

On Regular Mildly Generalized (RMG) Open sets in Topological Spaces

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Abstract: In this paper we introduce and study the new class of sets, namely Regular Mildly Generalized Open (briefly, RMG-open) sets, Regular Mildly Generalized neighborhoods (briefly, RMG-nhd), RMG-interior and RMG-closure in topological space and also some properties of new concept has been studied.

Keywords – RMG-closed sets, RMG-open sets, RMG-neighborhoods, RMG-interior, RMG-closure.

I. Introduction

Levine [9, 10] introduces a generalized open and semi-open sets in topological spaces. Regular open sets, pre-open sets, rg-open sets, Mildly-g-open sets have been introduced and studied by Stone [18], A.S. Mashhur et al [5], N. Palaniappan et al [14], J. K. Park et al [16] respectively. In this paper the concept of Regular Mildly Generalized (briefly RMG) open set is introduced and their properties are investigated and Regular Mildly Generalized neighborhood (briefly RMG-nhd), RMG-interior and RMG-closure in a topological spaces.

Throughout this paper X and Y represent the topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of topological space X , $cl(A)$ and $int(A)$ denote the closure of A and interior of A respectively. $X - A$ or A^c Denotes the complement of A in X . Now, we recall the following definitions.

II. Preliminaries

Definition 2.1 A subset A of X is called regular open (briefly r-open) [18] set if $A = int(cl(A))$ and regular closed (briefly r-closed) [18] set if $A = cl(int(A))$.

Definition 2.2 A subset A of X is called pre-open set [5] if $A \subseteq int(cl(A))$ and pre-closed [5] set if $cl(int(A)) \subseteq A$.

Definition 2.3 A subset A of X is called semi-open set [8] if $A \subseteq cl(int(A))$ and semi-closed [8] set if $int(cl(A)) \subseteq A$.

Definition 2.4 A subset A of X is called α -open [11] if $A \subseteq int(cl(int(A)))$ and α -closed [11] if $cl(int(cl(A))) \subseteq A$.

Definition 2.5 A subset A of X is called β -open [1] if $A \subseteq cl(int(cl(A)))$ and β -closed [1] if $int(cl(int(A))) \subseteq A$.

Definition 2.6 A subset A of X is called δ -closed [19] if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x \in X : int(cl(U)) \cap A \neq \emptyset, U \in \mathcal{A}\}$.

Definition 2.7 Let X be a topological space. The finite union of regular open sets in X is said to be π -open [4]. The complement of a π -open set is said to be π -closed.

Definition 2.8 A subset of a topological space (X, τ) is called

1. Generalized closed (briefly g-closed) [9] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
2. Generalized α -closed (briefly $g\alpha$ -closed) [6] if $\alpha-cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in X .
3. Weakly generalized closed (briefly wg-closed) [10] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
4. Strongly generalized closed (briefly g^* -closed) [15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X .
5. Weakly closed (briefly w-closed) [17] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X .
6. Mildly generalized closed (briefly mildly g-closed) [16] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X .
7. Regular weakly generalized closed (briefly rwg-closed) [10] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
8. Weakly π -generalized closed (briefly $w\pi g$ -closed) [13] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is π -open in X .
9. Regular weakly closed (briefly rw-closed) [2] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semiopen in X .
10. Generalized pre closed (briefly gp-closed) [7] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
11. A subset A of a space (X, τ) is called regular generalized closed (briefly rg-closed) [14] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open set in X .

12. π -generalized closed (briefly π g-closed)[3] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

The complements of the above mentioned closed sets are their respective open sets.

The semi-pre-closure (resp. semi-closure, resp. pre-closure, resp. α -closure) of a subset A of X is the intersection of all semi-pre-closed (resp. semi-closed, resp. pre-closed, resp. α -closed) sets containing A and is denoted by $spcl(A)$ (resp. $scl(A)$, resp. $pcl(A)$, resp. $cl(A)$).

Definition 2.3 Regular Mildly Generalized closed (briefly RMG-closed)[20] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is rg-open in X . We denote the family of all RMG-closed sets, RMG-open sets of X by $RMGC(X)$, $RMGO(X)$ respectively.

III. Regular Mildly Generalized Open (Briefly RMG-Open) Sets

In this section, we introduce and studied RMG-open sets in topological space and obtain some of their basic properties. Also we introduce RMG-neighborhood (briefly RMG-nhd) in topological spaces by using the notation of RMG-open sets.

Definition 3.1: A subset A of X is called Regular Mildly Generalized open (briefly, RMG-open) set in X . If $X - A$ is RMG-closed set in X . The family of all RMG-open sets is denoted by $RMGO(X)$.

Theorem 3.2: Every pre-open set is RMG-open set in X .

Proof: Let A be a pre-open set in X . Then by $X - A$ is pre-closed. By Theorem 3.2[20] every pre-closed set is RMG-closed, $X - A$ is RMG-closed set in X . Therefore A is RMG-open set in X .

The converse of the above theorem need not be true as seen from the following example.

Example 3.3: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $A = \{c\}$ is RMG-open set but not pre-open set in X .

Theorem 3.4: Every RMG-open set is Mildly-g-open set in X .

Proof: Let A be a RMG-open set in X . Then $X - A$ is RMG-closed. By Theorem 3.4[20] Every RMG-closed set is Mildly-g-closed, $X - A$ is RMG-closed. Therefore A is Mildly-g-open set in X .

The converse of above Theorem need not be true as seen from the following examples.

Examples 3.5: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then $A = \{a, c\}$ and $B = \{b, c\}$ are mildly-g-open set but not RMG-open set in X .

Corollary 3.6: 1. Every $g\alpha$ -open set is RMG-open set in X .

2. Every w -open set is RMG-open set in X .

3. Every open set is RMG-open set in X .

4. Every δ -open set is RMG-open set in X .

5. Every π -open set is RMG-open set in X .

6. Every regular open set is RMG-open set in X .

Proof:

1. Let A be a $g\alpha$ -open set in X . Then $X - A$ is $g\alpha$ -closed set. By Theorem 3.6.1[20] every $g\alpha$ -closed set is RMG-closed, $X - A$ is RMG-closed. Therefore A is RMG-open set in X .

2. Let A be a w -open set in X . Then $X - A$ is w -closed. By Theorem 3.6.2[20] every w -closed set is RMG-closed, $X - A$ is RMG-closed. Therefore A is RMG-open set in X .

3. Let A be a open set in X . Then $X - A$ is closed. By Theorem 3.6.3[20] every closed set is RMG-closed, $X - A$ is RMG-closed. Therefore A is RMG-open set in X .

4. Let A be a δ -open set in X . Then $X - A$ is δ -closed. By Theorem 3.6.4[20] every δ -closed set is RMG-closed, $X - A$ is RMG-closed. Therefore A is RMG-open set in X .

5. Let A be a π -open set in X . Then $X - A$ is π -closed. By Theorem 3.6.5[20] every π -closed set is RMG-closed, $X - A$ is RMG-closed. Therefore A is RMG-open set in X .

6. Let A be a regular open set in X . Then $X - A$ is regular closed. By Theorem 3.6.6[20] every regular closed set is RMG-closed, $X - A$ is RMG-closed. Therefore A is RMG-open set in X .

The converse of Corollary 3.6 is not true as shown in below examples.

Example 3.7: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$.

1. Let $A = \{c\}$ is RMG-open but not $g\alpha$ -open set in X .

2. Let $A = \{c\}$ is RMG-open but not w -open set in X .

3. Let $A = \{c\}$ is RMG-open but not open set in X .

4. Let $A = \{c\}$ is RMG-open but not δ -open set in X .

5. Let $A = \{c\}$ is RMG-open but not π -open set in X .

6. Let $A = \{c\}$ is RMG-open but not regular open set in X .

Corollary 3.8:

1. Every RMG-open set is wg-open set in X.
2. Every RMG-open set is $w\pi g$ -open set in X.
3. Every RMG-open set is rwg-open set in X.

Proof: 1. Let A be a RMG-open set in X. Then $X - A$ is RMG-closed. By Theorem 3.8.1[20] every RMG-closed set is wg-closed, $X - A$ is RMG-closed. Therefore A is wg-open set in X.

2. Let A be a RMG-open set in X. Then $X - A$ is RMG-closed. By Theorem 3.8.2[20] every RMG-closed set is $w\pi g$ -closed, $X - A$ is RMG-closed. Therefore A is $w\pi g$ -open set in X.

3. Let A be a RMG-open set in X. Then $X - A$ is RMG-closed. By Theorem 3.8.3[20] every RMG-closed set is rwg-closed, $X - A$ is rwg-closed. Therefore A is rwg-open set in X.

The converse of corollary 3.8 is not true as shown in below examples.

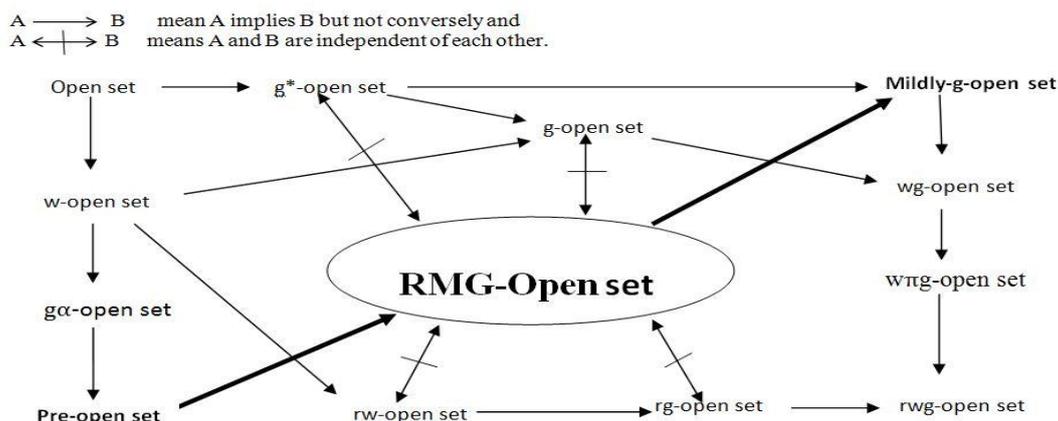
Example 3.9: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$.

1. Let $A = \{c\}$ is wg-open but not RMG-open set in X.
2. Let $A = \{b, d\}$ is $w\pi g$ -open but not RMG-open set in X.
3. Let $A = \{d\}$ is rwg-open but not RMG-open set in X.

Remark 3.10: The concept of semi-open, semi-pre-open, g-open, g^* -open, rg-open, rw-open, πg -open sets are independent with the concept of RMG-open sets as shown in the following example.

Example 3.11: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then the set $\{a, b, d\}$ is RMG-open set. However it can be verified that it is not g-open, not g^* -open, not πg -open, not rg-open, not rw-open., Also $\{b, d\}$ is both semi-open and semi pre open but not RMG-open .

Remark 3.12 From the above discussions and known results we have the following implications in the following diagram, by



Remark 3.13: The intersection of two RMG-open sets in X is need not be a RMG- open set in X.

Examples 3.14: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Now $A = \{a, b, d\}$ and $B = \{a, c, d\}$ are RMG-open sets in X, then $A \cap B = \{a, b, d\} \cap \{a, c, d\} = \{a, d\}$ which is not RMG- open set in X

Remark 3.15: The union of two RMG-open subsets of X is need not be RMG-open set in X.

Example 3.16: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Now $A = \{a\}$ and $B = \{c\}$ are RMG-open sets in X, then $A \cup B = \{a\} \cup \{c\} = \{a, c\}$ which is not RMG-open set in X.

Remark 3.17: Complement of a RMG-open set need not be RMG-open set in X.

Example 3.18: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $A = \{a, b\}$ is RMG-open set but $X - \{a, b\} = \{c, d\}$ is not RMG-open set in X.

Theorem 3.19: A subset A of a topological space X is RMG-open iff $F \subset \text{int}(\text{cl}(A))$ whenever $F \subset A$ and F is rg-closed set in X.

Proof: Assume A is RMG-open then $X - A$ is RMG-closed. Let F be a rg-closed set in X contained in A. Then $X - F$ is a rg-open set in X containing $X - A$. Since $X - A$ is RMG-closed, $\text{cl}(\text{int}(X - A)) \subset X - F$ this implies $X - \text{int}(\text{cl}(A)) \subset X - F$. Consequently $F \subset \text{int}(\text{cl}(A))$.

Conversely, let $F \subset \text{int}(\text{cl}(A))$ whenever $F \subset A$ and F is rg-closed in X. Let G be rg-open set containing $X - A$ then $X - G \subset \text{int}(\text{cl}(A))$. Hence $\text{cl}(\text{int}(X - A)) \subset G$. This prove that $X - A$ is RMG-closed and hence A is RMG-open set in X.

Theorem 3.20: If $A \subseteq X$ is RMG-closed set in X, then $\text{cl}(\text{int}(A)) - A$ is RMG-open set in X.

Proof: Let $A \subseteq X$ is RMG-closed and let F be a rg-closed set such that $F \subseteq \text{cl}(\text{int}(A)) - A$. Then by Theorem 3.21[20], $F = \emptyset$, that implies $F \subseteq \text{int}(\text{cl}(\text{cl}(\text{int}(X - A)))) - A$. This proves that $\text{cl}(\text{int}(A)) - A$ is RMG-open.

Theorem 3.21: Every singleton point set in a space X is either RMG-open or rg-closed.

Proof: Let X be a topological space. Let $x \in X$. To prove $\{x\}$ is either RMG-open or rg-closed. That is to prove $X - \{x\}$ is either RMG-closed or rg-open. Which follows from Theorem 3.23 of [20].

Theorem 3.22: If $\text{int}(\text{cl}(A)) \subseteq B \subseteq A$ and A is RMG-open, then B is RMG-open.

Proof: Let A be RMG-open and $\text{int}(\text{cl}(A)) \subseteq B \subseteq A$. Then $X - A \subseteq X - B \subseteq X - \text{int}(\text{cl}(A))$ that implies $X - A \subseteq X - B \subseteq \text{cl}(\text{int}(X - A))$. Since $X - A$ is RMG-closed, by Theorem 3.24 of [20] $X - B$ is RMG-closed. This proves that B is RMG-open.

The converse of above Theorem 3.24 need not be true in general.

Example 3.23: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Now $A = \{a, b, c\}$ and $B = \{b, c\}$. Now A and B both are RMG-open sets. But $\text{int}(\text{cl}(A)) \not\subseteq B \subseteq A$

Theorem 3.24: Let $A \subset Y \subset X$ and A is RMG-open set in X . Then A is RMG-open in Y provided Y is open set in X .

Proof: Let A be RMG-open in X and Y be a open sets in X . Let U be any rg-open in Y such that $A \subset U$. Then $U \subset A \subset Y \subset X$ by the lemma 3.26[20], U is rg-open in X . Since A is RMG-open in X , $U \subseteq \text{int}(\text{cl}(A))$. Also $\text{int}(\text{cl}(A)) \subset \text{int}_y(\text{cl}_y(A))$. Hence A is an RMG-open set in Y .

Theorem 3.25: If a subset A is RMG-open in X and if G is rg-open in X with $\text{int}(\text{cl}(A)) \cup (X - A) \subseteq G$ then $G = X$.

Proof: Suppose that G is an rg-open and $\text{int}(\text{cl}(A)) \cup (X - A) \subseteq G$. Now $(X - A) \subseteq (X - \text{int}(\text{cl}(A))) \cap X - (X - A)$ implies that $(X - G) \subseteq \text{cl}(\text{int}(X - A)) \cap A$. Suppose A is RMG-open. Since $X - G$ is rg-closed and $X - A$ is RMG-closed, then by Theorem 3.21 of [20], $X - G = \emptyset$ and hence $G = X$.

The converse of the above Theorem need not be true in general as shown in example 3.24.

Example 3.26: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Then

$\text{RMGO}(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$ and $\text{RGO}(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}\}$. Let $A = \{b, d\}$ is not an RMG-open set in X . However $\text{int}(\text{cl}(A)) \cup (X - A) = \{b, c\} \cup \{a, c\} = \{a, b, c\}$. So for some rg-open set G , such that $\text{int}(\text{cl}(A)) \cup (X - A) = \{a, b, c\} \subset G$ gives $G = X$ but A is not RMG-open set in X .

Theorem 3.27: Let X be a topological space and $A, B \subseteq X$. If B is RMG-open and $\text{int}(\text{cl}(B)) \subseteq A$, then $A \cap B$ is RMG-open in X .

Proof: Since B is RMG-open and $\text{int}(\text{cl}(B)) \subseteq A$, then $\text{int}(\text{cl}(B)) \subseteq A \cap B \subseteq B$, then by Theorem 3.24, $A \cap B$ is RMG-open set in X .

IV. Regular Mildly Generalized Neighborhoods (Briefly RMG-Nhd)

Definition 4.1 Let (X, τ) be a topological space and let $x \in X$. A subset N is said to be RMG- neighborhood (briefly, RMG-nhd) of x , if and only if there exists an RMG-open set G such that $x \in G \subset N$.

Definition 4.2(i) A subset N of X is a RMG-nhd of $A \subseteq X$ in topological space (X, τ) , if there exists an RMG-open set G such that $A \subset G \subset N$.

(ii) The collection of all RMG-nhd of $x \in X$ is called RMG-nhd system at $x \in X$ and shall be denoted by $\text{RMG-N}(x)$.

Theorem 4.3: Every neighborhood N of $x \in X$ is a RMG-nhd of x .

Proof: Let N be neighborhood of point $x \in X$. To prove that N is a RMG-nhd of x . By definition of neighborhood, there exists an open set G such that $x \in G \subset N$. As every open set is RMG-open, G is an RMG-open set in X . Then there exists a RMG-open set G such that $x \in G \subset N$. Hence N is RMG-nhd of x .

Remark 4.4: In general, a RMG-nhd N of x in X need not be nhd of x in X , as shown from example 4.5.

Example 4.5: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $\text{RMGO}(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$. The set $\{a, c\}$ is RMG-nhd of the point c , since the RMG-open set $\{c\}$ is such that $c \in \{c\} \subset \{a, c\}$. However, the set $\{a, c\}$ is not a neighbourhood of the point c , since no open set G exists such that $c \in G \subset \{a, c\}$.

Theorem 4.6: If a subset N of a space X is RMG-open, then N is a RMG-nhd of each of its points.

Proof: Suppose N is RMG-open. Let $x \in N$ we claim that N is a RMG-nhd of x . For N is a RMG-open set such that $x \in N \subset N$. Since x is an arbitrary point of N , it follows that N is a RMG-nhd of each of its points.

Remark 4.7: The converse of the above theorem is not true in general as seen from the following example 4.8.

Example 4.8: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $\text{RMGO}(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$. The set $\{a, c\}$ is a RMG-nhd of the point a , since the RMG-open set $\{a\}$ is such that $a \in \{a\} \subset \{a, c\}$. Also the set $\{a, c\}$ is a RMG-nhd of the point c , since the RMG-open

set $\{c\}$ is such that $c \in \{c\} \subset \{a, c\}$ i.e. $\{a, c\}$ is a RMG-nhd of each of its points. However the set $\{a, c\}$ is not a RMG-open set in X .

Theorem 4.9: Let X be a topological space. If F is a RMG-closed subset of X and $x \in (X - F)$, then there exists a RMG-nhd N of x such that $N \cap F = \emptyset$.

Proof: Let F be RMG-closed subset of X and $x \in (X - F)$. Then $(X - F)$ is an RMG-open set of X . By Theorem 4.6, $(X - F)$ contains a RMG-nhd of each of its points. Hence there exists a RMG-nhd N of x such that $N \subset X - F$. That is $N \cap F = \emptyset$.

Theorem 4.10: Let X be a topological space and for each $x \in X$, let $\text{RMG-N}(x)$ be the collection of all RMG-nhds of x . Then we have the following results.

- i) $\forall x \in X, \text{RMG} - \text{N}(x) \neq \emptyset$.
- ii) $N \in \text{RMG} - \text{N}(x) \Rightarrow x \in N$.
- iii) $N \in \text{RMG} - \text{N}(x)$ and $N \subset M \Rightarrow M \in \text{RMG} - \text{N}(x)$.
- iv) $N \in \text{RMG} - \text{N}(x) \Rightarrow \exists M \in \text{RMG} - \text{N}(x)$ such that $M \subset N$ and $M \in \text{RMG-N}(y)$ for every $y \in M$.

Proof: i) Since X is an RMG-open set, it is a RMG-nhd of every $x \in X$. Hence \exists at least one RMG-nhd(X) for each $x \in X$. Hence $\text{RMG} - \text{N}(x) \neq \emptyset$ for every $x \in X$.

ii) If $N \in \text{RMG-N}(x)$, then N is a RMG-nhd of x . So by definition of RMG-nhd $x \in N$.

iii) Let $N \in \text{RMG} - \text{N}(x)$ and $N \subset M$, then there is an RMG-open set G such that $x \in G \subset N$. Since $N \subset M$, $x \in G \subset M$ and so M is a RMG-nhd of x . Hence $M \in \text{RMG} - \text{N}(x)$.

iv) If $N \in \text{RMG} - \text{N}(x)$, then there exists an RMG-open set M such that $x \in M \subset N$. Since M is an RMG-open set, it is a RMG-nhd of each of its points. Therefore $M \in \text{RMG} - \text{N}(y)$ for all $y \in M$.

V. Regular Mildly Generalized Interior (Rmg-Interior) Operator.

In this section, the notation of RMG-interior is defined and some of its basic properties are studied.

Definition 5.1 (i): Let A be a subset of (X, τ) . A point $x \in A$ is said to be RMG-interior point of A if and only if A is RMG-neighbourhood of x . The set of all RMG-interior points of A is called the RMG-interior of A and is denoted by $\text{RMG-int}(A)$.

Definition (ii): Let (X, τ) be a topological space and $A \subset X$. Then $\text{RMG-int}(A)$ is the union of all RMG-open sets contained in A .

Theorem 5.2: Let A is a subset of (X, τ) , then $\text{RMG-int}(A) = \cup \{G : G \text{ is RMG-open, } G \subset A\}$.

Proof: Let A be a subset of (X, τ) . $x \in \text{RMG-int}(A)$

- $\Leftrightarrow x$ is a RMG-interior point of A .
- $\Leftrightarrow A$ is a RMG-nhd of point x .
- \Leftrightarrow There exists an RMG-open set G such that $x \in G \subset A$
- $\Leftrightarrow x \in \cup \{G : G \text{ is RMG-open, } G \subset A\}$.

Hence $\text{RMG-int}(A) = \cup \{G : G \text{ is RMG-open } G \subset A\}$.

Theorem 5.3: Let A and B are subsets of (X, τ) . Then

- i) $\text{RMG-int}(\emptyset) = \emptyset$ and $\text{RMG-int}(X) = X$.
- ii) $\text{RMG-int}(A) \subset A$.
- iii) If B is any RMG-open sets contained in A , then $B \subset \text{RMG-int}(A)$.
- iv) If $A \subset B$, then $\text{RMG-int}(A) \subset \text{RMG-int}(B)$.
- v) $\text{RMG-int}(\text{RMG-int}(A)) = \text{RMG-int}(A)$.

Proof: i) Obvious

- ii) Let $x \in \text{RMG-int}(A) \Rightarrow x$ is a RMG-interior point of A
 - $\Rightarrow A$ is a RMG-nhd of x
 - $\Rightarrow x \in A$

Thus $x \in \text{RMG-int}(A) \Rightarrow x \in A$. Hence $\text{RMG-int}(A) \subset A$.

iii) Let B be a any RMG-open set such that $B \subset A$. Let $x \in B$. Then since B is an RMG open set contained in A . x is an RMG-interior point of A . That is $x \in \text{RMG-int}(A)$. Hence $B \subset \text{RMG-int}(A)$.

iv) Let A and B are subsets of X such that $A \subset B$. Let $x \in \text{RMG-int}(A)$. Then x is an RMG-interior point of A and so A is a RMG-nhd of x . since $A \subset B$, B is also a RMG-nhd of x . This implies that $x \in \text{RMG-int}(B)$. Thus we have show that $x \in \text{RMG-int}(A) \Rightarrow x \in \text{RMG-int}(B)$. Hence $\text{RMG-int}(A) \subset \text{RMG-int}(B)$.

v) Since $\text{RMG-int}(A)$ is a RMG-open set in X , it follows that $\text{RMG-int}(\text{RMG-int}(A)) = \text{RMG-int}(A)$.

Theorem 5.4: If a subset A of space X is RMG-open, then $\text{RMG-int}(A)=A$.

Proof: Let A be a RMG-open subset of X and we know that $\text{RMG-int}(A) \subset A$. Since A is RMG-open set contained in A . From the Theorem 5.3(iii), $A \subset \text{RMG-int}(A)$ and hence we get $\text{RMG-int}(A)=A$.

The converse of the above theorem need not be true as seen in the following example.

Example 5.5: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $\text{RMGO}(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$. Note that $\text{RMG-int}(\{a, c\}) = \{a\} \cup \{c\} \cup \emptyset = \{a, c\}$. But $\{a, c\}$ is not a RMG-open set in X .

Theorem 5.6: If A and B are sub sets of X , then $\text{RMG-int}(A) \cup \text{RMG-int}(B) \subset \text{RMG-int}(A \cup B)$.

Proof: Since $A \subset A \cup B$ and $B \subset A \cup B$. Using the Theorem 5.3(iv), $\text{RMG-int}(A) \subset \text{RMG-int}(A \cup B)$ and $\text{RMG-int}(B) \subset \text{RMG-int}(A \cup B)$ This implies $\text{RMG-int}(A) \cup \text{RMG-int}(B) \subset \text{RMG-int}(A \cup B)$.

Theorem 5.7: If A and B are subsets of a space X , then $\text{RMG-int}(A \cap B) \subset \text{RMG-int}(A) \cap \text{RMG-int}(B)$.

Proof: Let A and B be subsets of X . Clearly $A \cap B \subset A$ and $A \cap B \subset B$. By Theorem 5.3(iv), $\text{RMG-int}(A \cap B) \subset \text{RMG-int}(A)$ and $\text{RMG-int}(A \cap B) \subset \text{RMG-int}(B)$. Hence $\text{RMG-int}(A \cap B) \subset \text{RMG-int}(A) \cap \text{RMG-int}(B)$.

Theorem 5.8: If A is a subset of X , then $\text{int}(A) \subset \text{RMG-int}(A)$.

Proof: Let A be a subset of a space X .

$x \in \text{int}(A) \Rightarrow x \in \cup \{G: G \text{ open, } G \subset A\}$.

\Rightarrow There exists an open set G such that $x \in G \subset A$

\Rightarrow There exists an RMG-open set G such that $x \in G \subset A$, as every open set is an RMG-open set in X .

$\Rightarrow x \in \cup \{G: G \text{ is RMG-open, } G \subset A\}$.

$\Rightarrow x \in \text{RMG-int}(A)$.

Thus, $x \in \text{int}(A) \Rightarrow x \in \text{RMG-int}(A)$. Hence $\text{int}(A) \subset \text{RMG-int}(A)$.

Remark 5.9: Containment relation in the above Theorem 5.8 may be proper as seen from the following example.

Example 5.10: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Then $\text{RMGO}(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$. Let $A = \{a, b\}$. Now $\text{RMG-int}(A) = \{a, b\}$ and $\text{int}(A) = \{a\}$. It follows that $\text{int}(A) \subset \text{RMG-int}(A)$ and $\text{int}(A) \neq \text{RMG-int}(A)$.

Remark 5.11: If A is sub set of X , then

i) $w\text{-int}(A) \subset \text{RMG-int}(A)$.

ii) $g\alpha\text{-int}(A) \subset \text{RMG-int}(A)$.

iii) $p\text{-int}(A) \subset \text{RMG-int}(A)$.

Theorem 5.12: If A is a subset of X , then $\text{RMG-int}(A) \subset \text{Mildly-g-int}(A)$.

Proof: let A be a subset of a space X .

$x \in \text{RMG-int}(A) \Rightarrow x \in \cup \{G: G \text{ RMG-open, } G \subset A\}$.

\Rightarrow There exists an RMG-open set G such that $x \in G \subset A$

\Rightarrow There exists an Mildly-g-open set G such that $x \in G \subset A$, as every RMG-open set is an Mildly-g-open set in X .

$\Rightarrow x \in \cup \{G: G \text{ is Mildly-g-open, } G \subset A\}$.

$\Rightarrow x \in \text{Mildly-g-int}(A)$.

Thus, $x \in \text{RMG-int}(A) \Rightarrow x \in \text{Mildly-g-int}(A)$. Hence $\text{RMG-int}(A) \subset \text{Mildly-g-int}(A)$.

Remark 5.13: If A is sub set of X , then $\text{RMG-int}(A) \subset \text{wg-int}(A)$.

VI. Regular Mildly Generalized Closure (Rmg-Closure) Operator.

Now we introduced the notation of RMG-closure in topological spaces by using the notation of RMG-closed sets and obtained some of their properties. For any $A \subset X$, it is proved that the complement of RMG-interior of RMG-closure of the complement of A .

Definition 6.1: Let A be a subset of a space (X, τ) . We defined the RMG-closure of A to be a intersection of all RMG-closed sets containing A . In symbol we have $\text{RMG-cl}(A) = \cap \{F: A \subset F \in \text{RMGC}(X)\}$.

Theorem 6.2: Let A and B are subsets of (X, τ) . Then

i) $\text{RMG-cl}(\emptyset) = \emptyset$ and $\text{RMG-cl}(X) = X$.

ii) $A \subset \text{RMG-cl}(A)$.

iii) If B is any RMG-closed sets contained in A , then $\text{RMG-cl}(A) \subset B$

iv) If $A \subset B$, then $\text{RMG-cl}(A) \subset \text{RMG-cl}(B)$.

v) $\text{RMG-cl}(\text{RMG-cl}(A)) = \text{RMG-cl}(A)$.

Proof: i) Obvious.

ii) By the definition of RMG-closure of A, it is obvious that $A \subseteq \text{RMG-cl}(A)$.

iii) Let B be any RMG-closed set containing A. Since $\text{RMG-cl}(A)$ is the intersection of all RMG-closed set containing A, $\text{RMG-cl}(A)$ is contained in every RMG-closed set containing A. Hence in particular $\text{RMG-cl}(A) \subseteq B$.

iv)]Let A and B be subsets of X. such that $A \subseteq B$. By the definition of RMG-closure, $\text{RMG-cl}(B) = \bigcap \{F : B \subseteq F \in \text{RMGC}(X)\}$. If $B \subseteq F \in \text{RMGC}(X)$, then $\text{RMG-cl}(B) \subseteq F$. Since $A \subseteq B$, $A \subseteq B \subseteq F \in \text{RMGC}(X)$, We have $\text{RMG-cl}(A) \subseteq F$. Therefore $\text{RMG-cl}(A) \subseteq \bigcap \{F : B \subseteq F \in \text{RMGC}(X)\} = \text{RMG-cl}(B)$. That is $\text{RMG-cl}(A) \subseteq \text{RMG-cl}(B)$.

v) Since $\text{RMG-cl}(A)$ is a RMG-closed set in X, it follows that $\text{RMG-cl}(\text{RMG-cl}(A)) = \text{RMG-cl}(A)$.

Theorem 6.3: If $A \subseteq X$ is RMG-closed, then $\text{RMG-cl}(A) = A$.

Proof: Let A be a RMG-closed subset of X. We know that $A \subseteq \text{RMG-cl}(A)$. Also $A \subseteq A$ and A is RMG-closed. By the theorem 6.2 (iii) $\text{RMG-cl}(A) \subseteq A$. Hence $\text{RMG-cl}(A) = A$.

The converse of the above theorem need not be true as seen from the following example.

Example 6.5: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $\text{RMGC}(X) = \{X, \emptyset, \{c\}, \{d\}, \{a, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Now $\text{RMG-cl}\{b\} = \{a, b, d\} \cap \{b, c, d\} \cap X = \{b, d\}$, but $\{b, d\}$ is not a RMG-closed subset in X.

Theorem 6.7: If A and B are sub sets of X, then $\text{RMG-cl}(A) \cup \text{RMG-cl}(B) \subseteq \text{RMG-cl}(A \cup B)$.

Proof: Let A and B are subsets of a space X. Clearly $A \subseteq A \cup B$ and $B \subseteq A \cup B$. By Theorem 6.2 (iv). $\text{RMG-cl}(A) \subseteq \text{RMG-cl}(A \cup B)$ and $\text{RMG-cl}(B) \subseteq \text{RMG-cl}(A \cup B)$. This implies that $\text{RMG-cl}(A) \cup \text{RMG-cl}(B) \subseteq \text{RMG-cl}(A \cup B)$.

Theorem 6.8: If A and B are subsets of a space X, then $\text{RMG-cl}(A \cap B) \subseteq \text{RMG-cl}(A) \cap \text{RMG-cl}(B)$.

Proof: Let A and B be subsets of X. Clearly $A \cap B \subseteq A$ and $A \cap B \subseteq B$. By Theorem 6.2(iv). $\text{RMG-cl}(A \cap B) \subseteq \text{RMG-cl}(A)$ and $\text{RMG-cl}(A \cap B) \subseteq \text{RMG-cl}(B)$. Hence $\text{RMG-cl}(A \cap B) \subseteq \text{RMG-cl}(A) \cap \text{RMG-cl}(B)$.

Theorem 6.9: Let A be a subset of X and $x \in X$. Then $x \in \text{RMG-cl}(A)$ if and only if $\bigcap V \cap A \neq \emptyset$ for every RMG-open set V containing x.

Proof: Let $x \in X$ and $x \in \text{RMG-cl}(A)$. To prove that $\bigcap V \cap A \neq \emptyset$ for every RMG-open set V containing x. Prove the results by contradiction. Suppose there exists a RMG-open set V containing x such that $\bigcap V \cap A = \emptyset$. Then $A \subseteq X - V$ and $X - V$ is RMG-closed. We have $\text{RMG-cl}(A) \subseteq X - V$. This shows that $x \notin \text{RMG-cl}(A)$. Which is contradiction. Hence $\bigcap V \cap A \neq \emptyset$ for every RMG-open set V containing x.

Conversely, let $\bigcap V \cap A \neq \emptyset$ for every RMG-open set V containing x. To prove that $x \in \text{RMG-cl}(A)$. We prove the result by contradiction. Suppose $x \notin \text{RMG-cl}(A)$. Then there exists a RMG-closed subset F containing A such that $x \notin F$. Then $x \in X - F$ and $X - F$ is RMG-open. Also $(X - F) \cap A = \emptyset$, which is a contradiction. Hence $x \in \text{RMG-cl}(A)$.

Theorem 6.10: Let A be a RMG-open set and B be any open set in X. If $A \cap B = \emptyset$, then $A \cap \text{RMG-cl}(B) = \emptyset$.

Proof: Suppose $A \cap \text{RMG-cl}(B) \neq \emptyset$ and $x \in A \cap \text{RMG-cl}(B)$. Then $x \in A$ and $x \in A \cap \text{RMG-cl}(B)$. by above Theorem 6.9 $A \cap B \neq \emptyset$ which is contrary to the hypothesis. Hence $A \cap \text{RMG-cl}(B) = \emptyset$.

Theorem 6.11: If A is a subset of (X, τ), Then $\text{RMG-cl}(A) \subseteq \text{cl}(A)$.

Proof: Let A be a subset of X. By definition of closure, $\text{cl}(A) = \bigcap \{F \subseteq X : A \subseteq F \in \mathcal{C}(X)\}$. If $A \subseteq F \in \mathcal{C}(X)$, then $A \subseteq F \in \text{RMGC}(X)$, because every closed set is RMG-closed. That is $\text{RMG-cl}(A) \subseteq F$. Therefore $\text{RMG-cl}(A) \subseteq \bigcap \{F \subseteq X : A \subseteq F \in \mathcal{C}(X)\} = \text{cl}(A)$. Hence $\text{RMG-cl}(A) \subseteq \text{cl}(A)$.

Remark 6.12: Containment relation in the above Theorem 5.34 may be proper as seen from the following example.

Example 6.13: Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then $\text{RMGC}(X) = \{X, \emptyset, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Let $A = \{c\}$. Now $\text{RMG-cl}(A) = \{c\}$ and $\text{cl}(A) = \{c, d\}$. It follows that $\text{RMG-cl}(A) \subseteq \text{cl}(A)$ and $\text{RMG-cl}(A) \neq \text{cl}(A)$.

Remark 6.14: If A be a subset of space X, then

i) $\text{RMG-cl}(A) \subseteq \text{w-cl}(A)$.

ii) $\text{RMG-cl}(A) \subseteq \text{g}\alpha\text{-cl}(A)$.

iii) $\text{RMG-cl}(A) \subseteq \text{p-cl}(A)$.

Theorem 6.1:5 If A is a subset of space (X, τ) , Then $\text{Mildly-g-cl}(A) \subseteq \text{RMG-cl}(A)$.

Proof: Let A be a subset of X . By definition of Mildly-g -closure, $\text{Mildly-g-cl}(A) = \bigcap \{F \subseteq X : A \subseteq F \in \text{Mildly-g-C}(X)\}$. If $A \subseteq F \in \text{RMGC}(X)$, then $A \subseteq F \in \text{Mildly-g-C}(X)$, because every RMG-closed set is Mildly-g-closed. That is $\text{Mildly-g-cl}(A) \subseteq F$. Therefore $\text{Mildly-g-cl}(A) \subseteq \bigcap \{F \subseteq X : A \subseteq F \in \text{RMC}(X)\} = \text{RMG-cl}(A)$. Hence $\text{Mildly-g-cl}(A) \subseteq \text{RMG-cl}(A)$.

Remark 6.16: If A subset of space X , then $\text{wg-cl}(A) \subseteq \text{RMG-cl}(A)$.

Lemma 6.17: Let A be a subset of a space X . Then

- i) $X - (\text{RMG-int}(A)) = \text{RMG-cl}(X - A)$.
- ii) $\text{RMG-int}(A) = X - (\text{RMG-cl}(X - A))$.
- iii) $\text{RMG-cl}(A) = X - (\text{RMG-int}(X - A))$.

Proof: Let $x \in X - (\text{RMG-int}(A))$. Then $x \notin \text{RMG-int}(A)$. That is every RMG-open set U containing x is such that $U \not\subseteq A$. That is every RMG-open set U containing x such that $U \cap (X - A) \neq \emptyset$. By the Theorem 6.9, $x \in \text{RMG-cl}(X - A)$ and therefore $X - (\text{RMG-int}(A)) \subseteq \text{RMG-cl}(X - A)$.

Conversely, let $x \in \text{RMG-cl}(X - A)$, Then by Theorem 6.9, every RMG-open set U containing x such that $U \not\subseteq A$. This implies by definition of RMG-interior of A , $x \notin \text{RMG-int}(A)$. That is $x \in X - (\text{RMG-int}(A))$ and $\text{RMG-cl}(X - A) \subseteq \text{RMG-cl}(A^c) \subseteq (\text{RMG-int}(A))^c$. Thus $(\text{RMG-int}(A))^c = \text{RMG-cl}(A^c)$.

- ii) Follows by taking complements in (i).
- iii) Follows by replacing A by $X - A$ in (i).

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