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On Designs arising from Corona Product $H \circ K_3$

[1]Sumathi M. P and [2]Anwar Alwardi and
[1]*Department of Mathematics, Mahajana First Grade College, Mysore 570006, India*
[2] *University of Aden, Aden, Yemen*

Abstract: *In this paper, we determine the partially balanced incomplete block designs and association scheme which are formed by the minimum dominating sets of the graphs $C_3 \circ K_3$, we determine the number of minimum dominating sets of graph $G = C_n \circ K_3$ and prove that the set of all mini- mum dominating sets of $G = C_n \circ K_3$ forms a partially balanced incomplete block design with two association scheme. Finally we generalize the results for the graph $H \circ K_3$.*

Key Words: *Minimum dominating sets, association schemes, PBIB designs.*
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I. Introduction

By a graph, we mean a finite undirected graph without loops or multiple lines. For a graph $G = (V, E)$, let V and E respectively denote the vertex set and the edge set of graph G. For any vertex $u \in V$, $N(u) = \{v \in V : uv \in E\}$ is called the open neighbourhood of u in V, and the closed neighbourhood of u in G is $N[u] = N(v) \cup \{u\}$. The degree of u in G, $deg(u) = |N(u)|$. The open Neighborhood of a set of vertices S in G is
$N(S) = \cup_{v \varepsilon S} N(v)$ and the closed neighbourhood of the set S is
$N[S] = N(S) \cup S$. A subset $D \subseteq V$ is called dominating set of $G = (V, E)$ if
$N[D] = V$. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set.
A dominating set D is called minimal dominating set if no proper subset $S \subset D$
is a dominating set.
The PBIBD with m-association scheme which are arising from dominating sets has been studied extensively by many for example see [8],[1]. In this paper, We study the PBIBD and the association scheme which can be obtained from the minimum dominating sets in $(C_n \circ K_3)$ graph. Finally we generalize the results for the graph $H \circ K_3$.

II. PBIBD arising from minimum dominating sets of $(C_n \circ K_3)$

Definition1. Given v objects a relation satisfying the following conditions is said to be an association scheme with m classes:
(i) Any two objects are either first associates, or second associates…., or mth associates, the relation of association being symmetric.
(ii) Each object α has n_i ith associates, the number n_i being independent of α.
(iii) If two objects α and β are ith associates, then the number of objects which are jth associates of α and kth associates of β is p^i_{jk}

and is independent of the
pair o f ith associates α and β. Also $p^i_{jk} = p^i_{kj}$

If we have association scheme for the v objects we can define a PBIBD as the following definition.

Definition2. The PBIBD design is arrangement of v objects into b sets (called blocks) of size k where $k < v$ such that
(i) Every object is contained in exactly r blocks.
(ii) Each block contains k distinct objects.
(iii) Any two objects which are ith associates occur together in exactly λ_i blocks.

Theorem3. ***From ($C_3 \circ K_3$) we can get PBIBD with parameters ($v = 12, k = 3, r = 16, b = 64, \lambda_1 = 0, \lambda_2 = 4$) and association scheme of 2-classes***

With $P_1 = \begin{bmatrix} p^1_{11} & p^1_{12} \\ p^1_{21} & p^1_{22} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$ ***and*** $P_2 = \begin{bmatrix} p^2_{11} & p^2_{12} \\ p^2_{21} & p^2_{22} \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 3 & 4 \end{bmatrix}$.

Proof. let $G = (V, E)$ be a corona graph $C_3 \circ K_3$.

By labelling $\{v_1, v_2, v_3, v_1', v_2', v_3', v_1'', v_2'', v_3'', v_1''', v_2''', v_3'''\}$ as in Figure.1, we can Define PBIBD as follows:

The point set is the vertices and the block set is the minimum dominating sets $\{v_1, v_2, v_3\}$, $\{v_1', v_2', v_3'\}$, $\{v_1'', v_2'', v_3''\}$, $\{v_1''', v_2''', v_3'''\}$, $\{v_1, v_2, v_3'\}$, $\{v_1, v_2, v_3''\}$, $\{v_1, v_2, v_3'''\}$, $\{v_1', v_2, v_3\}$, $\{v_1'', v_2, v_3\}$, $\{v_1''', v_2, v_3\}$, $\{v_1', v_2, v_3'''\}$, $\{v_1''', v_2', v_3\}$, $\{v_1, v_2''', v_3'\}$, $\{v_1, v_2', v_3\}$, $\{v_1, v_2'', v_3\}$, $\{v_1, v_2''', v_3\}$, $\{v_1', v_2', v_3\}$, $\{v_1', v_2', v_3''\}$,

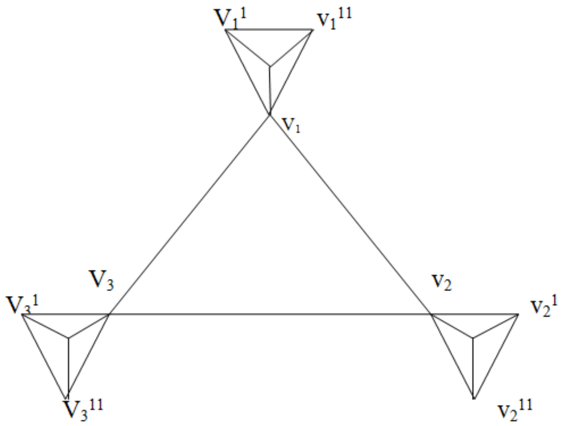

Figure 1: $C_3 \circ K_3$

$\{v_1', v_2', v_3'''\}$, $\{v_1, v_2', v_3'\}$, $\{v_1'', v_2', v_3'\}$, $\{v_1''', v_2', v_3'\}$, $\{v_1', v_2, v_3'\}$, $\{v_1', v_2'', v_3'\}$,

$\{v_1', v_2'', v_3'\}$, $\{v_1'', v_2'', v_3\}$, $\{v_1'', v_2'', v_3'\}$, $\{v_1'', v_2'', v_3'''\}$, $\{v_1, v_2'', v_3''\}$, $\{v_1', v_2'', v_3''\}$,

$\{v_1''', v_2'', v_3''\}$, $\{v_1'', v_2, v_3''\}$, $\{v_1'', v_2', v_3''\}$, $\{v_1'', v_2''', v_3''\}$, $\{v_1''', v_2''', v_3\}$, $\{v_1''', v_2''', v_3'\}$,

$\{v_1''', v_2''', v_3''\}$, $\{v_1, v_2''', v_3'''\}$, $\{v_1', v_2''', v_3'''\}$, $\{v_1''', v_2, v_3'''\}$, $\{v_1''', v_2', v_3'''\}$, $\{v_1''', v_2'', v_3'''\}$,

$\{v_1, v_2', v_3''\}$, $\{v_1, v_2', v_3'''\}$, $\{v_1', v_2, v_3''\}$, $\{v_1', v_2, v_3'''\}$, $\{v_1'', v_2, v_3'\}$, $\{v_1'', v_2, v_3'''\}$,

$\{v_1'', v_2''', v_3'''\}$, $\{v_1'', v_2, v_3'\}$, $\{v_1''', v_2, v_3'\}$, $\{v_1'', v_2', v_3\}$, $\{v_1'', v_2', v_3\}$, $\{v_1', v_2'', v_3\}$,

$\{v_1''', v_2'', v_3\}$, $\{v_1''', v_2'', v_3'''\}$, $\{v_1', v_2'', v_3\}$, $\{v_1', v_2''', v_3\}$, $\{v_1, v_2'', v_3'\}$, $\{v_1, v_2'', v_3'\}$,

$\{v_1, v_2', v_3''\}$, $\{v_1, v_2''', v_3''\}$ and $\{v_1''', v_2''', v_3''\}$.

We define the association scheme as follows, for any $\alpha, \beta \in V(G)$, α is first associate of β if α and β appear in zero or three minimum dominating sets and α is second associate of β otherwise, see Table 1.

Elements	First Associates	Second Associates
v_1	$v_1^{1}, v_1^{11}, v_1^{111}$	$v_2, v_2^{1}, v_2^{11}, v_2^{111}, v_3, v_3^{1}, v_3^{11}, v_3^{111}$
v_1^{1}	v_1, v_1^{11}, v_1^{111}	$v_2, v_2^{1}, v_2^{11}, v_2^{111}, v_3, v_3^{1}, v_3^{11}, v_3^{111}$
v_1^{11}	v_1, v_1^{1}, v_1^{111}	$v_2, v_2^{1}, v_2^{11}, v_2^{111}, v_3, v_3^{1}, v_3^{11}, v_3^{111}$
v_1^{111}	v_1, v_1^{1}, v_1^{11}	$v_2, v_2^{1}, v_2^{11}, v_2^{111}, v_3, v_3^{1}, v_3^{11}, v_3^{111}$
v_2	$v_2^{1}, v_2^{11}, v_2^{111}$	$v_1, v_1^{1}, v_1^{11}, v_1^{111}, v_3, v_3^{1}, v_3^{11}, v_3^{111}$
v_2^{1}	v_2, v_2^{11}, v_2^{111}	$v_1, v_1^{1}, v_1^{11}, v_1^{111}, v_3, v_3^{1}, v_3^{11}, v_3^{111}$
v_2^{11}	v_2, v_2^{1}, v_2^{111}	$v_1, v_1^{1}, v_1^{11}, v_1^{111}, v_3, v_3^{1}, v_3^{11}, v_3^{111}$
v_2^{111}	v_2, v_2^{1}, v_2^{11}	$v_1, v_1^{1}, v_1^{11}, v_1^{111}, v_3, v_3^{1}, v_3^{11}, v_3^{111}$
v_3	$v_3^{1}, v_3^{11}, v_3^{111}$	$v_1, v_1^{1}, v_1^{11}, v_1^{111}, v_2, v_2^{1}, v_2^{11}, v_2^{111}$
v_3^{1}	v_3, v_3^{11}, v_3^{111}	$v_1, v_1^{1}, v_1^{11}, v_1^{111}, v_2, v_2^{1}, v_2^{11}, v_2^{111}$
v_3^{11}	v_3, v_3^{1}, v_3^{111}	$v_1, v_1^{1}, v_1^{11}, v_1^{111}, v_2, v_2^{1}, v_2^{11}, v_2^{111}$
v_3^{111}	v_3, v_3^{1}, v_3^{11}	$v_1, v_1^{1}, v_1^{11}, v_1^{111}, v_2, v_2^{1}, v_2^{11}, v_2^{111}$

Table 1:

Theorem 4. Let G ∼= (Cn ◦ K3). Then the number of minimum dominating sets of G is 4^n.

Proof. Let G ∼= (Cn ◦ K3). Then γ(G) = n. We need to find out all the sets of size n. For this, we have many possibilities :

case1. All the vertices of the minimum dominating set are from inside that is from Cn. Then there is only one minimum dominating set.

case2. The vertices of minimum dominating set i.e, not from the vertices of Cn.
The number of ways to select minimum dominating sets of size n from outside is 3n.

case3. We select some vertices of minimum dominating sets from inside and some from outside. So we start by selecting one vertex from inside and (n − 1)

vertices from outside. There are $\binom{n}{1} 3^{n-1}$ ways. Similarly 2 vertex from inside

(n − 2) vertices from outside. There are $\binom{n}{2} 3^{n-1}$ ways. By continuing in same way till (n − 1) vertices from inside and one from outside, there are $\binom{n}{n-1}$ 3 ways.

Hence the total number of minimum dominating sets is

$$3^n + \binom{n}{1} 3^{n-1} + \binom{n}{2} 3^{n-2} + \ldots\ldots + \binom{n}{n-1} 3 + 1$$

$$= \sum_{i=0}^{n} \binom{n}{1} 3^{n-i}$$

$$= 4^n.$$

Theorem 5. Let $G \cong (C_n \circ K_3)$. Any two vertices in G either belong to zero minimum dominating set or 4^{n-2} minimum dominating sets.

Proof. By labeling the vertices of the graph G as $\{v_1, v_2, \ldots, v_n, v_1', v_1'', v_1''', v_2', v_2'', v_2''', \ldots v_n', v_n'', v_n'''\}$

Where $\{v_1, v_2, \ldots, v_n\}$ are the vertices of c_n and $\{v_1', v_1'', v_1''', v_2', v_2'', v_2''' \ldots v_n', v_n'', v_n'''\}$ are the vertices of the copies k_3.

Suppose $A = \{v_1, v_2, \ldots, v_n\}$ and $B = \{v_1', v_1'', v_1''', v_2', v_2'', v_2''', \ldots v_n', v_n'', v_n'''\}$.

Let u, v be any two vertices, we have the following cases:

Case1. u and v belong to A then there are 4^{n-2} minimum dominating sets containing u and v.

Case2. u and v belong to B then there are 4^{n-2}ways to select minimum dominating sets containing u and v.

Case3. Let $u \in A$ and $v \in B$ we have two subcases:

Case(i). Let u and v in the same triangle then there does not exists any Minimum dominating sets containing u and v.

Case(ii). If u and v are from the different triangle then there are 4^{n-2} ways to select minimum dominating sets.

Theorem6. Let $G \cong (C_n \circ K_3)$. Then every vertex $v \in V(G)$ contained in 4^{n-1} Minimum dominating sets.

Proof. Let $G \cong (C_n \circ K_3)$. The vertices of G can be partitioned into n sets, each set containing 3 vertex as the triangles $\Delta_1, \Delta_2 \ldots \Delta_n$. Let $v \in V(G)$ be any vertex such that $v \in \Delta_i$ for some $1 \le i \le n$. Any minimum dominating set containing v will contain $(n-1)$ vertices from the other triangle Δ_j where $i = j$. But it is not allowed to take two vertex from the same triangle so we need to take one vertex from each triangle.

Hence the ways to select $n-1$ vertices from the Δ_j triangles $i = j$ is 4^{n-1}.

Finally, we can generalize Theorem 6 as following.

Theorem7. For any graph $G \cong (H \circ K_3)$, there is PBIBD and association scheme associate with G as the following parameters,

($v = 4n$, $k = n$, $r = 4^{n-1}$, $b = 4^n$, $\lambda_1 = 0, \lambda_2 = 4^{n-2}$) and

$$P_1 = \begin{bmatrix} p_{11}^1 & p_{12}^1 \\ p_{21}^1 & p_{22}^1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 4(n-1) \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} p_{11}^2 & p_{12}^2 \\ p_{21}^2 & p_{22}^2 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 3 & 4(n-2) \end{bmatrix}.$$

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