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On The Fuzzy Implicative Ideal of a BH-Algebra

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Abstract: *In this paper, we study a fuzzy implicative ideal of a BH-algebra. We give some properties of this fuzzy ideal and link it with other types of fuzzy ideals and fuzzy subset of a BH-algebra.*
Keywords: *fuzzy completely closed ideal, fuzzy ideal, fuzzy implicative ideal, fuzzy set, fuzzy sub-implicative ideal, fuzzy p- ideal, implicative BH-algebra, implicative BH-algebra.*
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I. Introduction

In 1966, K. Iseki introduced the notion of a BCI–algebra which was a generalization of a BCK-algebra [9]. In this year, K. Iseki introduced the notion of an ideal of a BCK–algebra [8]. In 1998, Jun et al, Roh and kim introduced the notion of BH-algebra, which is a generalization of BCH-algebras [13]. Then, Y. B. Jun, E. H. Roh, H. S. Kim and Q. Zhang discussed more properties on BH-algebras [13]. In 2011, H. H. Abbass and H. M.A. Saeed generalized the notion of a closed ideal, p-ideal and implicative ideal to a BH-algebra and BCA-part to a BH-algebra [4]. In 2012, H. H. Abbass and H. D. Dahham introduced the notion of a completely closed ideal and completely closed ideal with respect to an element of a BH-algebra [3]. In 1965, L. A. Zadeh introduced the notion of a fuzzy subset of a set as a method for representing uncertainty in real physical world. Since then its application had been growing rapidly over many disciplines [9]. In 2001, Q. Zhang, E. H. Roh and Y. B. Jun studied the fuzzy theory in BH-algebras [11]. In 2011, H. H. Abbass and H. M.A. Saeed generalized the notion of a fuzzy closed ideal, fuzzy p-ideal and fuzzy implicative ideal to a BH-algebra and BCA-part [4]. In 2012, H. H. Abbass and H. D. Dahham introduced the notion of a fuzzy completely closed ideal of a BH-algebra [3].

In this paper, we define the notion of fuzzy implicative ideal of BH-algebra, We state and prove some theorems which determine the relationshipsamong this notion and the other types of fuzzy ideals and fuzzy subsets of a BH-algebra

II. Preliminaries

In this section, we give some basic concepts about BCI-algebra, BH-algebra. We state and prove some theorems which determine the relationship between this notion and the other types of fuzzy ideals and fuzzy subset of a BH-algebra, fuzzy ideals of BH-algebra with some theorems, propositions and examples.

Definition (2.1) [8]
A BCI-algebra is an algebra (X,*, 0), where X is a nonempty set, * is a binary operation and 0 is a constant, satisfying the following axioms:
$\forall$ x, y, z$\in$ X:

i. ((x *y) * (x * z)) * (z * y) = 0.
ii. (x * (x * y)) * y = 0.
iii. x * x = 0.
iv. x * y =0 and y * x = 0 imply x = y.

Definition (2.2) [13]
A **BH-algebra** is a nonempty set X with a constant 0 and a binary operation * satisfying the following conditions:

i. x * x = 0, $\forall$ x$\in$X.
ii. x * y = 0 and y * x = 0 imply x = y, $\forall$ x, y$\in$ X.
iii. x *0 = x**,** $\forall$ x$\in$X**.**

Definition (2.3) [13]
Let I be a nonempty subset of a BH-algebra X. Then I is called an **ideal** of X if it satisfies:

i. 0$\in$I.
ii. x*y$\in$I and y $\in$I imply x$\in$I.

Definition (2.4) [4]
A nonempty subset I of a BH-algebra X is called an **implicative ideal** of X if:
i. $0 \in I$.
ii. $(x*(y*x))*z \in I$ and $z \in I$ imply $x \in I$, $\forall$ x, y, z $\in$ X.

Proposition (2.5) [4]
Every implicative ideal of a BH-algebra X is an ideal of X.

Definition (2.6)[9]
Let X be a non-empty set and I be the closed interval [0, 1] of the real line (real numbers).A **fuzzy set A in X (a fuzzy subset of X)** is a function from X into I.
fuzzy sets in X.

Definition (2.7)[11]
A fuzzy subset A of a BH-algebra X is said to be a **fuzzy ideal** if and only if:
i. $A(0) \geq A(x)$, $\forall$ $x \in X$.
ii. $A(x) \geq \min\{A(x*y), A(y)\}$, $\forall$ $x, y \in X$.

Definition (2.8) [4]
A **fuzzy subset** A of a BH-algebra X is called a **fuzzy implicative ideal** of X if it satisfies:
i. $A(0) \geq A(x)$, $\forall$ $x \in X$.
ii. $A(x) \geq \min\{A((x*(y*x))*z), A(z)\}$, $\forall$ $x, y, z \in X$.

Example (2.9)
Consider the BH-algebra X= {0, 1, 2} with the binary operation '*' defined by the following table**:**

*	0	1	2
0	0	0	1
1	1	0	1
2	2	2	0

The fuzzy subset A of X by defined $A(x) = \begin{cases} 1 & ; \quad x = 0,1 \\ 0.5 & ; \quad x = 2 \end{cases}$ is a **fuzzy implicative ideal** of X.

Proposition (2.10) [4]
In BH-algebra of X, every fuzzy implicative ideal is a fuzzy ideal, but the converse is not true, in general.

III. The Relationship between Fuzzy Implicative Ideal and Fuzzy Sub-implicative Ideal of a BH-Algebra.

In this section, we link the fuzzy implicative ideal with fuzzy sub-implicative ideal of a BH-algebra with some theorems, propositions and examples.

Definition (3.1) [12]
A fuzzy set A of a BCI-algebra X is called a **fuzzy sub-implicative ideal** of X if it satisfies:
i. $A(0) \geq A(x)$, $\forall$ $x \in X$.
ii. $A(y*(y*x)) \geq \min\{A(((x*(x*y))*(y*x))*z), A(z)\}$, $\forall$ $x, y, z \in X$.
We generalize the concept of a **fuzzy sub-implicative ideal** to a BH-algebra.

Definition (3.2)
A fuzzy set A of a BH-algebra X is called a **fuzzy sub-implicative ideal** of X if it satisfies:
i. $A(0) \geq A(x)$, $\forall$ $x \in X$.
ii. $A(y*(y*x)) \geq \min\{A(((x*(x*y))*(y*x))*z), A(z)\}$, $\forall$ $x, y, z \in X$.

Proposition(3.3)
Let X be a BH-algebra. Then every fuzzy sub-implicative ideal of X is fuzzy ideal of X.
Proof: Let A be a fuzzy sub-implicative ideal of X. Then
i. $A(0) \geq A(x)$, $\forall$ $x \in X$. [By definition (3.2)(i)]
ii. Let $x, z \in X$. Then $A(x) = A(x*0) = A(x*(x*x)) \geq \min\{A(((x*(x*x))*(x*x))*z)), A(z)\}$
[Since A is a fuzzy sub-implicative of X. By definition (3.2)(ii)]
$= \min\{A(((x*0)*0)*z)), A(z)\}$ [Since X is BH-algebra , x*x=0]
$= \min\{A((x*0)*z), A(z)\}$ [Since X is BH- algebra, x*0=x]
$= \min\{A(x*z), A(z)\}$ [Since X is BH- algebra, x*0=x]
$\Rightarrow$ $A(x) \geq \min\{A(x*z), A(z)\}$. Therefore, A is a fuzzy ideal of X.■

Remark (3.4)
The converse of proposition (3.3) is not correct in general, as in the following example.
Example (3.5)
Consider the BH-algebra X= {0, 1, 2, 3} with the binary operation '*' defined by the following table**:**

*	0	1	2	3
0	0	0	0	3
1	1	0	0	3
2	2	2	0	3
3	3	3	3	0

The fuzzy subset A defined by $A(x) = \begin{cases} 0.5 & ; x = 0 \\ 0.2 & ; x \neq 0 \end{cases}$ is a fuzzy ideal of X, but A is not a fuzzy sub-implicative ideal of X. Since if x=2, y=1, z=0, then
A(1*(1*2))=A(1*0)=A(1)=0.2 < min{A(((2*(2*1))*(1*2))*0)), A(0)} = min{A(((2*2)*(0))*0)), A(0)}
= min{A(0*0),A(0)}=min{A(0), A(0)} =A(0)=0.5

Theorem (3.6)
Let X be a BH-algebra and let A be a fuzzy ideal of X. Then A is a fuzzy sub-implicative ideal of X if and only if A(y*(y*x)) ≥ A((x*(x*y))*(y*x)) (b_1).
Proof: Let A be a fuzzy sub-implicative ideal of X and x, y∈X. Then
A(y*(y*x)) ≥ min{A(((x*(x*y))*(y*x))*0),A(0)}
= min{A((x*(x*y))*(y*x)),A(0)}[Since X is a BH-algebra ;x*0=x]
= A((x*(x*y))*(y*x))[Since A is a fuzzy ideal of X, A(0) ≥A(x)].
Then the condition (b_1) is satisfied.
Conversely, Let A be a fuzzy ideal of X. Then
i. A(0) ≥ A(x), ∀ x∈X. [By definition (2.7)(i)]
ii. Let x, y, z ∈ X. Then A((x*(x*y))*(y*x)) ≥ min{A(((x*(x*y))*(y*x))*z),A(z)}
[Since A is a fuzzy ideal of X. By definition (2.7)(ii)]
⇒ A(y*(y*x)) ≥ A((x*(x*y))*(y*x)) [By (b_1)]
⇒ A(y*(y*x)) ≥ min{A(((x*(x*y))*(y*x))*z),A(z)}.
Therefore, A is a fuzzy sub-implicative ideal of X.■

Definition (3.7) [7]
A BCI-algebra is said to be an implicative if it satisfies the condition,
(x*(x*y))*(y*x) =y*(y*x); ∀ x, y ∈X.
We generalize the concept of an **implicative** to a **BH-algebra**.

Definition (3.8)
A BH-algebra is said to be an implicative if it satisfies the condition,
(x*(x*y))*(y*x) = y*(y*x); ∀ x, y∈X.

Example (3.9)
Consider the BH-algebra X= {0, 1, 2} with the binary operation '*' defined by the following table**:**

*	0	1	2
0	0	0	2
1	1	0	2
2	2	2	0

Then (X,*, 0) is an implicative BH-algebra.

Theorem(3.10)[13]
Every BH-algebra satisfying the condition: ((x*y) * (x*z))*(z*y) =0, ∀ x, y, z∈X is a BCI-algebra
Theorem (3.11) [15]
A BCI-algebra X is an implicative if and only if every fuzzy closed ideal of X is a fuzzy implicative ideal of X.
Theorem (3.12)
Let X be a BH-algebra and satisfies the condition:
((x*y)*(x*z))*(z*y) = 0, ∀x, y, z ∈ X (b_2).
Then X is an implicative if and only if every fuzzy closed ideal of X is a fuzzy implicative ideal of X.
Proof: Directly from theorems (3.10) and (3.11). ■

Theorem(3.13)
Let X be an implicative BH-algebra. Then every fuzzy ideal of X is a fuzzy sub-implicative ideal of X.
Proof:
Let A be a fuzzy ideal of X. Then
i. $A(0) \geq A(x), \forall x \in X$. [By definition (2.7)(i)]
ii. Let $x, y, z \in X$. Then $A(y*(y*x)) \geq \min\{A((y*(y*x))*z), A(z)\}$
$\geq \min\{A(((x*(x*y))*(y*x))*z)), A(z)\}$[Since X is an implicative]
$\Rightarrow A(y*(y*x)) \geq \min\{A(((x*(x*y))*(y*x))*z)), A(z)\}$.
Therefore, A is a fuzzy sub-implicative ideal of X.■

Definition (3.14) [3]
A BCH-algebra X is called **medial** if $x * (x * y) = y, \forall x, y \in X$.
We generalize the concept of **medial** to BH-algebra.

Definition(3.15)
A BH-algebra X is called **medial** if $x * (x * y) = y, \forall x, y \in X$.

Corollary (3.16)
In a medial BH-algebra, every fuzzy ideal of X is a fuzzy sub-implicative ideal of X.
Proof: Let A be a fuzzy ideal of X. Then
i. $A(0) \geq A(x), \forall x \in X$. [By definition (2.7)(i)]
ii. Let $x, y \in X$. Then $A(y*(y*x))=A(x)$ [Since X is a medial, $x*(x*y)=y$]
$\geq \min\{A(x*z),A(z)\}$ [Since A is a fuzzy ideal of X. By definition (2.7)(ii)]
$\geq \min\{A((y*(y*x))*z), A(z)\}$ [Since X is a medial, $x=y*(y*x)$]
$\geq \min\{A(((x*(x*y))*(y*x))*z)), A(z)\}$. [Since X is a medial, $x*(x*y)= y$]
$\Rightarrow A(y*(y*x)) \geq \min\{A(((x*(x*y))*(y*x))*z)), A(z)\}$.
Therefore, A is a fuzzy sub-implicative ideal of X.■

Remark (3.17)
The following example shows that notions of fuzzy implicative ideal of X and fuzzy sub-implicative ideal of X are independent.

Example (3.18)
Consider the BH-algebra X= {0, 1, 2, 3} with the binary operation '*' defined by the following table:

*	0	1	2	3
0	0	0	1	1
1	1	0	0	1
2	2	2	0	1
3	3	0	1	0

The fuzzy subset A of X defined by $A(x) = \begin{cases} 0.8 & ; x = 0,3 \\ 0.5 & ; x = 1,2 \end{cases}$ is a fuzzy implicative ideal of X, since $A(x) \geq A(x*(y*x)), \forall x, y \in X$. But it is not a fuzzy sub-implicative ideal of X. Since
if x=3, y= 2, z=0, then $A(2*(2*3))=A(2)=0.5 < \min\{A(((3*(3*2))*(2*3))*0), A(0)\}$
$= \min\{A((3*1)*1), A(0)\} = \min\{A(0*1), A(0)\} = \min\{A(0), A(0)\}=A(0)=0.8$

Remark (3.20)
If A is a fuzzy sub-implicative ideal of a BH-algebra of X, then A may not be a fuzzy implicative ideal of X, as in the following example.

Example (3.21)
Let X be the BH-algebra in example (2.9). Then the fuzzy ideal A which defined $A(x) = \begin{cases} 0.8 & ; x = 0,1 \\ 0.5 & ; x = 2 \end{cases}$ is a fuzzy sub-implicative ideal of X. But A is not a fuzzy implicative ideal of X. Since if x=2, y= 0, z=0, then $A(2)=0.5 < \min\{A(((2*(0*2))*0), A(0)\}$
$= \min\{A(2*2), A(0)\}= \min\{A(0), A(0)\}=A(0)=0.8$
We provide conditions for a fuzzy sub-implicative ideal to be a fuzzy implicative ideal of X.

Proposition (3.22)
Let X be a BH-algebra. Then every fuzzy sub-implicative ideal of X is a fuzzy ideal of X.
Proof: Let A be a fuzzy sub-implicative ideal of X. To prove A is a fuzzy ideal of X.
i. $A(0) \geq A(x), \forall x \in X$. [Since A is a sub-implicative ideal of X.]
ii. Let $x, y, z \in X$ such that $\min\{A(x*z),A(z)\}=\min\{A((x*0)*z),A(z)\}$[Since X is BH-algebra; $x*0=x$]

= min {A((x*(0*0))*z), A(z)}   [Since X is BH-algebra; x*x=0]
= min{A(((x*(x*x))*(x*x))*z) ,A(z)} [Since X is BH-algebra; x*x=0]
$\Rightarrow$ A(x*(x*x)) ≥ min {A(((x*(x*x))*(x*x))*z), A(z)}[Since A is a fuzzy sub-implicative of X. By definition (3.2)(ii)]
Now, A(x*(x*x)) =A(x*0)=A(x)   [Since X is BH-algebra ; x*x=0, x*0=x]
$\Rightarrow$A(x) ≥ min{A(((x*(x*x))*(x*x))*z) ,A(z)}= min{A(x*z),A(z)}
$\Rightarrow$A(x) ≥ min{A(x*z),A(z)}.Therefore, A is a fuzzy ideal of X.■

Remark (3.23)
The converse the proposition (3.22) is not true, in general. As in the following example.

Example (3.24)
Consider the BH-algebra X= {0, 1, 2, 3} with the binary operation '*' defined by the following table**:**

*	0	1	2	3
0	0	0	0	0
1	1	0	0	2
2	2	2	0	1
3	3	3	3	0

The fuzzy subset A defined by $A(x) = \begin{cases} 1 & ; \ x = 0 \\ 0.5 & ; x = 1,2,3 \end{cases}$ is a fuzzy ideal of X, but A is not a fuzzy sub-implicative ideal of X. Since if x=2, y=1, z=0, then
A(1*(1*2))=A(1*0)=A(1)= 0.5 < min{A(((2*(2*1))*(1*2))*0) ,A(0)}
= min{A((2*2))*(1*2)) ,A(0)} = min{A(0),A(0)}=A(0)=1.

Theorem (3.25)
If X is a BH-algebra of X satisfies the condition:
∀ x, y, z∈X ; A(y*z) ≥ A((x *(x *y))*z) (b_3),
then every fuzzy ideal of X is a fuzzy sub-implicative ideal of X.

Proof: Let A be a fuzzy ideal of X. Then
i. Let x, y, z∈X. Then A((x*(x*y))*(y*x)) ≥ min{A(((x*(x*y))*(y*x))*z), A(z)} [Since A is a fuzzy ideal of X. By definition (2.7)(ii)]
Put z =y*x in b3. Then A(y*(y*x)) ≥ A((x*(x*y))*(y*x))
$\Rightarrow$ A(y*(y*x)) ≥ min {A(((x*(x*y))*(y*x))*z), A(z)}.
Therefore, A is a fuzzy sub-implicative ideal of X. ■

Theorem (3.26)
Let X be an implicative BH-algebra. Then every fuzzy ideal of X is a fuzzy sub-implicative ideal of X.

Proof: Let A be a fuzzy ideal of X. Then
i. A(0) ≥ A(x),∀x∈ X. [By definition (2.7)(i)]
ii. Let x, y, z∈X. Then A((x*(x*y))*(y*x)) ≥ min{A(((x*(x*y))*(y*x))*z),A(z)}
[Since A is a fuzzy ideal of X. By definition (2.7)(ii)]
$\Rightarrow$ A((x*(x*y))*(y*x))=A(y*(y*x))   [Since X is an implicative]
$\Rightarrow$ A(y*(y*x)) ≥ min {A(((x*(x*y))*(y*x))*z),A(z)}. Then A is a fuzzy sub-implicative ideal of X.■

Proposition (3.27)[5]
Every medial BH-algebra is an implicative BH-algebra

Corollary(3.28)
If X is a medial BH-algebra, then every fuzzy ideal of X is fuzzy sub-implicative of X.

Proof: Directly by theorem (3.26) and proposition (3.27).■

Theorem (3.29) [5]
Let X be a BH-algebra and let A be a fuzzy ideal. Then A is a fuzzy implicative ideal of X if and only if A satisfies the following inequality:
∀ x, y∈X; A(x) ≥ A(y*(y*x)) (b_4).

Theorem (3.30)
Let X be a medial BH-algebra satisfies the condition:
∀ x, y∈X; A((x*(x*y))*(y*x)) ≥ A(x *(y *x)) (b_5).
Then every fuzzy sub-implicative ideal is a fuzzy implicative ideal of X.

Proof: Let A be a fuzzy sub-implicative ideal of X. To prove A is a fuzzy implicative ideal of X.
i. A(0) ≥ A(x), ∀ x∈X.   [By definition (3.2)(i)]
ii. Let x, y, z ∈X. Then A(x*(y*x)) ≥ min {A((x*(y*x))*z) , A(z)}

[Since A is a fuzzy ideal of X. By proposition (3.3)]
Now, A((x*(x*y))*(y*x)) ≥ A(x *(y *x)), [By (b_5)]
and A(y*(y*x)) ≥ min{ A(((x*(x*y))*(y*x))*z), A(z)} [Since A is a fuzzy sub-implicative ideal of X.]
if z=0, then A(y*(y*x)) ≥ min{ A(((x*(x*y))*(y*x))*0), A(0)}
⇒ A(y*(y*x)) ≥ min{ A((x*(x*y))*(y*x)), A(0)} [Since X is a BH-algebra, x*0=x.]
⇒ A(y*(y*x)) ≥ A((x*(x*y))*(y*x)). [Since A(0) ≥ A(x)]
⇒ A(y*(y*x)) ≥ A(x *(y *x))
Now, we have A(x) = A(y*(y*x))[Since X be a medial; y*(y*x) = x]
⇒ A(x) ≥ A(y*(y*x)). Therefore, A is a fuzzy implicative ideal of X. [By theorem (3.29)].■

Theorem (3.31)
Let X be a implicative BH-algebra. Then every fuzzy implicative ideal of X is a fuzzy sub-implicative ideal of X.
Proof: Let A be a fuzzy implicative ideal of X. To prove A is a fuzzy sub-implicative ideal of X.
i. A(0) ≥ A(x), ∀ x∈X. [By definition (2.8)(i)]
ii. Let x, y, z∈X. Then A((x*(x*y))*(y*x)) ≥ min{A(((x*(x*y))*(y*x))*z), A(z)}
[Since A is a fuzzy ideal of X. By proposition (2.10)]
⇒ A((x*(x*y))*(y*x)) =A(y*(y*x)) [Since X is an implicative.]
⇒ A(y*(y*x)) ≥ min {A(((x*(x*y))*(y*x))*z),A(z)}.
Therefore, A is a fuzzy sub-implicative ideal of X.■

Corollary (3.32)
Let X be a medial BH-algebra. Then every implicative ideal of X is a sub-implicative ideal of X.
Proof: Directly by theorem (3.31) and proposition (3.27).■

IV. The Relationship Between Fuzzy Implicative Idealand Fuzzy Completely Closed Idealof a BH-algebra

In this section, we link the fuzzy implicative ideal with fuzzy completely closed ideal of a BH-algebra with some theorems, propositions and examples.
Definition (4.1) [4]
Afuzzy ideal A of a BH-algebra X is said to be **fuzzy closed** if A (0 * x) ≥ A(x), ∀x∈X.

Definition (4.2) [3]
Let X be a BH-algebra and A be a fuzzy ideal of X. Then A is called a **fuzzy completely closed ideal** of X if A (x*y) ≥ min{A(x), A(y)}, ∀x,y∈X.
Proposition(4.3)
Let X be an implicative BCI-algebra .Then every fuzzy completely closed ideal of X is a fuzzy implicative ideal of X.
Proof: Let A be a fuzzy completely closed ideal of X .To prove A is a fuzzy implicative ideal of X.
⇒ A is a fuzzy ideal of X. [By definition (4.2)]
i. A(0) ≥ A(x), ∀ x∈X. [By definition (2.7)(i)]
ii. Let y ∈X, if x=0
⇒ A(0*y) ≥ min{A(0) ,A(y)}=A(y). [By definition (4.2)]
⇒ A(0*y) ≥ A(y).
⇒ A is a fuzzy closed ideal of X. [By definition (4.2)]
Therefore, A is a fuzzy implicative ideal of X. [By theorem (3.12)].■

Proposition(4.4)
Let X be an implicative BH-algebra satisfies (b_2). Then every fuzzy completely closed ideal of X is a fuzzy implicative ideal of X.
Proof: Let A be a fuzzy completely closed ideal of X . Then A is a fuzzy ideal of X. [By definition (4.2)]
To prove A is a fuzzy closed ideal of X.
Now, let y ∈X. if x=0,
⇒ A(0*y) ≥ min{A(0) ,A(y)}=A(y) [A(0) ≥ A(y)]
⇒ A(0*y) ≥ A(y)
⇒ A is a fuzzy closed ideal of X. Therefore, A is a fuzzy implicative ideal of X. By theorem (3.12)].■

Corollary (4.5)
Let X be a medial BH-algebra satisfies (b_2). Then every fuzzy completely closed ideal of X is a fuzzy implicative ideal of X.
Proof: Directly by theorem (3.27) and proposition (4.4).■

V. The Relationship Between Fuzzy Implicative Idealand Fuzzy P-Idealof BH-algebra

In this section, we link the fuzzy implicative ideal with fuzzy p- ideal of a BH-algebra with some theorems, propositions and examples.

Definition (5.1) [4]
A fuzzy set A of a BH-algebra X is called a **fuzzy p-ideal** of X if it satisfies:
i. A(0) ≥ A(x), ∀ x∈X.
ii. A(x) ≥ min {A((x *z) *(y *z)), A(y)}, ∀ x, y, z ∈ X.

Example (5.2)[4]
Let X be a BH-algebra in example (2.9), then the fuzzy ideal A which defined by$A(x) = \begin{cases} 1 \ ; \ x = 0,1 \\ 0.5 \ ; \ x = 2 \end{cases}$. Then A is a **fuzzy p-ideal** of X.

Definition (5.3) [4]
Let X be a BH-algebra. Then the set X_+={x∈X | 0*x=0} is called the **BCA-part** of X.

Theorem (5.4)
Let X=X_+ be a BH-algebra. Then every fuzzy p-ideal of X is a fuzzy implicative ideal of X.
Proof: Let A be a fuzzy p-ideal of X. Then
i. A(0) ≥ A(x), ∀ x ∈ X. [By definition (5.1)(i)]
ii. Let x, y∈X. Then A(a) ≥ min{A((a*c)*(b*c)),A(b)}. Put a=x, b=0, c=y*x
A(x) ≥ min {A((x*(y*x))*(0*(y*x)),A(0)}
= min {A((x*(y*x))*0),A(0)} [Since X=X_+]
= min {A(x*(y*x)),A(0)} [Since X is BH-algebra ; x*0=x]
= A(x*(y*x)) [Since A(0) ≥ A(x), ∀ x ∈ X.]
⇒ A(x) ≥ A(x*(y*x)). Therefore, A is a fuzzy implicative ideal of X. [By theorem (3.29)] ■

Remark(5.5)
In the following example, we see that the converse of theorem (5.4) may not be true in general.

Example (5.6)
Consider X = {0, 1, 2} with the b

*	0	1	2
0	0	0	0
1	1	0	1
2	2	2	0

inary operation '*' defined by the following table**:**

Define a fuzzy subset by $A(x) = \begin{cases} 0.7 \ ; x = 0 \\ 0.5 \ ; \ x = 1 \\ 0.2 \ ; \ x = 2 \end{cases}$. Then A is a fuzzy implicative ideal of X, but A is not a fuzzy p-ideal of X. Because if x=2, y=1, z=2, then A(2) =0.2 < min{A((2 * 2) * (1* 2)) , A(1)}
= min {A(0*1) , A(0)}= min {A(0) , A(0)}= A(0)=0.7

VI. The Relationship between Fuzzy Implicative Ideal and Some Fuzzy and Ordinary Sets of BH-algebra

In this section, we link the fuzzy implicative ideal and some fuzzy and ordinary sets of BH-algebra with some theorems, propositions and examples.

Definition (6.1)[10]
Let A be a fuzzy set in X, ∀α∈ [0, 1], the set A_α= { x ∈ X, A(x) ≥α } is called a **level subset of A.**

Theorem (6.2)
Let X be a BH-algebra and A be a fuzzy ideal of X. Then A is a fuzzy implicative ideal of X if and only if the level subset A_α is an implicative ideal of X, ∀α∈[0, $\sup_{x\in X} A(x)$].

Proof:
Let A be a fuzzy implicative ideal of X. To prove A_α is an implicative ideal of X.
i. A(0) ≥ A(x), ∀ x∈X. [By definition (2.8)(i)]
⇒ A(0) ≥ α, ∀ α∈[0, A(0)]. Then 0 ∈ A_α.

ii. Let x, y, z $\in$X such that $(x*(y*x))*z\in A_\alpha$ and $z\in A_\alpha$
$\Rightarrow$ $A((x*(y*x))*z) \geq \alpha$ and $A(z) \geq \alpha$ [By definition(6.1) of A_α]
$\Rightarrow$ min { A((x*(y*x))*z) , A(z) } $\geq$ α
ButA(x) $\geq$ min{A((x*(y*x))*z),A(z)}[Since A is a fuzzy implicative ideal of X. By definition (2.8)(ii)]
$\Rightarrow A(x) \geq \alpha \Rightarrow x \in A_\alpha$. Therefore, A_α is an implicative ideal of X.
Conversely, Let A_α be an implicative ideal of X,$\forall\alpha\in$[0, A(0)] and Let $\alpha= \sup_{x\in X} A(x)$].. To prove that A is a fuzzy implicative ideal of X.
i. $0\in A_\alpha$. [Since A_α is an implicative ideal of X]
$\Rightarrow$A(0) $\geq$ α. Then A(0) $\geq$ A(x), $\forall$ x$\in$X. [Since A(0)=1]
ii. Let x , y , z $\in$ X such that min{ A((x*(y*x))*z) , A(z) }= α
$\Rightarrow$ $A((x*(y*x))*z) \geq \alpha$ and $A(z) \geq \alpha$
$\Rightarrow$ $(x*(y*x))*z \in A_\alpha$ and $z\in A_\alpha$
$\Rightarrow$ $x\in A_\alpha$. [Since A_α be an implicative ideal of X.]
$\Rightarrow$ $A(x) \geq \alpha$
$\Rightarrow$ A(x) $\geq$ min {A((x*(y*x))*z) , A(z)}. Therefore, A is a fuzzy implicative ideal of X.■
Corollary (6.3)
Let X be a BH-algebra, Then A is a fuzzy implicative ideal A of X if and only if the set X_A is an implicative ideal of X, where X_A={x ∈ X |A(x)=A(0)}
Proof: Let A be a fuzzy implicative ideal of X. To prove X_A is an implicative ideal of X.
i. If x=0, then A(x)=A(0). Then $0 \in X_A$.
ii. Let x, y, z ∈ X such that $(x*(y* x))* z \in X_A$ and $z \in X_A$.
$\Rightarrow$ A((x*(y* x))* z)=A(0) and A(z)= A(0).
We have A(x) $\geq$ min{A((x*(y*x))*z), A(z)} [Since A is a fuzzy implicative ideal of X]
= min {A(0), A(0)}=A(0)
$\Rightarrow$ A(x) $\geq$ A(0).
$\Rightarrow$ A(x) = A(0). [Since A is a fuzzy implicative ideal of X, A(0) $\geq$ A(x)]
$\Rightarrow$ $x \in X_A$. Therefore, X_A is an implicative ideal of X. ■
Conversely, Let X_A be an implicative ideal of X. To prove A is a fuzzy implicative ideal of X.
Since $X_A= A_\alpha$, α = A(0). Therefore, A is a fuzzy implicative ideal of X.■

Proposition (6.4)
Let X be a BH-algebra and A be a fuzzy subset of X defined by $A(x) = \begin{cases}\alpha_1 & ; \; x\in X_A \\ \alpha_2 & ; \; \text{otherwise}\end{cases}$, where $\alpha_1, \alpha_2 \in [0, 1]$ such that $\alpha_1 > \alpha_2$. Then A is a fuzzy implicative ideal of X if and only if X_A is an implicative ideal of X.
Proof: Let A be a fuzzy implicative ideal of X. To prove X_A is an implicative ideal of X.
i. $A(0) =\alpha_1$. $\Rightarrow 0 \in X_A$. [Since A(0) $\geq$ A(x) ; $\forall$x ∈X. By definition (2.8)(i)]
ii. Let x , y, z $\in X_A$ such that $(x*(y*x))*z\in X_A$ and $z\in X_A$
$\Rightarrow$ $A((x*(y*x))*z)=A(0)=\alpha_1$ and $A(z)=A(0)=\alpha_1$
$\Rightarrow$A(x) $\geq$ min{A((x*(y*x))*z), A(z)}=α_1 [Since A is a fuzzy implicative ideal of X, by definition(2.8)(ii)]
$\Rightarrow A(x) =\alpha_1$
$\Rightarrow x \in X_A$. Then X_A is an implicative ideal of X.
Conversely, Let X_A be an implicative ideal of X. To prove A is a fuzzy implicative ideal of X.
i. Since $0 \in X_A$, then $A(0) =\alpha_1$.
$\Rightarrow A(0) =\alpha_1\geq A(x)$. Then A(0) $\geq$ A(x), $\forall$ x$\in$X.
ii. Let x, y, z ∈X. Then we have four cases:
Case1: If $(x*(y*x))*z \in X_A$ and $z \in X_A$
$\Rightarrow x \in X_A$. [Since X_A is an implicative ideal of X]
$\Rightarrow A((x*(y*x))*z)=\alpha_1 \Rightarrow A(x) \geq$ min{A((x*(y*x))*z), A(z)}.
Case2: If $(x*(y*x))*z \in X_A$ and $z\notin X_A$
$\Rightarrow A((x*(y*x))*z)=\alpha_1$ and $A(z)=\alpha_2$
$\Rightarrow$ min {A((x*(y*x))*z), A(z)}=α_2
$\Rightarrow$ A(x) $\geq$ min{A((x*(y*x))*z), A(z)}.
Case3: If $(x*(y*x))*z \notin X_A$ and $z \in X_A$
$\Rightarrow A((x*(y*x))*z)=\alpha_2$ and $A(z)=\alpha_1$
$\Rightarrow$ min {A((x*(y*x))*z), A(z)}=α_2
$\Rightarrow$ A(x) $\geq$ min {A((x*(y*x))*z), A(z)}.
Case4: If $(x*(y*x))*z\notin X_A$ and $z \notin X_A$

$\Rightarrow$ A((x*(y*x))*z)=α_2 and A(z)=α_2
$\Rightarrow$ min {A((x*(y*x))*z), A(z)}= α_2
$\Rightarrow$ A(x) $\geq$ min{A((x*(y*x))*z), A(z)}. Therefore, A is a fuzzy implicative ideal of X. ■

Proposition (6.5)
Let X be a BH-algebra and let A be a fuzzy subset of X. Then A is a fuzzy implicative ideal of X if and only if $A^{\#}$(x) = A(x) + 1- A(0) is a fuzzy implicative ideal of X.
Proof: Let A be a fuzzy implicative ideal of X. To prove $A^{\#}$ be a fuzzy implicative ideal of X.
i. $A^{\#}$(0)= A(0)+1-A(0).
$\Rightarrow$ $A^{\#}$(0)=1.
$\Rightarrow$ $A^{\#}$ (0) $\geq$ $A^{\#}$(x), $\forall$x$\in$X.
ii. Let x, y , z $\in$X. Then
$A^{\#}$(x)=A(x)+1-A(0) $\geq$ min{A((x*(y*x))*z),A(z)}+1-A(0) [Since A is a fuzzy implicative of X.]
= min {A((x*(y*x))*z)+1- A(0),A(z) +1-A(0)}
= min {$A^{\#}$((x*(y*x))*z), $A^{\#}$(z)}.
Then $A^{\#}$ is a fuzzy implicative ideal of X.

Conversely, Let $A^{\#}$ be a fuzzy implicative ideal of X. To prove A be a fuzzy implicative ideal of X.
i. Let x$\in$X. Then we have A(0)=$A^{\#}$(0)-1+A(0) $\geq$ $A^{\#}$(x)-1+A(0)=A(x)
$\Rightarrow$ A(0) $\geq$ A(x), $\forall$x$\in$X.
ii. Let x, y, z $\in$X. Then
A(x) = $A^{\#}$(x)-1+ A(0) $\geq$ min{ $A^{\#}$((x*(y*x))*z), $A^{\#}$(z)}-1+ A(0)
[Since $A^{\#}$ is a fuzzy implicative ideal of X. By definition (2.8)(ii)]
= min{ $A^{\#}$((x*(y*x))*z)-1+ A(0), $A^{\#}$(z) +1-A(0)}
= min{ A((x*(y*x))*z), A (z)}. Then A is a fuzzy implicative ideal of X.■
Remark(6.6)
Let A be a fuzzy subset of a BH-algebra X and w∈X. The set { x∈X|A(w)≤A(x)} is denoted by ↑A(w).

Proposition (6.7)
Let A be a fuzzy ideal of a BH-algebra X and w ∈X. If A satisfies the condition:
$\forall$ x, y∈X, A(x) $\geq$ A (x*(y*x)) (b_4), then ↑A(w) is an implicative ideal of X.
Proof: Let A be a fuzzy ideal of X. Then
i. A(0)≥A(x), ∀x ∈X. [By definition (2.7)(i)]
⇒A(0)≥A(w) [Since w∈X.]. Then 0 ∈↑A(w).
ii. Let x, y, z ∈ X such that (x*(y*x))*z ∈↑A(w) and z ∈↑A(w)
⇒ A(w) ≤A((x*(y*x))*z) and A(w) ≤A(z)
⇒A(w)≤min{A((x*(y*x))*z),A(z)}≤A(x*(y*x))[Since A is a fuzzy ideal of X. By definition (2.7)(ii)]
But A(x*(y*x))≤A(x). [By (b_4)]
⇒ A(w) ≤A(x).
⇒ x ∈↑A(w). Therefore, ↑A(w) is an implicative ideal of X.

Proposition (6.8)
Let X be a BH-algebra and w∈X. If A is a fuzzy implicative ideal of X, then ↑A(w) is an implicative ideal of X.
Proof: Let A be a fuzzy implicative ideal of X. To prove that ↑A(w) is an implicative ideal of X.
i. A(0)≥A(x), ∀x ∈X. [By definition (2.8)(i)]
⇒A(0)≥A(w) [Since w∈X.] . Then 0 ∈↑A(w).
ii. Let x, y, z ∈ X such that (x*(y*x))*z ∈↑A(w) and z ∈↑A(w),
⇒ A(w) ≤A((x*(y*x))*z) and A(w) ≤A(z),
⇒ A(w) ≤ min{ A((x*(y*x))*z) , A(z)} .
But min { A((x*(y*x))*z) , A(z)}≤A(x). [By definition (2.8)(ii)]
⇒ A(w) ≤A(x). Then x ∈↑A(w). Thus↑A(w) is an implicative ideal of X. ■

Definition (6.9)[1]
if {A_α, $\alpha\in\Lambda$} is a family of fuzzy sets in X, then
$\bigcap_{i\in I} A_i$ (x) = inf { A_i (x), i$\in$ I}, $\forall$ x $\in$X and $\bigcup_{i\in I} A_i$ (x) = sup { A_i (x), i$\in$ I}, $\forall$ x $\in$X. which are also

Proposition (6.10)[3]
Let $\{A_\alpha|\alpha\in\lambda\}$ be a family of fuzzy ideals of a BH-algebra X. Then $\bigcap_{\alpha\in\lambda} A_\alpha$ is a fuzzy ideal of X.

Proposition (6.11)
Let $\{A_\alpha\ |\alpha\in\lambda\}$ be a family of fuzzy implicative ideals of a BH-algebra X. Then $\bigcap_{\alpha\in\lambda} A_\alpha$ is a fuzzy implicative ideal of X.
Proof: Let $\{A_\alpha|\alpha\in\lambda\}$ be a family of fuzzy implicative ideals of X.
i. Let $x\in X$. Then $\bigcap_{\alpha\in\lambda} A(0) = \inf\{ A_\alpha(0)\ |\alpha\in\lambda\})\ \geq \inf\{ A_\alpha(x)\ |\alpha\in\lambda\} = \bigcap_{\alpha\in\lambda} A_\alpha(x)$

[Since A_α is a fuzzy implicative ideal of X, $\forall\alpha\in\lambda$. By definition (2.8)(i)]. Then $\bigcap_{\alpha\in\lambda} A_\alpha(0)\ \geq \bigcap_{\alpha\in\lambda} A_\alpha(x)$

ii. Let $x,y,z\in X$. Then,we have $\bigcap_{\alpha\in\lambda} A_\alpha(x) = \inf\{A_\alpha(x)\ |\ \alpha\in\lambda\} \geq \inf\{\min\{A_\alpha((x*(y*x))*z),\ A_\alpha(z)\ |\ \alpha\in\lambda\}\}$
[Since A_α is a fuzzy implicative ideal of X,$\forall\alpha\in\lambda$. By definition (2.8)(ii)]]
$= \min\{\inf\{A_\alpha((x*(y*x))*z), A_\alpha(z)\ |\ \alpha\in\lambda\}\}$
$= \min\{\inf\{A((x*(y*x))*z)\ |\ \alpha\in\lambda\}, \inf\{A_\alpha(z)\ \ |\ \alpha\in\lambda\}\}$
$= \min\{ \bigcap_{\alpha\in\lambda} A((x*(y*x))*z)\ |\ \ \in\lambda\}, \bigcap_{\alpha\in\lambda} A_\alpha(z)\ |\ \ \alpha\in\lambda\}\}$

$\Rightarrow \bigcap_{\alpha\in\lambda} A_\alpha(x) \geq \min\{ \bigcap_{\alpha\in\lambda} A_\alpha((x*(y*x))*z)\ |\ \alpha\in\lambda\}, \bigcap_{\alpha\in\lambda} A_\alpha(z)\ |\ \ \in\lambda\}\}$

Therefore, $\bigcap_{\alpha\in\lambda} A_\alpha$ is a fuzzy implicative ideal of X. ■

Theorem (6.12)
Let $\{A_\alpha\ |\alpha\in\lambda\}$ be a family of fuzzy ideals of a BH-algebra X satisfies (b_4). Then $\bigcap_{\alpha\in\lambda} A_\alpha$ is a fuzzy implicative ideal of X.
Proof: Directly from proposition (6.10) and theorem (3.29)]. ■

Proposition (6.13) [3]
Let $\{A_\alpha|\alpha\in\lambda\}$ be a chain of fuzzy ideals of a BH-algebra X. Then $\bigcup_{\alpha\in\lambda} A_\alpha$ is a fuzzy ideal of X.

Proposition (6.14)
Let $\{A_\alpha\ |\alpha\in\lambda\}$ be a chain of fuzzy implicative ideals of a BH-algebra X. Then $\bigcup_{\alpha\in\lambda} A_\alpha$ is a fuzzy implicative ideal of X.

Proof: Let $\{A_\alpha\ |\alpha\in\lambda\}$ be a chain of fuzzy implicative ideal of X.
i. Let $x\in X$. Then $\bigcup_{\alpha\in\lambda} A_\alpha(0) = \sup\{ A_\alpha(0)\ |\alpha\in\lambda\})\ \geq\ \sup\{ A_\alpha(x)\ |\alpha\in\lambda\} = \bigcup_{\alpha\in\lambda} A_\alpha(x)$
[Since A_α is a fuzzy implicative ideal of X, $\forall\alpha\in\lambda$. By definition (2.8)(i)]
$\Rightarrow \bigcup_{\alpha\in\lambda} A(0)\ \ \geq \bigcup_{\alpha\in\lambda} A_\alpha(x)$.
ii. Let x, y, z $\in$X. Then, we have

$$\bigcup_{\alpha\in\lambda} A_\alpha(x) = \sup\{A_\alpha(x)\ |\alpha\in\lambda\} \geq\ \sup\{ \min\{A_\alpha((x*(y*x))*z),A_\alpha(z)\ |\ \ \alpha\in\lambda\}\}$$

[Since A_α is a fuzzy implicative ideal of X,$\forall\alpha\in\lambda$. By definition (2.8)(ii)]
= min {sup {A_α ((x*(y*x))*z),A_α(z) | $\alpha\in\lambda$}} [Since A_α is a chain, $\alpha\in\lambda$]
= min {sup {A_α ((x*(y* x))*z) | $\alpha\in\lambda$}, sup{A_α(z) | $\alpha\in\lambda$}}

$$= \min \{ \bigcup_{\alpha\in\lambda} A((x*(y*x))*z) \mid \alpha\in\lambda\}, \bigcup_{i\in\Gamma} A(z) \mid \in\lambda\}\}$$

$$\Rightarrow \bigcup_{\alpha\in\lambda} A_\alpha (x) \geq \min \{ \bigcup_{\alpha\in\lambda} A((x*(y*x))*z) ; \alpha\in\lambda\}, \bigcup_{\alpha\in\lambda} A_\alpha(z); \alpha\in\lambda\}\}$$

Therefore, $\bigcup_{\alpha\in\lambda} A_\alpha$ is a fuzzy implicative ideal of X. ■

Theorem (6.15)
Let {A_α |$\alpha\in\lambda$} be a chain of fuzzy ideals of a BH-algebra X satisfies (b_4). Then $\bigcup_{\alpha\in\lambda} A_\alpha$ is a fuzzy implicative ideal of X.
Proof: Directly from proposition (6.13) and theorem (3.29).■

Remark (6.16) [14]
Let X and Y be BH-algebras. A mapping f: X→Y is called a **homomorphism** if f(x*y) = f(x)*f(y), ∀ x, y∈X. A homomorphism f is called a **monomorphism** (resp., **epimorphism**), if it is injective (resp**.,** surjective). A bijective homomorphism is called an isomorphism. Two BH-algebras X and Y are said to be **isomorphic**, written X ≅ Y, if there exists an isomorphism f: X→Y. For any homomorphism f: X →Y, the set {x∈X:f(x)=0'} is called the **kernel** of f, denoted by ker(f), and the set {f(x):x∈X} is called the **image** of f, denoted by Im(f). Notice that f(0)=0', ∀ homomorphism f.

Definition (6.17) [2]
Let X and Y be any two sets, A be any fuzzy set in X and f: X → Y be any function. The set $f^{-1}(y)$ = {x∈X | f(x) = y}, ∀ y ∈Y. The fuzzy set B in Y defined by B(y) $=\begin{cases}\sup\{A(x) \mid x\in f^{-1}(y)\}; & \text{if } f^{-1}(y)\neq\emptyset \\ 0; & \text{otherwise}\end{cases}$, ∀ y ∈Y, is called the **image** of A under f and is denoted by f(A).
Definition (6.18) [2]
Let X and Y be any two sets, f: X → Y be any function and B be any fuzzy set in f(A). The fuzzy set A in X defined by: A(x)=B(f(x)), ∀x∈X is called the **preimage** of B under f and is denoted by f^{-1}(B).
Proposition(6.19) [2]
Let f: (X,*, 0)→(Y,*',0') be a BH-epimorphism. If A is a fuzzy ideal of X, then f(A) is a fuzzy ideal of Y.

Proposition (6.20)
Let f: (X,*, 0) → (Y,*', 0') be a BH-epimorphism. If A is a fuzzy implicative ideal of X, then f(A) is a fuzzy implicative ideal of Y.
Proof: Let A be a fuzzy implicative ideal of X. Then
i. Let y ∈Y. Then there exists x ∈X.
(f(A))(0')=sup {A(x_1) | x_1∈ f^{-1}(0')}=A(0) ≥ sup{A(x) | x ∈X}≥ sup {A(x_1) | x = f^{-1}(y)}=(f(A))(y)
[Since A is a fuzzy implicative ideal of X. By definition (2.8)(i)]]
⇒ (f(A))(0') ≥(f(A))(y), ∀ y ∈Y. By proposition(6.19) and byproposition(2.5)]
ii. Let y_1, y_2, y_3 ∈ Y. Then there exist f(x_1)=y_1, f(x_2)=y_2, f(z)=$y_3$such that x_1, x_2, z ∈X
⇒(f(A))(y_1) = sup {A(x_1) | x ∈ f^{-1}(y_1)}
≥ sup{ A(((x_1*(x_2 ∗ x_1)**)***z),A(z)|(x_1*(x_2 ∗ x_1)**)***z) ∈f^{-1}((y_1*'(y_2*'y_1))*'y_3)} and z ∈ f^{-1}(y_3)}}
[Since A is a fuzzy implicative ideal of X. By definition(2.8)(ii)]]
≥ min { sup{A(((x_1*(x_2 ∗ x_1)**)***z)|(x_1*(x_2 ∗ x_1)**)***z)∈
f^{-1} ((y_1*'(y_2*'y_1))*'y_3)*'t))}} , sup {A(z) | x ∈ f^{-1}(y_3)}}
= min {((f(A))(f((x_1*(x_2 ∗ x_1)**)***z)**,** (f(A))(f(z))}
= min{((f(A))(f((x_1)*(f(x_2)*'f((x_1)**))***'f(z)))**,**(f(A))(f(z))} [Since f is an epimorphism. By remark (6.16)]
= min {(f(A))((y_1*'(y_2*' y_1))*' y_3), (f(A))(y_3)}
⇒(f(A))(y_1)≥ min{(f(A))((y_1*'(y_2*'y_1))*'y_3),(f(A))(y_3)}.
Therefore, f(A) is a fuzzy implicative ideal of Y.■

Proposition(6.21) [2]
Let f:(X,*,0)→(Y,*',0') be BH-homomorphism. If B be a fuzzy ideal of Y, then $f^{-1}(B)$ is a fuzzy ideal of X.

Proposition (6.22)
Let f: (X, *, 0) → (Y, *', 0') be a BH-homomorphism. If B be a fuzzy implicative ideal of Y, then $f^{-1}(B)$ is a fuzzy implicative ideal of X.

Proof: Let B be a fuzzy implicative ideal of Y. To prove $f^{-1}(B)$ is a fuzzy implicative ideal of X.

i. Let x ∈ X. Then $(f^{-1}(B))(0)= B(f(0))= B(0') \geq B(f(x)) =(f^{-1}(B))(x)$
[Since B is a fuzzy implicative ideal of Y. By definition (2.8)(i)]]
$\Rightarrow (f^{-1}(B))(0) \geq (f^{-1}(B))(x), \forall x \in X$. . By proposition(6.21) and byproposition(2.5)]

ii. Let x , y , z ∈X. Then $(f^{-1}(B))(x)= B(f(x))$
$\geq \min \{B((f (x)*'(f(y)*'f(x)))*'f(z)), B(f (z))\}$ [By definition(2.8)(ii)]
$= \min \{B(f (((x*(y*x))*z)),B(f (z))\}$ [Since f is a homomorphism]
$= \min \{(f^{-1}(B))(((x*(y*x))*z)), f^{-1}(B)(z)\}$. [Since $(f^{-1}(B))(x)= B(f(x))$]
$\Rightarrow (f^{-1}(B))(x) \geq \min \{(f^{-1}(B))(((x*(y*x))*z)) , (f^{-1}(B))(z)\}$.
Thus $f^{-1}(B)$ is a fuzzy implicative ideal of X. ■

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