	PCV
α_1	1	582.177	-1.59	1	586.993	-0.77	1	595.879	0.73	1	599.949	1.42
γ_1	1	592.219	0.11	1	591.890	0.05	1	591.228	-0.05	1	590.895	- 0.11
W	1	562.162	-4.96	1	576.861	-2.48	1	606.259	2.48	1	620.957	4.96
a_1	1	576.861	-2.48	1	584.211	-1.24	1	598.909	1.24	1	606.259	2.48
a_2	1	966.150	63.32	1	778.855	31.66	1	404.265	-31.66	1	216.969	-63.32
b_1	1	589.723	-0.31	1	590.641	-0.15	1	592.479	0.15	1	593.397	0.31
b_2	1	81.258	-86.26	1	336.409	-43.13	1	846.711	43.13	1	1101.86	86.26
D_1	1	483.304	-18.30	1	537.432	-9.15	1	645.688	9.15	1	699.816	18.30

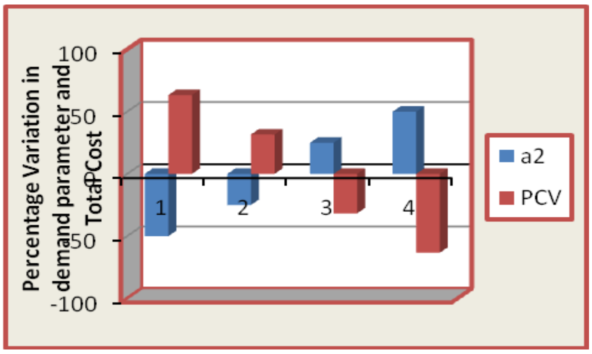

Fig 11.% Variation of total cost with the parameter 'a_2'

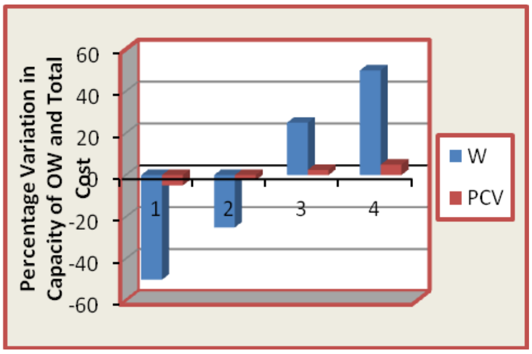

Fig 12. % Variation of total cost with the parameter 'W'

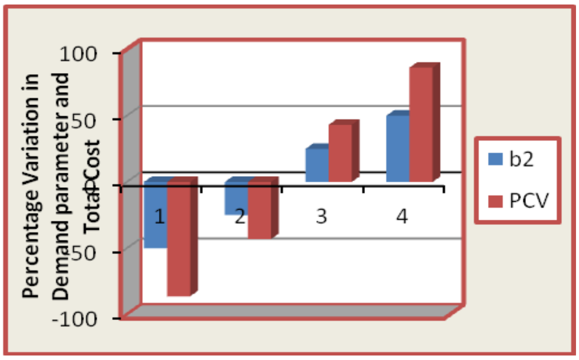

Fig 13 % Variation of total cost with the parameter 'b_2'

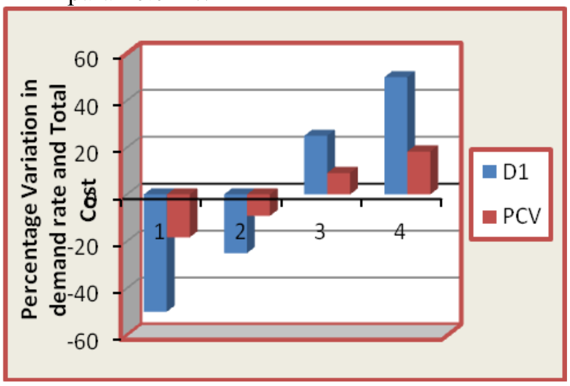

Fig 14.% Variation of total cost with the parameter 'D_1'

4.9.1.2 Observations for Model 1 (Case 2: $\eta_1 = \mu_1$): The following observations are made based on the above findings:

The values of percentage variation in total costs are the highly sensitive to the following parameters:

- When 'a_2' increases by 50%, the value of percentage variation in total cost decreases by over 63%.
- When 'b_2' decreases by 50%, the value of percentage variation in total cost decreases by over 86%.
- When 'D_1' decreases by 50%, the value of percentage variation in total cost decreases by over 18%.

The values of percentage variation in total costs are quite sensitive to the following parameters.

- When 'W' decreases by 50%, the value of percentage variation in total cost decreases by over 4%.
- When 'a_1' and'α_1' increases by 50%, the value of percentage variation in total cost increases by over 2% and 1% respectively.

The values of percentage variation in total costs are not so sensitive to the following parameters.

- When 'γ_1' decreases by 50%, the value of percentage variation in total cost increases by over 0.1%.
- When 'b_1' decreases by 50%, the value of percentage variation in total cost decreases by over 0.3%.

IX. Concluding Remarks

In this chapter we successfully provide a rigorous and efficient method to derive the optimal solution for the inventory models with deteriorating items and trapezoidal type demand rate under the learning effect on set up cost. We study two alternative inventory models for determining the optimal replenishment schedule for two warehouse inventory problem under shortages, in which the inventory deteriorates at a variable rate over time. In this model deterioration rate at any item is assumed to follow three parameter Weibull distribution function of time. This deterioration rate is suitable for items with and without life-period.

The nature of demand of seasonal and fashionable products is increasing-steady-decreasing and becomes asymptotic. The demand pattern assumed here is found to occur not only for all types of seasonal products but also for fashion apparel, computer chips of advanced computers, spare parts, etc. The procedure presented here may be applied to many practical situations. Retailers in supermarket face this type of problem to deal with highly perishable seasonal products. Thus, to make a better combination of increasing-steady-decreasing demand pattern for perishable seasonal products.

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