IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 5 Ver. I (Sep. - Oct.2016), PP 34-37
www.iosrjournals.org

Completeness

Dr.Pinheiro[1]

Abstract: *In this paper, we investigate the concept of completeness. We studied the concept whilst still attending college, but that wasfrom a mathematical perspective. In 2000, we got to have contact with The Logicians' understanding of the concept through the hands of one of the most important modern icons of Philosophy, Dr. Graham Priest. We recently mentioned his ways of applying the concept, and that was in our last paper with the APM journal. There seems to be a bit of discrepancy. Because of that, it is worth studying the subtleties involved. It seems that reserved words should not be recreated in meaning, so that if there is any chance The Logicians' completeness does not coincide with The Mathematicians' completeness, the sense that last appeared should be dropped in favour of coherence.*
Keywords: *Logic, priest, completeness, mathematics, gödel*

I. Introduction

> *The completeness question for the first order predicate calculus was stated precisely and in print for the firsttime in 1928 by Hilbert and Ackermann in their text Grundzüge der theoretischenLogik [3], a text with whichGödel would have been quite familiar. [6]*
> *The question Hilbert and Ackermann pose is whether a certain explicitly given axiom system for the first orderpredicate calculus "...is complete in the sense that from it all logical formulas that are correct for each domain of individuals can be derived..." ([4], p. 48).*
>
> (Pinheiro, 2016)

This is the mathematical definition we have learned at school: The axiomatic system is complete if we can derive all formulas that are correct for each allowed domain from it.
Basically, mathematicians would have a lot of difficulties because their minds would go: Give me a formula that is correct and I will then prove that it can be derived from our axiomatic system.
The person who listens to them would then say: Here is formula X.
They would then say: Here is the proof that it can be derived from our axiomatic system.
Now, how can we be sure that any other randomly chosen formula that is correct could be derived from the same system?
How can we know how many formulas are correct without writing them down?

If we consider a really small set of Classical Logic symbols, so say $=>$, Λ, and $\mathbf{V}$, and a few letters, so say A, B, and C, we already have, considering a special arrangement:

Choice of the first element: 3 (letters)
Choice of the second element: 3 (symbols)
Choice of the third element: 3 (letters)

3^3 or 27 possible formulas. We must take away from the counting the situation where we have the same letter before and after the symbol, so that we must take away three for each letter and each symbol, what gives us 9 situations to take away. We must also take away the situation involving .and. or .or. and permutation, so that we have combination of three by twos, 3, to take away here for each one of them, therefore 6 to take away. In total, we must take away 15 cases, what still gives us 12 possible cases.
As we add complexity to the logical formula, figures grow quite a lot: If we simply add an implication before the previously considered formula and a letter, with all we had before being put inside of brackets, we get three choices for every antecedent and therefore 3x12=36 possible cases.
The mathematician would then go crazy: How can I possibly check on all correct formulas?
Priest and his fellows seem to have come up with a solution to all the mathematicians' nightmares in what regards this topic: You simply depart from a generic couple of letters organized in a way to have a symbol in the middle and you then assume that the formula is correct. After that, you see if it is possible going back to the

[1]IICSE University. drmarciapinheiro@gmail.com

value of the letter via allowed inferential rules (allowed in that particular logical system we study). You would then have to just rewrite things in the proper order to show that it is possible to derive that particular formula from the axiomatic system.

In the example we have just provided, our efforts would resume to checking three formulas (A => B, A ∧B, and AvB).

Wow! That is miraculous, right?
We went from 12 to 3 in a single life: Oh, gosh!
So, is this the miracle of all times or there is something wrong going on there?
That was the question in my mind in that 2000 as I watched Priest skilfully running his fingers through the board and doing his weaving: He would go upwards from the lowest levels of the thing he drew, which was like a tree. Trees I was used to: That came from Systems Analysis.
The resulting unit, the formula, let's say, was at the bottom.
The main question that emerges in our mind is: Can we really cover all formulas like that, like all formulas that could be correct and made out of the allowed symbols for this system?
In this paper, we shall endeavour to try to answer such a question.

II. Development

We observe that in (Pinheiro, 2016) we proved that there is no counter-example to the claim *Arithmetic is complete*. We actually only have circumstantial evidence on the contrary.
To prove that a system is incomplete, it suffices that we find a counter-example to the claim that it is complete, so that it suffices that we find one formula that cannot be derived from the system we have at hand, basically.
To prove that a system is complete, however, we would have to prove that ALL allowed formulas that are true can be derived from the system we have.
Whilst logicians like Priest are using the sigmatoids (Pinheiro, 2015)*completeness* and *soundness* at waste and creating numerous logical systems from day to night, mathematicians would have the rights to feel at least suspicious: Is that something that we are allowed to do?
The book Priest showed us in that 2000 (Priest, 2001) brought the following:

1.11.5 COMPLETENESS LEMMA: Let b be an open complete branch of a tableau. Let ν be the interpretation induced by b. Then:

if A is on b, $\nu(A) = 1$

if $\neg A$ is on b, $\nu(A) = 0$

1.11.6 COMPLETENESS THEOREM: For finite Σ, if $\Sigma \models A$ then $\Sigma \vdash A$.

1.3.3 Let Σ be any set of formulas (the premises); then A (the conclusion) is a *semantic consequence* of Σ ($\Sigma \models A$) iff there is no interpretation that makes all the members of Σ true and A false, that is, every interpretation that makes all the members of Σ true makes A true. '$\Sigma \not\models A$' means that it is not the case that $\Sigma \models A$.

1.3.4 A is a *logical truth* (*tautology*) ($\models A$) iff it is a semantic consequence of the empty set of premises ($\phi \models A$), that is, every interpretation makes A true.

The detail that most matters here is that completeness, for Dr. Priest, means that whatever makes the set of premises to the left side true also makes the set of premises to the right true, but, in Mathematics, in what comes to completeness, we would need to guarantee is the opposite direction: For every premise that is true to the right, it is possible to find a set of premises that are true from which we can derive them to the left. The concepts are therefore incompatible. The fact that the set of facts X makes whatever is to the left and to the right true does not mean that it is not possible to find another true assertion to the right that does not find derivation on whatever is available to the left. Therefore they have not solved Gödel's main problem. This is at most a proof of coherence of some sort and, more than likely, could not be called completeness, since that is a reserved name, basically.

1.1.5 It is also standard to define two notions of validity. The first is *semantic*. A valid inference is one that *preserves truth*, in a certain sense. Specifically, every interpretation (that is, crudely, a way of assigning truth values) that makes all the premises true makes the conclusion true. We use the metalinguistic symbol '$\models$' for this. What distinguishes different logics is the different notions of interpretation they employ.

(Priest, 2001)

Semantic validity appeared because people like Dr. Corcoran (Corcoran, 1999) were looking for some relationship that matters between antecedent and consequent in Classical Logic: They wanted to have some connection in terms of meaning between one thing and another. The name *Semantic Validity* is ALSO equivocated therefore. There was an expectation and a search that mattered to those people: They were looking for an almost spiritual connection between left and right. If the definition of Semantic Validity is the one we just presented here, and this is the definition Dr. Priest uses, we cannot guarantee any semantic attachment because we can, for instance, have a tautology to the right side and therefore any interpretation that makes the left side true will make the right side also true. That does not imply any semantic connection whatsoever.

1.1.6 The second notion of validity is *proof-theoretic*. Validity is defined in terms of some purely formal procedure (that is, one that makes reference only to the symbols of the inference). We use the metalinguistic symbol '$\vdash$' for this notion of validity. In our case, this procedure will (mainly) be one employing tableaux. What distinguish different logics here are the different tableau procedures employed.

(Priest, 2001)

Here we are just talking about the rules of the system and mathematicians would therefore not have any problems with this.

The Completeness Theorem here exhibited states that, for a finite set of premises, *whenever thoseand the right side are true* (observe that they use *if..then*, not the CL symbol)*there is a proof of whatever appears to the right side that can be made out of these premises*.

In this case, say we have A -> B, C to the left side and C to the right.

Assuming that A->B is true in CL is saying that it is never the case that v(A)=1 and v(B)=0. Assuming that C is true in CL is saying that v(C)=1. With this, we obviously get the right side to have value 1 or to be true, what would then tell us that there is a deduction originating in the left side that leads to the right side.

On the other hand, assume that D is also true and we have no other premises to count on to the left side, so say D is a tautology, like regardless of the values we use from this system, D is always true. That will obviously mean that D is true for every interpretation of sigma. Assume that A, B, C, and D have nothing in common. Now, we do not have completeness in the mathematical sense in this system, which involves only A->B, and C, but we do have completeness in the logical sense according to the definition we hold.

What we are actually checking here is if whenever the truth of a set of premises does not lead to the falsity of another we are sure to have a proof of the latter inside of that set of premises, all being just a matter of organizing things properly. We are then only worried about the case in which we have 1 -> 1, what is odd. In this case, we will guarantee that there is a proof of whatever comes to the right. 0 -> 1 would also be OK, but we are not guaranteeing this case.

Ex-Falso is excluded because if we have that both A->B and ~(A->B) are true, that is not acceptable in CL. They cannot both be true at the same time. Notwithstanding, if we had that to the left side and assumed it was all true we would have explosion and therefore ALSO whatever is in the right side. The truth of whatever is to the left is still leading to the truth of whatever is to the right. Yet, we should not have a proof of whatever is to the right in this way because, first of all, we cannot have both being true at the same time. Otherwise anything and

everything we say would be provable in the system: It suffices getting the thing we want to prove and its negation.
As another point, Priest talks about many-valued logics, what would mean that it would be possible to have both cases validated with no conflict if the logical system chosen were Special K, let's say: v(A->B)=1 and v(~(A->B))=1. The truth of those could therefore imply the truth of the right side, so say it is A->B. Now, how do we actually prove the right side?
A proof could imply the application of more than one rule, not just one, as a start of conversation. Is that a proof of A->B or just a fact?
Would we have to consider their definition of completeness inside of each logical system, according to their rules? We get the impression that they use CL to interpret their axiom in what regards completeness instead.
In the same sense, if all is implied, we don't seem to have a special proof of whatever is to the right side. It is only the application of a rule.
Any contradiction would be a proof in this case. We can create never ending ones by simply swapping one element.
This Completeness Theorem is either incomplete or equivocated.
What Priest does with his hands on the board, however, could be the actual thing: He comes from the basic formula, the most basic units of all, and he goes back to the root of those formulas, to the most elementary parts, so say A->B. We assume this premise is true, and therefore that v(A->B)=1 if we are in CL.
For that to be true, either B is true or A is false. If our universe is CL, provided we can evaluate all As and Bsas true or false, we are OK.
How do we evaluate a contradiction if we find it in place of A, however? Is that true or false? We would have to say it is impossible if we talk about CL, and therefore we are stuck with being unable to provide a truth-value to it.
When he moves his hands therefore, in the weaving thing, he is not proving completeness either, very unfortunately.
Oh, well, in this case, the most he could be doing is checking the syntax involved for coherence, we assume.
Proving completeness, already said Gödel somehow, is something really difficult to do, so that logicians should refrain from using this mathematical sigmatoid from now onwards, we reckon.

III. Conclusion

The inadequate use of the mathematical sigmatoid completeness led us to get very confused about the teachings of Priest in 2000. The reason is quite obvious: Logicians ARE NOT proving completeness.
In that being a drama for mathematicians for almost one century (Pinheiro, 2016), it would be unlikely that the logicians had solved all in so simple moves, but, even so, given the amount of work they have produced, those from nonclassical logic, we must think of the possibility and therefore take it all very seriously.
Logic, as much as Mathematics, should never be a joke.
The sigmatoid should definitely be withdrawn from the logical jargon until more adequate theorems and lemmas are found, in case that ever happens.
What they meant, more than likely, was syntactic coherence at most, we think.
Truth be said, if they had really found ways to prove CL completeness, Gödel's incompleteness theorems would not be a problem anymore, so that we had plenty of material evidence to count on when we decided to challenge their statements.
When we go from words to symbols, things become quite dangerous, as said by us many times. The opposite move is much more guaranteed.

References

[1]. Pinheiro, M. R. (2015). Words for Science. *Indian Journal of Applied Research*, *5*(5), 19–22. Retrieved from https://www.worldwidejournals.com/ijar/articles.php?val=NjQ0MQ==&b1=853&k=214
[2]. Pinheiro, M. R. (2016). Gödel and the Incompleteness of Arithmetic. *Advances in Pure Mathematics*, *6*(8), 9. Retrieved from http://file.scirp.org/Html/4-5301134_68410.htm
[3]. Priest, G. (2001). *An Introduction to Non-Classical Logic*. Cambridge University Press.
[4]. Corcoran, J. (1999). Information-Theoretic Logic and Transformation-Theoretic Logic. *Fragments of Science*, 25–36. Retrieved from http://philpapers.org/archive/CORILA.pdf