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Minimum Spanning Tree of City to City Road Network in Nigeria

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Abstract: *This paper deals with the construction of minimum spanning tree of city – to – city network in Nigeria. Nodes in our network are the 36 state capitals in Nigeria and the FCT, and the arcs are the proposed major roads that link the state capitals. The distances between states are computed using the Lad/Long converter software which makes use of the latitude and longitude of each state capital. We employed Prim's algorithm to determine the minimum spanning tree with Yenagoa as the starting point. The result gives a total distance of 5, 128.5 km.*
Keywords: *Spanning tree, Network, Prim's algorithm, Latitude and Longitude*

I. Introduction

A lot of researches have been done on designing optimal network for purposes of distribution, communication, machine scheduling, gas and pipelines, etc. The tool commonly used for the optimal design of such networks is the Minimal Spanning Tree. A Minimal Spanning Tree problem is one of the most fundamental and intensively studied problems in network optimization problem with many theoretical and practical applications (Ahuja, et al, 1993), (Taha, 2006), (Winston, 2004), (Dippon, 1999), (Seth, 2002), (Nahla, 2011), (Mares, 2008). (Rothfard, 1970) used minimum spanning tree to design optimal offshore natural gas pipeline systems

A number of algorithms exist for the determination of Minimum Spanning Tree; these include Kruskal algorithm, Prim's algorithm and Boruvka algorithm, (Agarval, 2010). The Prim's algorithm was developed in 1957 by a computer scientist, Robert C. Prim and rediscovered by Edsger Diijkstra in the same year. For this reason the algorithm is sometimes known as DJP algorithm (Wilkipedia, 2010). An optimal design of natural gas pipeline of Amaco East Cross field gas pipeline project in Alberta – Canada with a total distance of 66km was reduced to 49.9km by using hamster program software. Steiner minimum spanning tree algorithm was later used to reduce the distance to 48.84km (Dott, 1997).

A study on the optimal design of oil pipeline network for the South Gabon oil field having 33 nodes and 129 possible arcs reduces the total distance of 188.2 miles to 156.2 miles using Prim's algorithm (Brimberg et al, 2003). (Donkor et al, 2011) used Prim's algorithm to determine the Minimum Spanning Tree of length 712km of the West African gas pipeline from Nigeria through Benin and Togo to Ghana.

In this paper, we construct a Minimum Spanning Tree covering 36 state capitals and FCT, namely: Yenagoa, Port – Harcourt, Oweri, Asaba, Awka, Umuhia, Uyo, Calabar, Ababkiliki, Enugu, Lokoja, Lafia, Makurdi, Jalingo, Yola, Maiduguri, Damaturu, Gombe, Bauchi, Jos, Kaduna, Minna, Ilorin, Oshogbo, Adoekiti, Akure, Benin city, Ikeja, Abeokuta, Ibadan, Berrin - Kebbi, Sokoto, Gusau, Katsina, Kano, Dutse and Abuja.

II. Methodology

City – to – City road Network in Nigeria

A network is a collection of points (nodes) linked by arcs (branches). Let N be a set of finite number of nodes, and A be a set of arcs linking the nodes. Then a network is defined by the pair (N, A). In a network, the arc linking two distinct nodes i and j is denoted by (i, j). In every network, there is a flow of some type along its arcs. For instance, in road network, the flows are the vehicle; in communication network, the flows are the messages along the wires; in pipeline network, the flows are oil products, etc. If flows are allowed in only one direction along an arc, the arc is said to be directed or oriented. If all the arcs in a network are directed, then the network is called a directed network, otherwise it is undirected (Hillier & Lieberman, 2001).

A sequence of arcs linking two distinct nodes forms a path in a network. A path forms a loop or cycle in a network if it connects a node to itself. Two nodes in a network are connected if there is at least one path linking them. When all the nodes in a network are linked by at least one path, the network is said to be connected (Taha, 2006).In this paper, a city – to – city road network in Nigeria is constructed first. Then Prim's algorithm is employed to obtain a minimum spanning tree. Figure 1 shows City – to – City road network in Nigeria, while Fig. 2 shows minimum spanning tree of the network.

Tree

A tree is any subset of a network not containing a loop. Given a set of nodes N, a tree can be grown by linking any two nodes by an arc, and subsequently adding new arc in such a way that it links a node already linked to other nodes to a new node not previously linked to any other node. When nodes are linked this way, the problem of creating a chain is avoided and the number of nodes linked will be 1 greater than the number of arcs. (Taha, 2002) A tree is spanning tree if all the nodes in a network are linked and are connected. Every spanning tree has exactly (m – 1) arcs in a network of m nodes, since this is the minimum number of arcs needed to have a connected network and the maximum number possible without having a chain.

The Minimal Spanning Tree problem

A spanning tree is a group of (m – 1) arcs that links all the m nodes of the network and contains no chain. A spanning tree of minimum length in a network is a minimum spanning tree. The minimal spanning tree problem found its applications in the creation of a network of paved roads that links several rural towns, where the road between two towns may pass through one or more other towns.

The Minimal Spanning Tree algorithm (The Prim's Algorithm)

The steps are as follows:

Step 0: set C = ø and $C' = \{1, 2, \cdots, m\}$

Step 1: Start with any node, say node i, in C'_0 and connect it to node j that is closest to node i. Set $C = \{i,\ j\}$ and $C' = \{1, 2, \cdots, m\} \setminus \{i,\ j\}$.

Step 2: Now choose node l in C' that is closest to some node k in C. Then connects node k to node l. Set C = {i, j, k} and C' = C'\C

Step 3: Repeat step 2 until C' = ø. Ties for the closest node and arc to be included in the minimum spanning tree may be broken arbitrarily. (Hillier and Lieberman, 2001).

Method of Data collection

The data used for this study is generated using Lad/Long converter software which makes use of latitude and longitude of cities in Nigeria to determine the distances between cities. Table 1 shows latitude and longitude of each state capital, while Table 2 shows city – to – city distances.

City-to-City Road Network in Nigeria.

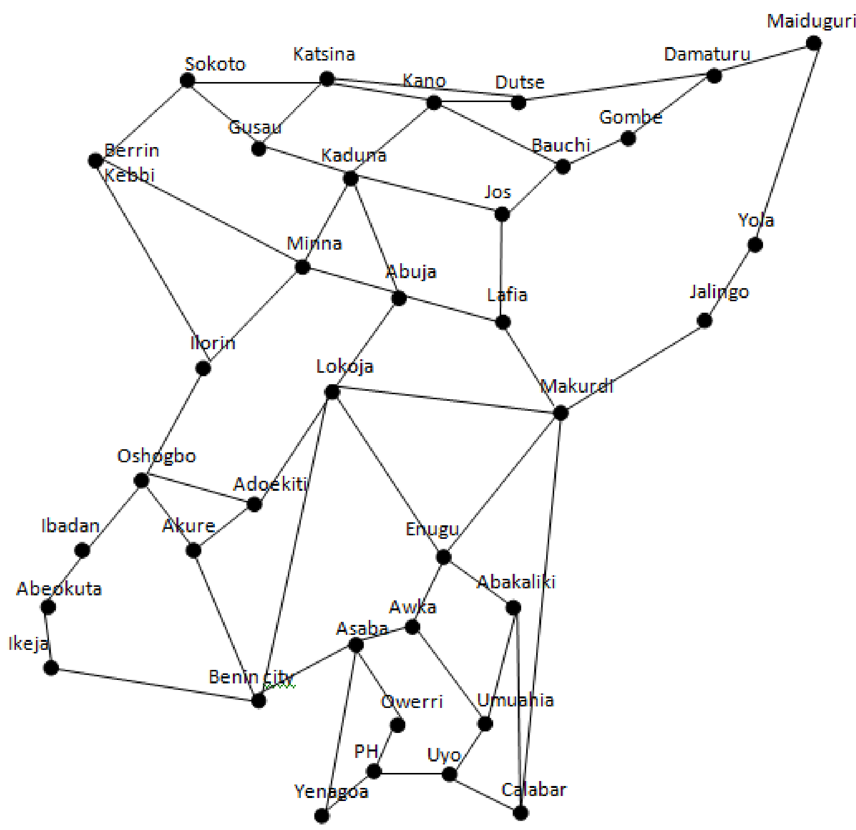

Fig.1

Table 1: Latitudes and Longitudes of State capitals in Nigeria

S/N	STATE CAPITALS	LATITUDE	LONGITUDE	S/N	STATE CAPITALS	LATITUDE	LONGITUDE
1	Ikeja	6.5833°N	3.3333°E	23	Lafia	8.4917°N	8.5167°E
2	Abeokuta	7.1608°N	3.3483°E	24	Jos	9.9333°N	8.8833°E
3	Ibadan	7.3964°N	3.9167°E	25	Bauchi	10.5000°N	10.0000°E
4	Oshogbo	7.7660°N	4.5667°E	26	Gombe	10.2500°N	11.1667°E
5	Ado – Ekiti	7.6211°N	5.2214°E	27	Jalingo	8.9000°N	11.3667°E
6	Akure	7.2500°N	5.1950°E	28	Yola	9.2300°N	12.4600°E
7	Benin City	6.3176°N	5.6145°E	29	Maiduguri	11.8333°N	13.1500°E
8	Asaba	6.1978°N	6.7285°E	30	Damaturu	11.7444°N	11.9611°E
9	Awka	5.0000°N	7.8333°E	31	Dutse	11.7011°N	9.3419°E
10	Enugu	6.4527°N	7.5103°E	32	Kano	12.0000°N	8.5167°E
11	Abakaliki	6.3333°N	8.1000°E	33	Katsina	12.2500°N	7.5000°E
12	Umuahia	5.5333°N	7.4833°E	34	Gusau	12.1500°N	6.6667°E
13	Calabar	4.7500°N	8.3250°E	35	Sokoto	13.0667°N	5.2333°E
14	Uyo	5.0500°N	7.9333°E	36	Berrin – Kebbi	11.5000°N	4.0000°E
15	Owerri	5.4850°N	7.0350°E	37	Kaduna	10.5167°N	7.4333°E
16	Port – Harcourt	6.3176°N	7.0000°E				
17	Yenagoa	4.7500°N	6.3333°E				
18	Ilorin	8.5000°N	4.5500°E				
19	Minna	9.6139°N	6.5569°E				
20	Abuja	9.0667°N	7.4833°E				
21	Lokoja	7.8167°N	6.7500°E				
22	Makurdi	7.7306°N	8.5361°E				

Table 2: City – to – City Road Network showing Distances in KM

CITY	YENAGOA	PH	OWERRI	ASABA	AWKA	UMUAHIA	UYO	CALABAR	ABAKALIKI	ENUGU	LOKOJA	ABUJA	LAFIA	MAKURDI
YENAGOA		80.3		136.6										
PH	80.3		81.3				108.6							
OWERRI				86.2										
ASABA					180.7									
AWKA				180.7		70.8				165.3				
UMUAHIA					70.8		73.2							
UYO		108.6				73.2		44.9						
CALABAR							44.9		115.7					309.9
ABAKALIKI								115.7		85.5				
ENUGU					165.3			85.5	85.5		173.2			181.6
LOKOJA										173.2		160.6		196.9
ABUJA											160.6		130.2	
LAFIA												130.2		84.6
MAKURDI								309.9		181.6	196.9		84.6	
JALINGO														337.3
YOLA														
MAIDUGURY														
DAMATURU														
GOMBE														
BAUCHI														
JOS													165.2	
KADUNA												161.2		
MINNA												118.4		
ILORIN														
OSHOGBO														
ADOEKITI											381.8			
AKURE														
BENIN CITY				123.8							208.4			
IKEJA														
ABEOKUTA														
IBADAN														
BERIN KEBI														
SOKOTO														
GUSAU														
KATSINA														
KANO														
DUTSE														

CITY	JALINGO	YOLA	MAIDUGURY	DAMATURU	GOMBE	BAUCHI	JOS	KADUNA	MINNA	ILORIN	OSHOGBO	ADOEKITI	AKURE
YENAGOA													
PH													
OWERRI													
ASABA													
AWKA													
UMUAHIA													
UYO													
CALABAR													
ABAKALIKI													
ENUGU													
LOKOJA												381.8	
ABUJA								161.2	118.4				
LAFIA							165.2						
MAKURDI	337.3												
JALINGO		125.5											
YOLA	125.5		298.9										
MAIDUGURY		298.9		129.7									
DAMATURU			129.7		187.3								
GOMBE						130.5							
BAUCHI					130.5		137.4						
JOS						137.4		171.3					
KADUNA							171.3		138.7				
MINNA								138.7		252.6			
ILORIN									252.6		81.5		
OSHOGBO										81.5		73.9	89.9
ADOEKITI											73.9		
AKURE											89.9	203.8	
BENIN CITY													113.5
IKEJA													
ABEOKUTA													
IBADAN										141.1	82.6		
BERIN KEBI									349.2	338.8			
SOKOTO													
GUSAU								199.8					
KATSINA													
KANO						232.2		282.7					
DUTSE				285									

CITY	BENIN CITY	IKEJA	ABEOKUTA	IBADAN	BERIN KEBI	SOKOTO	GUSAU	KATSINA	KANO	DUTSE
YENAGOA										
PH										
OWERRI										
ASABA	123.8									
AWKA										
UMUAHIA										
UYO										
CALABAR										
ABAKALIKI										
ENUGU										
LOKOJA	208.4									
ABUJA										
LAFIA										
MAKURDI										
JALINGO										
YOLA										
MAIDUGURY										
DAMATURU										285
GOMBE										
BAUCHI									232.2	
JOS										
KADUNA							199.8		282.7	
MINNA					349.2					
ILORIN				141.1	338.8					
OSHOGBO				82.6						
ADOEKITI										
AKURE	113.5									
BENIN CITY		253.6								
IKEJA	253.6		64.2	110.9						
ABEOKUTA		67.9		64.2						
IBADAN		110.9	67.9							
BERIN KEBI						219.6				
SOKOTO					219.6		185.8	361.9		
GUSAU						185.8		91.2		
KATSINA						361.9	91.2		113.9	209.3
KANO								113.9		95.6
DUTSE								209.3	95.6	

III. Result

The result of applying the Prim's algorithm to the network in figure 1 is as shown in figure 2. From the Minimum Spanning Tree in fig. 2 a total distance of 5,128.5km is obtained.

Minimum Spanning Tree Network of city to City Distance in Nigeria.

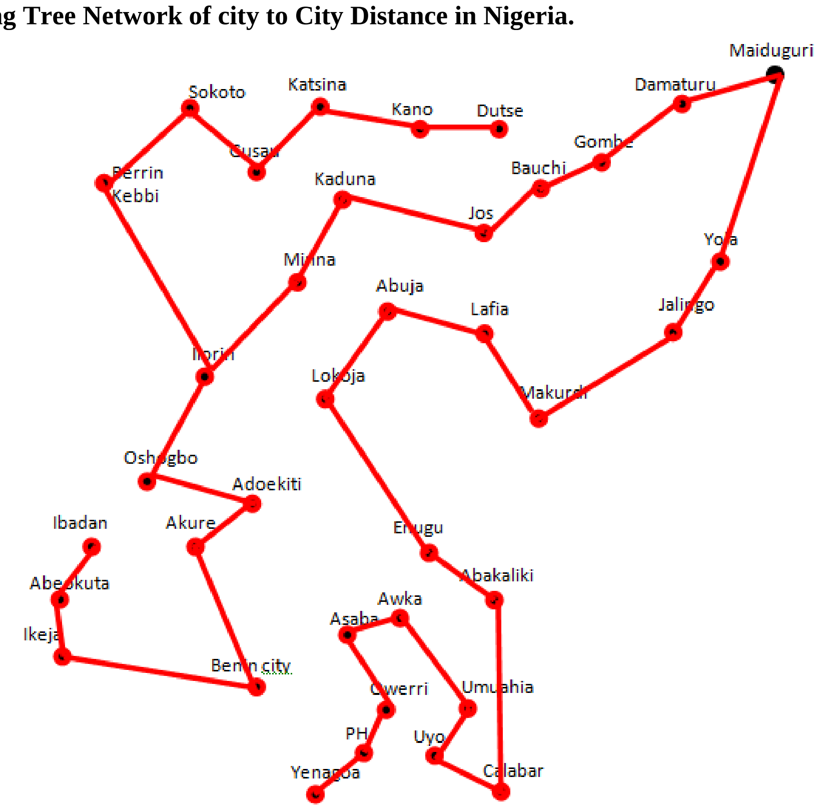

Fig.2

IV. Conclusion

The optimal road network so constructed can be used for various purposes in Nigeria, for example, designing of telecommunication network, transportation network, network of high – voltage electrical power transmission lines, network of gas and petroleum pipelines, etc. Although most of these networks have been in existence in Nigeria, the network design in this work would be needful in other applications yet to be considered.

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