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# On Regular Mildly Generalized (RMG) Open Sets in Topological Spaces

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***Abstract:*** *In this paper we introduce and study the new class of sets, namely Regular Mildly Generalized Open (briefly, RMG-open) sets, Regular Mildly Generalized neighborhoods (briefly, RMG-nhd), RMG-interior and RMG-closure in topological space and also some properties of new concept has been studied.*
***Keywords:*** *RMG-closed sets, RMG-open sets, RMG-neighborhoods, RMG-interior, RMG-closure.*

## I. Introduction

Levine [9, 10] introduces a generalized open and semi-open sets in topological spaces. Regular open sets, pre-open sets, rg-open sets, Mildly-g-open sets have been introduced and studied by stone [18], A.S. Mashhur.et.al[5], N. Palaniappan et. al[14], J. K. Park et. al[16] respectively. In this paper the concept of Regular Mildly Generalized (briefly RMG) open set is introduced and their properties are investigated and Regular Mildly Generalized neighborhood (briefly RMG-nhd), RMG-interior and RMG- closure in a topological spaces.

Throughout this paper X and Y represent the topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of topological space X, cl(A) and int(A) denote the closure of A and interior of A respectively. $X - A$ or $A^c$ Denotes the complement of A in X. Now, we recall the following definitions.

## II. Preliminaries

**Definition 2.1** A subset A of X is called regular open (briefly r-open) [18] set if A = int(cl(A)) and regular closed (briefly r-closed) [18] set if A = cl(int(A)).

**Definition 2.2** A subset A of X is called pre-open set [5] if A⊆int(cl(A)) and pre-closed [5] set if cl(int(A))⊆A.

**Definition 2.3** A subset A of X is called semi-open set [8] if A⊆cl(int(A)) and semi-closed [8] set if int(cl(A)) ⊆A.

**Definition 2.4** A subset A of X is called α-open [11] if A⊆int(cl(int(A))) and α-closed [11] if cl(int(cl(A))) ⊆A.

**Definition 2.5** A subset A of X is called β-open [1] if A⊆cl(int(cl(A))) and β-closed [1] if int(cl(int(A))) ⊆A.

**Definition 2.6** A subset A of X is called δ-closed [19] if A = $cl_\delta(A)$, where $cl_\delta(A)$ = {x ∈X : int(cl(U)) ∩ A ≠ ∅, U∈A}.

**Definition 2.7** Let X be a topological space. The finite union of regular open sets in X is said to be π-open [4]. The compliment of a π-open set is said to be π-closed.

**Definition 2.8** A subset of a topological space (X, τ) is called

1. Generalized closed (briefly g-closed) [9] if cl(A) ⊆U whenever A⊆U and U is open in X.
2. Generalized α-closed (briefly gα-closed) [6] if α-cl(A) ⊆U whenever A⊆U and U is α-open in X.
3. Weakly generalized closed (briefly wg-closed) [10] if cl(int(A)) ⊆U whenever A⊆U and U is open in X.
4. Strongly generalized closed (briefly g*-closed) [15] if cl(A) ⊆U whenever A⊆U and U is g-open in X.
5. Weakly closed (briefly w-closed) [17] if cl(A) ⊆U whenever A⊆U and U is semi-open in X.
6. Mildly generalized closed (briefly mildly g-closed) [16] if cl(int(A)) ⊆U whenever A⊆U and U is g-open in X.
7. Regular weakly generalized closed (briefly rwg-closed) [10] if cl(int(A)) ⊆U whenever A⊆U and U is regular open in X.
8. Weakly π-generalized closed (briefly wπg -closed)[13] if cl(int(A)) ⊆U whenever A⊆U and U is π-open in X.
9. Regular weakly closed (briefly rw-closed)[2] if cl(A)⊆U whenever A⊆U and U is regular semiopen in X.
10. Generalized pre closed (briefly gp-closed)[7] if pcl(A) ⊆U whenever A⊆U and U is open in X.
11. A subset A of a space (X, τ) is called regular generalized closed (briefly rg-closed) [14] if cl(A) ⊆U whenever A⊆U and U is regular open set in X.

12. π-generalized closed (briefly πg-closed)[3] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

The complements of the above mentioned closed sets are their respective open sets.

The semi-pre-closure (resp. semi-closure, resp. pre-closure, resp. α-closure) of a subset A of X is the intersection of all semi-pre- closed (resp. semi- closed, resp. pre- closed, resp. α-closed) sets containing A and is denoted by spcl(A)(resp. scl(A), resp. pcl(A), resp. cl(A)).

**Definition 2.3** Regular Mildly Generalized closed (briefly RMG-closed)[20] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is rg-open in X. We denote the family of all RMG-closed sets, RMG-open sets of X by RMGC(X), RMGO(X) respectively.

## III. Regular Mildly Generalized Open (briefly RMG-open) Sets

In this section, we introduce and studied RMG-open sets in topological space and obtain some of their basic properties. Also we introduce RMG-neighborhood (briefly RMG-nhd) in topological spaces by using the notation of RMG-open sets.

**Definition 3.1:** A subset A of X is called Regular Mildly Generalized open (briefly, RMG-open) set in X. If $X - A$ is RMG-closed set in X. The family of all RMG- open sets is denoted by RMGO(X).

**Theorem 3.2:** Every pre-open set is RMG-open set in X.

**Proof:** Let A be a pre-open set in X. Then by $X - A$ is pre-closed. By Theorem 3.2[20] every pre-closed set is RMG-closed, $X - A$ is RMG-closed set in X. Therefore A is RMG-open set in X.

The converse of the above theorem need not be true as seen from the following example.

**Example3.3:** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$.Then A={c} is RMG-open set but not pre-open set in X.

**Theorem3.4:** Every RMG-open set is Mildly-g-open set in X.

**Proof:** Let A be a RMG-open set in X. Then $X - A$ is RMG -closed. By Theorem 3.4[20] Every RMG-closed set is Mildly-g-closed, $X - A$ is RMG-closed. Therefore A is Mildly-g-open set in X.

The converse of above Theorem need not be true as seen from the following examples.

**Examples3.5:** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$.Then A ={a c} and B={b, c} are mildly-g-open set but not RMG-open set in X.

**Corollary 3.6: 1**. Every gα-open set is RMG-open set in X.

**2.** Every w-open set is RMG-open set in X.

**3**. Every open set is RMG-open set in X.

**4.** Every δ-open set is RMG-open set in X.

**5.** Every π-open set is RMG-open set in X.

6. Every regular open set is RMG-open set in X.

**Proof:**

1. Let A be a gα -open set in X. Then $X - A$ is gα –closed set. By Theorem 3.6.1[20] every gα -closed set is RMG- closed, $X - A$ is RMG-closed .Therefore A is RMG-open set in X.

2. Let A be a w-open set in X. Then $X - A$ is w-closed. By Theorem 3.6.2[20] every w-closed set is RMG-closed, $X - A$ is RMG-closed .Therefore A is RMG-open set in X.

3. Let A be a open set in X. Then $X - A$ is closed. By Theorem 3.6.3[20] every closed set is RMG-closed, $X - A$ is RMG-closed .Therefore A is RMG-open set in X.

4. Let A be a δ-open set in X. Then $X - A$ is δ-closed. By Theorem 3.6.4[20] every δ- closed set is RMG-closed, $X - A$ is RMG-closed .Therefore A is RMG-open set in X.

5. Let A be a π-open set in X. Then $X - A$ is π-closed. By Theorem 3.6.5[20] every π- closed set is RMG-closed, $X - A$ is RMG-closed .Therefore A is RMG-open set in X.

6. Let A be a regular open set in X. Then $X - A$ is regular closed. By Theorem 3.6.6[20] every regular closed set is RMG-closed, $X - A$ is RMG-closed .Therefore A is RMG-open set in X.

The converse of Corollary 3.6 is not true as shown in below examples.

**Example 3.7:** Let X= {a, b, c, d} with topology τ={X, ∅, {a}, {b}, {a, b}, {b, c}, {a, b, c}}.

1. Let A= {c} is RMG-open but not gα-open set in X.
2. Let A= {c} is RMG-open but not w-open set in X.
3. Let A= {c} is RMG-open but not open set in X.
4. Let A= {c} is RMG-open but not δ-open set in X.
5. Let A= {c} is RMG-open but not π-open set in X.
6. Let A= {c} is RMG-open but not regular open set in X.

**Corollary 3.8:**

1. Every RMG-open set is wg-open set in X.

2. Every RMG-open set is wπg-open set in X.
3. Every RMG-open set is rwg-open set in X.

**Proof: 1**. Let A be a RMG-open set in X. Then X − A is RMG-closed. By Theorem 3.8.1[20] every RMG-closed set is wg-closed, X − A is RMG-closed. Therefore A is wg-open set in X.

**2**. Let A be a RMG-open set in X. Then X − A is RMG-closed. By Theorem 3.8.2[20] every RMG-closed set is wπg-closed, X − A is RMG-closed. Therefore A is wπg-open set in X.

**3**. Let A be a RMG-open set in X. Then X − A is RMG-closed. By Theorem 3.8.3[20] every RMG- closed set is rwg-closed, X − A is rwg-closed. Therefore A is rwg-open set in X.

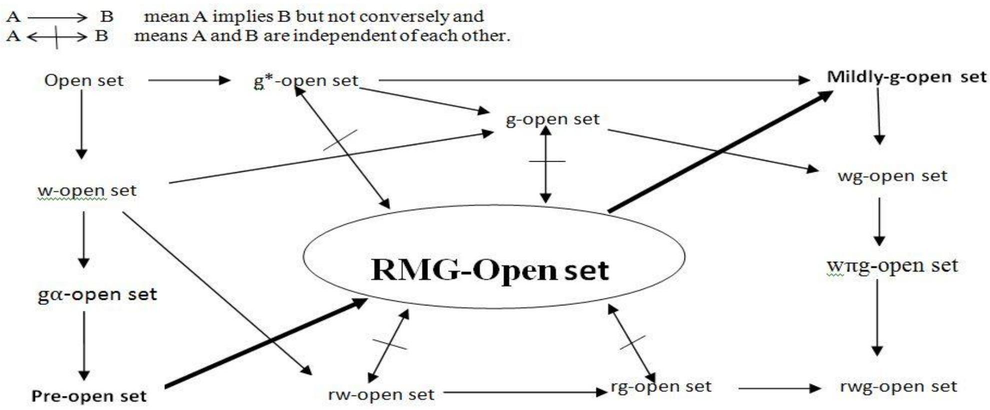
The converse of corollary 3.8 is not true as shown in below examples.

**Example3.9:** Let X= {a, b, c, d} with topology τ={X, ∅, {a}, {b}, {a, b},{a, b, c}}.
**1.** Let A= {c} is wg-open but not RMG-open set in X.
**2.** Let A= {b, d} is wπg-open but not RMG-open set in X.
**3.** Let A= {d} is rwg-open but not RMG-open set in X.

**Remark 3.10:** The concept of semi-open, semi-pre-open, g-open, g*-open, rg-open, rw-open, πg-open sets are independent with the concept of RMG-open sets as shown in the following example.

**Example 3.11:** Let X= {a, b, c, d} with topology τ={X, ∅, {a}, {b}, {a, b}, {b, c}, {a, b, c}}. Then the set {a, b, d} is RMG-open set. However it can be verified that it is not g-open, not g*-open, not πg-open, not rg-open, not rw-open,, Also {b, d} is both semi-open and semi pre open but not RMG-open .

**Remark 3.12**From the above discussions and known results we have the following implications in the following diagram, by




**Remark s3.13:** The intersection of two RMG-open sets in X is need not be a RMG- open set in X.

**Examples 3.14:** Let X= {a, b, c, d} with topology τ={X, ∅, {a},{b, c}, {a, b, c}}. Now A={a, b, d} and B={a, c, d} are RMG-open sets in X, then A∩B={a, b, d}∩{a, c, d}={a, d} which is not RMG- open set in X

**Remark 3.15:** The union of two RMG-open subsets of X is need not be RMG-open set in X.

**Example 3.16:** Let X= {a, b, c, d} with topology τ={X, ∅, {a}, {b}, {a, b}, {b, c}, {a, b, c}}. Now A= {a} and B= {c} are RMG-open sets in X, then A ∪ B= {a}∪{c}={a, c} which is not RMG-open set in X.

**Remark 3.17:** Complement of a RMG-open set need not be RMG-open set in X.

**Example 3.18:** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $A = \{a, b\}$ is RMG-open set but $X - \{a, b\} = \{c, d\}$ is not RMG-open set in X.

**Theorem 3.19:** A subset A of a topological space X is RMG-open iff F⊂int(cl(A)) whenever F⊂A and F is rg-closed set in X.

**Proof:** Assume A is RMG-open then X − A is RMG-closed. Let F be a rg-closed set in X contained in A. Then X − F is a rg-open set in X containing X − A . Since X − A is RMG-closed, cl(int(X − A))⊂ X − F this implies X −int(cl(A))⊂ X − F . Consequently F⊂int(cl(A)).

Conversely, let F⊂int(cl(A)) whenever F⊂A and F is rg-closed in X. Let G be rg-open set containing X − A then X − G ⊂int(cl(A)). Hence cl(int(X − A))⊂G. This prove that X − A is RMG-closed and hence A is RMG-open set in X.

**Theorem 3.20:** If A⊆X is RMG-closed set in X, then cl(int(A)) −A is RMG-open set in X.

**Proof:** Let A⊆X is RMG-closed and let F be a rg-closed set such that F⊆cl(int(A))−A. Then by Theorem 3.21[20], F= ∅, that implies F⊆int(cl(cl(int(X − A )))) −A. This proves that cl(int(A))−A is RMG-open.

**Theorem 3.21:** Every singleton point set in a space X is either RMG-open or rg-closed.

**Proof:** Let X be a topological space. Let x∈X. To prove {x} is either RMG-open or rg-closed. That is to prove X-{x} is either RMG-closed or rg-open. Which follows from Theorem 3.23 of [20].

**Theorem 3.22:** If int(cl(A)) ⊆B⊆A and A is RMG-open, then B is RMG-open.

**Proof:** Let A be RMG-open and int(cl(A))⊆B⊆A. Then $X - A \subseteq X - B \subseteq X - \text{int}(\text{cl}(A))$ that implies $X - A \subseteq X - B \subseteq \text{cl}(\text{int}(X - A))$. Since $X - A$ is RMG-closed, by Theorem 3.24 of [20] $X - B$ is RMG-closed. This proves that B is RMG-open.

The converse of above Theorem 3.24 need not be true in general.

**Example 3.23:** Let X={a, b, c, d} with topology τ={X, ∅, {a}, {b}, {a, b}, {a, b, c}}. Now A={a, b, c} and B={b, c}. Now A and B both are RMG-open sets. But int(cl(A))⊈B⊆ A

**Theorem 3.24:** Let A ⊂ Y ⊂ X and A is RMG-open set in X. Then A is RMG-open in Y provided Y is open set in X.

**Proof:** Let A be RMG-open in X and Y be a open sets in X. Let U be any rg-open in Y such that A⊂U. Then U ⊂ A ⊂ Y ⊂ X by the lemma 3.26[20], U is rg-open in X. Since A is RMG-open in X, U⊆int(cl(A). Also int(cl(A)) ⊂ $\text{int}_y(\text{cl}_y(A))$. Hence A is an RMG-open set in Y.

**Theorem 3.25:** If a subset A is RMG-open in X and if G is rg-open in X with $\text{int}(\text{cl}(A) \cup (X - A) \subseteq G$ then G=X.

**Proof:** Suppose that G is an rg-open and $\text{int}(\text{cl}(A)) \cup (X - A) \subseteq G$. Now $(X - A) \subseteq (X - \text{int}(\text{cl}(A))) \cap X - (X - A)$ implies that $(X - G) \subseteq \text{cl}(\text{int}(X - A)) \cap A$. Suppose A is RMG-open. Since $X - G$ is rg-closed and $X - A$ is RMG-closed, then by Theorem 3.21of [20], $X - G = \emptyset$ and hence $G = X$.

The converse of the above Theorem need not be true in general as shown in example 3.24.

**Example3.26:** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Then $\text{RMGO}(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$ and $\text{RGO}(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}\}$. Let $A = \{b, d\}$ is not an RMG-open set in X. However $\text{int}(\text{cl}(A)) \cup (X - A) = \{b, c\} \cup \{a, c\} = \{a, b, c\}$. So for some rg-open set G, such that $\text{int}(\text{cl}(A)) \cup (X - A) = \{a, b, c\} \subset G$ gives $G = X$ but A is not RMG-open set in X.

**Theorem3.27:** Let X be a topological space and A, B⊆X. If B is RMG-open and $\text{int}(\text{cl}(B)) \subseteq A$, then $A \cap B$ is RMG-open in X.

**Proof:** Since B is RMG-open and $\text{int}(\text{cl}(B)) \subseteq A$, then $\text{int}(\text{cl}(B)) \subseteq A \cap B \subseteq B$, then by Theorem 3.24, $A \cap B$ is RMG-open set in X.

## IV. Regular Mildly Generalized Neighborhoods (briefly RMG-nhd)

**Definition 4.1** Let $(X, \tau)$ be a topological space and let $x \in X$. A subset N is said to be RMG- neighborhood (briefly, RMG-nhd) of x, if and only if there exists an RMG-open set G such that $x \in G \subset N$.

**Definition 4.2(i)** A subset N of X is a RMG−nhd of $A \subseteq X$ in topological space$(X, \tau)$, if there exists an RMG-open set G such that $A \subset G \subset N$.

**(ii)** The collection of all RMG-nhd of $x \in X$ is called RMG-nhd system at $x \in X$ and shall be denoted by RMG-N(x).

**Theorem 4.3:** Every neighborhood N of $x \in X$ is a RMG-nhd of x.

**Proof:** Let N be neighborhood of point $x \in X$. To prove that N is a RMG-nhd of x. By definition of neighborhood, there exists an open set G such that $x \in G \subset N$. As every open set is RMG-open, G is an RMG-open set in X. Then there exists a RMG-open set G such that $x \in G \subset N$. Hence N is RMG-nhd of x.

**Remark4.4:** In general, a RMG-nhd N of x in X need not be nhd of x in X, as shown from example 4.5.

**Example4.5:** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ .Then $\text{RMGO}(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$. The set {a, c} is RMG-nhd of the point c, since the RMG-open set {c} is such that $c \in \{c\} \subset \{a, c\}$. However, the set {a, c} is not a neighbourhood of the point c, since no open set G exists such that $c \in G \subset \{a, c\}$.

**Theorem4.6:** If a subset N of a space X is RMG-open, then N is a RMG-nhd of each of its points.

**Proof:** Suppose N is RMG-open. Let $x \in N$ we claim that N is a RMG-nhd of x. For N is a RMG-open set such that $x \in N \subset N$. Since x is an arbitrary point of N, it follows that N is a RMG-nhd of each of its points.

**Remark4.7:** The converse of the above theorem is not true in general as seen from the following example 4.8.

**Example4.8:** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $\text{RMGO}(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$. The set {a, c} is a RMG-nhd of the point a, since the RMG-open set {a} is such that $a \in \{a\} \subset \{a, c\}$. Also the set {a, c} is a RMG-nhd of the point c, since the RMG-open set {c} is such that $c \in \{c\} \subset \{a, c\}$ i.e. {a, c} is a RMG-nhd of each of its points. However the set {a, c} is not a RMG-open set in X.

**Theorem4.9:** Let X be a topological space. If F is a RMG-closed subset of X and $x \in (X - F)$, then there exists a RMG-nhd N of x such that $N \cap F = \emptyset$.

**Proof:** Let F be RMG-closed subset of X and $x \in (X - F)$. Then $(X - F)$ is an RMG-open set of X. By Theorem 4.6, $(X - F)$ contains a RMG-nhd of each of its points. Hence there exists a RMG-nhd N of x such that $N \subset X - F$. That is $N \cap F = \emptyset$.

**Theorem 4.10:** Let X be a topological space and for each $x \in X$, let RMG-N(x) be the collection of all RMG-nhds of x. Then we have the following results.

i)$\forall x \in X$, $RMG - N(x) \neq \emptyset$.

ii)$N \in RMG - N(x) \Rightarrow x \in N$.

iii)$N \in RMG - N(x)$and $N \subset M \Rightarrow M \in RMG - N(x)$.

iv)$N \in RMG - N(x) \Rightarrow \exists\ M \in RMG - N(x)$ such that $M \subset N$ and $M \in$RMG-N(y) for every $y \in M$.

**Proof:** i) Since X is an RMG-open set, it is a RMG-nhd of every $x \in X$. Hence $\exists$ at least one RMG-nhd(X) for each $x \in X$. Hence $RMG - N(x) \neq \emptyset$ for every $x \in X$.

ii) If $N \in$ RMG-N(x), then N is a RMG-nhd of x. So by definition of RMG-nhd $x \in N$.

iii) Let $N \in RMG - N(x)$and $N \subset M$, then there is an RMG-open set G such that $x \in G \subset N$. Since $N \subset M$, $x \in G \subset$ M and so M is a RMG-nhd of x. Hence $M \in RMG - N(x)$.

iv) If $N \in RMG - N(x)$, then there exists an RMG-open set M such that $x \in M \subset N$. Since M is an RMG-open set, it is a RMG-nhd of each of its points. Therefore $M \in RMG - N(y)$ for all $y \in M$.

## V. Regular Mildly Generalized Interior (RMG-Interior) Operator.

In this section, the notation of RMG-interior is defined and some of its basic properties are studied.

**Definition 5.1 (i):** Let A be a subset of $(X, \tau)$. A point $x \in A$ is said to be RMG-interior point of A if and only if A is RMG-neighbourhood of x. The set of all RMG-interior points of A is called the RMG-interior of A and is denoted by RMG-int(A).

**Definition (ii):** Let $(X, \tau)$ be a topological space and $A \subset X$. Then RMG-int (A) is the union of all RMG – open sets contained in A.

**Theorem 5.2:** Let A is a subset of $(X, \tau)$, then RMG-int(A)=$\cup$\{G:G is RMG-open, $G \subset A$\}.

**Proof:** Let A be a subset of $(X, \tau)$. $x \in$RMG-int(A)

$\Leftrightarrow$ x is a RMG-interior point of A.

$\Leftrightarrow$ A is a RMG-nhd of point x.

$\Leftrightarrow$ There exists an RMG-open set G such that $x \in G \subset A$

$\Leftrightarrow$ $x \in \cup$\{G:G is RMG-open, $G \subset A$\}.

Hence RMG-int(A) =$\cup$\{G:G is RMG-open $G \subset A$\}.

**Theorem 5.3:** Let A and B are subsets of $(X, \tau)$. Then

i) RMG-int($\emptyset$)= $\emptyset$ and RMG-int(X) =X.

ii) RMG-int(A) $\subset$A.

ii) If B is any RMG-open sets contained in A, then $B \subset$RMG-int(A).

iv) If $A \subset B$, then RMG-int(A) $\subset$RMG-int(B).

v) RMG-int (RMG-int(A))=RMG-int(A).

**Proof:** i) Obvious

ii) Let $x \in$RMG-int(A) $\Rightarrow$ x is a RMG-interior point of A

$\Rightarrow$ A is a RMG-nhd of x

$\Rightarrow$ $x \in A$

Thus $x \in$RMG-int(A) $\Rightarrow x \in A$. Hence RMG-int(A)$\subset$A.

iii) Let B be a any RMG-open set such that $B \subset A$. Let $x \in B$. Then since B is an RMG open set contained in A. x is an RMG-interior point of A. That is $x \in$RMG-int(A). Hence $B \subset$RMG-int(A).

iv) Let A and B are subsets of X such that $A \subset B$. Let $x \in$RMG-int(A). Then x is an RMG-interior point of A and so A is a RMG-nhd of x. since $A \subset B$, B is also a RMG-nhd of x. This implies that $x \in$RMG-int(B). Thus we have show that $x \in$RMG-int(A)$\Rightarrow x \in$RMG-int(B). Hence RMG-int(A) $\subset$RMG-int(B).

v) Since RMG-int(A) is a RMG-open set in X, it follows that RMG-int(RMG-int(A))=RMG-int(A).

**Theorem 5.4:** If a subset A of space X is RMG-open, then RMG-int(A)=A.

**Proof:** Let A be a RMG-open subset of X and we know that RMG-int(A) $\subset$A. Since A is RMG-open set contained in A. From the Theorem 5.3(iii), $A \subset$RMG-int(A) and hence we get RMG-int(A)=A.

The converse of the above theorem need not be true as seen in the following example.

**Example 5.5:** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $RMGO(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$. Note that RMG-int({a, c})={a}∪ {c}∪ ∅ ={a, c}. But {a, c} is not a RMG-open set in X.

**Theorem 5.6:** If A and B are sub sets of X, then RMG-int(A)∪ RMG-int(B)⊂ RMG-int(A ∪B).

**Proof:** Since A⊂A∪ B and B⊂A∪B. Using the Theorem 5.3(iv), RMG-int(A) ⊂RMG-int(A∪ B) and RMG-int(B) ⊂RMG-int(A∪B)This implies RMG-int(A)∪ RMG-int(B)⊂ RMG-int(A ∪B).

**Theorem5.7:** If A and B are subsets of a space X, then RMG-int(A∩B)⊂RMG-int(A) ∩RMG-int(B).

**Proof:** Let A and B be subsets of X. Clearly A∩B⊂A and A∩B⊂B. By Theorem 5.3(iv). RMG-int(A∩B)⊂ RMG-int(A) and RMG-int(A∩B)⊂RMG-int(B). Hence RMG-int(A∩B)⊂RMG-int(A) ∩RMG-int(B).

**Theorem 5.8:** If A is a subset of X, then int(A)⊂RMG-int(A).

**Proof:** Let A be a subset of a space X.

x∈int(A) ⇒ x ∈∪{G:G open, G⊂A}.

⇒ There exists an open set G such that x∈G⊂A

⇒ There exists an RMG-open set G such that x∈G⊂A, as every open set is an RMG-open set in X.

⇒ x ∈∪{G:G is RMG-open, G⊂A}.

⇒x ∈RMG-int(A).

Thus, x∈ int(A)⇒x ∈RMG-int(A). Hence int(A) ⊂RMG-int(A).

**Remark 5.9:** Containment relation in the above Theorem 5.8 may be proper as seen from the following example.

**Example 5.10:** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Then $RMGO(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$. Let A={a, b}. Now RMG-int(A) ={a, b} and int(A) ={a}. It follows that int(A) ⊂RMG-int(A) and int(A) ≠RMG-int(A).

**Remark5.11:** If A is sub set of X, then

i) w-int(A)⊂RMG-int(A).

ii) gα-int(A)⊂RMG-int(A).

iii) p-int(A)⊂RMG-int(A).

**Theorem5.12:** If A is a subset of X, then RMG-int(A)⊂Mildly-g-int(A).

**Proof:** let A be a subset of a space X.

x∈RMG-int(A) ⇒ x ∈∪{G:G RMG-open, G⊂A}.

⇒ There exists an RMG-open set G such that x∈G⊂A

⇒ There exists an Mildly-g-open set G such that x∈G⊂A, as every RMG-open set is an Mildly-g-open set in X.

⇒ x ∈∪{G:G is Mildly-g-open, G⊂A}.

⇒ x∈Mildly-g-int(A).

Thus, x ∈RMG-int(A) ⇒x ∈Mildly-g-int(A). Hence RMG-int(A) ⊂Mildly-g-int(A).

**Remark5.13:** If A is sub set of X, then RMG-int(A)⊂wg-int(A).

## VI. Regular Mildly Generalized Closure (RMG-Closure) Operator.

Now we introduced the notation of RMG-closure in topological spaces by using the notation of RMG-closed sets and obtained some of their properties. For any A X, it is proved that the complement of RMG-interior of RMG-closure of the complement of A.

**Definition 6.1:** Let A be a subset of a space (X, $\tau$). We defined the RMG-closure of A to be a intersection of all RMG-closed sets containing A. In symbol we have RMG-cl(A)=∩{F:A⊂F∈RMGC(X)}.

**Theorem 6.2:** Let A and B are subsets of (X, $\tau$). Then

i) RMG-cl(∅)= ∅ and RMG-cl(X) =X.

ii) A⊂RMG-cl(A).

iii) If B is any RMG-closed sets contained in A, then RMG-cl(A)⊂B

iv) If A⊂B, then RMG-cl(A) ⊂RMG-cl(B).

v) RMG-cl(RMG-cl(A))=RMG-cl(A).

**Proof:** i) Obvious.

ii) By the definition of RMG-closure of A, it is obvious that A⊂RMG-cl(A).

iii) Let B be any RMG-closed set containing A. Since RMG-cl(A) is the intersection of all RMG-closed set containing A, RMG-cl(A) is contained in every RMG-closed set containing A. Hence in particular RMG-cl(A)⊂ B.

iv) ]Let A and B be subsets of X. such that A⊂B. By the definition of RMG-closure, RMG-cl(B) =∩{F:B⊂F∈ RMGC(X)}. If B⊂F∈RMGC(X), then RMG-cl(B)⊂F. Since A⊂B, A⊂B⊂F∈RMGC(X), We have RMG-cl(A) ⊂F. Therefore RMG-cl(A)⊂ ∩{F:B⊂F∈RMGC(X)} =RMG-cl(B). That is RMG-cl(A)⊂RMG-cl(B).

v) Since RMG-cl(A) is a RMG-closed set in X, if it follows that RMG-cl(RMG-cl(A))=RMG-cl(A).

**Theorem 6.3:** If A⊂X is RMG-closed, then RMG-cl(A)=A.

**Proof:** Let A be a RMG-closed subset of X. We know that A⊂RMG-cl(A). Also A⊂A and A is RMG-closed. By the theorem 6.2 (iii) RMG-cl(A)⊂A. Hence RMG-cl(A)=A.

The converse of the above theorem need not be true as seen from the following example.

**Example 6.5:** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then RMGC(X) = $\{X, \emptyset, \{c\}, \{d\}, \{a, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Now RMG-cl{b}={a, b, d}∩{b, c, d}∩X= {b, d}, but {b, d} is not a RMG-closed subset in X.

**Theorem 6.7:** If A and B are sub sets of X, then RMG-cl(A)∪ RMG-cl(B)⊂RMG-cl(A∪B).

**Proof:** Let A and B are subsets of a space X. Clearly A⊂A∪B and B⊂A∪B. By Theorem6.2 (iv). RMG-cl(A) ⊂RMG-cl(A∪B) and RMG-cl(B)⊂RMG-cl(A∪B). This implies that RMG-cl(A)∪RMG-cl(B)⊂RMG-cl(A∪B).

**Theorem 6.8:** If A and B are subsets of a space X, then RMG-cl(A∩B)⊂RMG-cl(A) ∩RMG-cl(B).

**Proof:** Let A and B be subsets of X. Clearly A∩B⊂A and A∩B⊂B. By Theorem 6.2(iv). RMG-cl(A∩B)⊂ RMG-cl(A) and RMG-cl(A∩B)⊂RMG-cl(B). Hence RMG-cl(A∩B)⊂RMG-cl(A) ∩RMG-cl(B).

**Theorem 6.9:** Let A be a subset of X and x∈X. Then x∈RMG-cl(A) if and only if V∩A≠ ∅ for every RMG-open set V containing x.

**Proof:** Let x∈X and x∈RMG-cl(A). To prove that V∩A≠ ∅ for every RMG-open set V containing x. Prove the results by contradiction. Suppose there exists a RMG-open set V containing x such that V∩A= ∅. Then A⊂X-V and X-V is RMG-closed. We have RMG-cl(A)⊂X-V. This shows that x∉RMG-cl(A). Which is contradiction. Hence V∩A≠ ∅ for every RMG-open set V containing x.

Conversely, let V∩A≠ ∅ for every RMG-open set V containing x. To prove that x∈RMG-cl(A). We prove the result by contradiction. Suppose x∉RMG-cl(A). Then there exists a RMG-closed subset F containing A such that x∉F. Then x∈X-F and X-F is RMG-open. Also (X-F) ∩A= ∅, which is a contradiction. Hence x∈ RMG-cl(A).

**Theorem 6.10:** Let A be A RMG-open set and B be any open set in X. If A∩B= ∅, then A∩RMG-cl(B)= ∅.

**Proof:** Suppose A∩RMG-cl(B)≠ ∅ and x∈ A∩RMG-cl(B). Then x∈A and x∈ A∩RMG-cl(B). by above Theorem 6.9 A∩B≠ ∅ which is contrary to the hypothesis. Hence A∩RMG-cl(B)≠ ∅.

**Theorem 6.11:** If A is a subset of (X, $\tau$), Then RMG-cl(A)⊂cl(A).

**Proof:** Let A be a subset of X. By definition of closure, cl(A)=∩{F⊂X:A⊂F∈C(X)}. If A⊂F∈C(X), then A⊂F ∈RMGC(X), because every closed set is RMG-closed. That is RMG-cl(A)⊂F. Therefore RMG-cl(A)⊂ ∩{F⊂ X:A⊂F∈C(X)}=cl(A).Hence RMG-cl(A)⊂cl(A).

**Remark 6.12:** Containment relation in the above Theorem 5.34 may be proper as seen from the following example.

**Example 6.13:** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then RMGC(X) = $\{X, \emptyset, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Let A={c}. Now RMG-cl(A) ={c} and cl(A) ={c, d}. It follows that RMG-cl(A)⊂cl(A) and RMG-cl(A)≠cl(A).

**Remark6.14:** If A be a subset of space X, then

i) RMG-cl(A)⊂w-cl(A).

ii) RMG-cl(A)⊂g$\alpha$-cl(A).

iii) RMG-cl(A)⊂p-cl(A).

**Theorem 6.1:5** If A is a subset of space (X, τ), Then Mildly-g-cl(A)⊂RMG-cl(A).

**Proof:** Let A be a subset of X. By definition of Mildly-g -closure, Mildly-g-cl(A)=∩{F⊂X:A⊂F∈Mildly-g-C (X)}. If A⊂F∈RMGC(X), then A⊂F∈Mildly-g-(X), because every RMG-closed set is Mildly-g-closed. That is Mildly-g-cl(A)⊂F. Therefore Mildly-g-cl(A)⊂ ∩{F⊂X:A⊂F∈ RMC(X)}=RMG-cl(A). Hence Mildy-g-cl(A)⊂ RMG-cl(A).

**Remark 6.16:** If A subset of space X, then wg-cl(A)⊂RMG-cl(A).

**Lemma 6.17:** Let A be a subset of a space X. Then

i) X−(RMG-int(A))= RMG-cl(X−A).

ii) RMG-int(A)=X− (RMG-cl(X−A)).

iii) RMG-cl(A)= X− (RMG-int(X−A)).

**Proof:** Let x∈X− (RMG-int(A)). Then x∉RMG-int(A). That is every RMG-open set U containing x is such that U⊄A. That is every RMG-open set U containing x such that U∩(X−A) ≠ ∅. By the Theorem 6.9, x∉RMG-cl(X−A) and therefore X− (RMG-int(A))⊂ RMG-cl(X−A).

Conversely, let x∈RMG-cl(X−A), Then by Theorem 6.9, every RMG-open set U containing x such that U⊄A. This implies by definition of RMG-interior of A, x∉RMG-int(A). That is x∈X− (RMG-int(A)) and RMG-cl(X−A) and RMG-cl($A^C$)⊂(RMG-int(A))$^C$. Thus (RMG-int(A))$^C$ = RMG-cl($A^C$).

ii) Follows by taking complements in (i).

iii) Follows by replacing A by X−A in (i).

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