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Global Triple Connected Domination Number of A Graph

G.Mahadevan[1], A.Selvam Avadayappan[2], Twinkle Johns[3]
[1]*Department of Mathematics, Gandhigram Rural University, Gandhigram*
[2,3]*Department of Mathematics, V.H.N.S.N. College, Virdhunagar*

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I. Introduction

Throughout this paper, we consider finite, simple connected and undirected graph G (V,E) V denotes its vertex set while E its edge set. The number of vertices in G is denoted by P. Degree of a vertex v is denoted by d(v), the maximum degree of a graph G, denoted by Δ. A graph G is connected if any two vertices of G are connected by a path. A maximal connected subgraph of a graph G is called a component of G. The number of components of G is denoted by $\omega(G)$. The complement $\bar{G}$ of G is the graph with vertex set V in which two vertices are adjacent if and only if they are not adjacent in $\bar{G}$. We denote a cycle on p vertices by C_p, a path on P vertices by P_p, complete graph on P vertices by K_p. The friendship graph F_n can be constructed by joining n copies of the cycle C_3 with a common vertex. A wheel graph can be constructed by connecting a single vertex to all the vertices of $C_{p-1.}$ A helm graph, denoted by H_n is a graph obtained from the wheel by W_n by attaching a pendent vertex to each vertex in the outer cycle of W_n. If S is a subset of V, then $\langle S\rangle$ denotes the vertex induced subgraph of G induced by S. The cartesian product $G_1 \times G_2$ of two graphs G_1 and G_2 was defined as the graph with vertex set $V_1 \times V_2$ and any two distinct vertices (u_1, v_1) and (u_2,v_2)are adjacent only if either u_1= u_2 and $v_1v_2 \in E_2$ or $u_1u_2 \in E_1$ and v_1= $v_{2.}$ If S is a subset of V, then $\langle S\rangle$ denotes the vertex induced subgraph of G is induced by S. The open neighbourhood of a set S of vertices of a graph G is denoted by N(S) is the set of all vertices adjacent to some vertex in S and N(S) $\cup S$ is called the closed neighbourhood of S, denoted by N[s]. The diameter of a connected graph G is the maximum distance between two vertices in G and is denoted by diam (G). A cut vertex of a graph G is a vertex whose removal increases the number of components. A vertex cut or separating set of a connected graph G is the set of vertices whose removal increases the number of components. The connectivity or vertex connectivity of a graph G is denoted by κ (G) (where G is not complete) is the size of a smallest vertex cut. A connected subgraph H of a connected graph G is called a H-cut if ω(G-H) $\geq$ **2.** The chromatic number of a graph G is denoted by $\chi(G)$ is the smallest number of colors needed to color all the vertices of a graph in which adjacent vertices receive different color. For any real number x, $[x]$ denotes the largest integer less than or equal to x. A subset S of vertices in a graph G = (V,E) is a dominating set if every vertex in V-S is adjacent to at least one vertex in S. A dominating set S of a connected graph G is called a connected dominating set if the induced sub graph < S > is connected. A set S is called a global dominating set of G if S is a dominating set of both G and $\overline{G}$. A subset S of vertices of a graph G is called a global connected dominating set if S is both a global dominating and a connected dominating set. The global connected domination number is the minimum cardinality of a global connected dominating set of G and is denoted by γ_{gc}(G). Many authors have introduced different types of domination parameters by imposing conditions on the dominating set. Recently, the concept of triple connected graphs has been introduced by Paul Raj Joseph J.et.al., By considering the existence of a path containing any three vertices of G. They have studied the properties of triple connected if any three vertices lie on a path in G. All paths, cycles, complete graphs and wheels are some standard examples of triple connected graphs. In this paper, we use this idea to develop the concept of connected dominating set and global triple connected domination number of a graph.

Theorem 1.1[2] A tree T is triple connected if and only if T$\cong$ P_P; P$\geq$3.

Theorem 1.2[2] A connected graph G is not triple connected if and only if there exist a H cut with ω(G-H)$\geq$ 3 such that $|V(H) \cap N(H \cap C_i)| = 1$ for atleast three components $C_{1,} C_{2,} C_{3,}$ of G-H.

Notation1.3 Let G be a connected graph with m vertices $v_{1,} v_{2,} \dots \dots . . v_{m.}$ The graph G(n_1Pl_1,n_2pl_2,n_3pl_3,…….n_mpl_m) where n_i, l_i≥0 and $1\leq i \leq m$, is obtained from G by pasting n_1 times a pendent vertex Pl_1 on the vertex v_1, n_2 times a pendent vertex of Pl_2 on the vertex v_2 and so on.

Example 1.4 Let v_1, v_2, v_3, v_4 be the vertex of C_4, the graph $C_4(2P_2,P_3,3P_2,P_2)$ is obtained from C_4 by pasting two times a pendent vertex of P_2 on v_1, 1 times a pendent vertex of P_3 on v_2,3 times a pendent vertex of P_2 on v_3 and I times a pendent vertex of P_2 on $v_{4.}$

II. Global Triple Connected Domination Number

Definition 2.1A subset S of V of a nontrivial graph G is said to be a global triple connected dominating set, if S is a global dominating set and the induced subgraph $\langle S \rangle$ is triple connected. The minimum cardinality taken over all triple connected dominating set is called the triple connected domination number of G and is denoted by $\gamma_{gtc}(G)$.

Example 2.2 For the graph K_5, S= $\{v_1,v_2,v_3,v_{4,}\ v_5\}$ forms a global triple connected dominating set

Observation 2.3 Global triple connected dominating set does not exist for all graphs and if exists, then $\gamma_{gtc}(G) \geq 3$ and $\gamma_{gtc}(G) \leq p$.

Example 2.4. For this Graph G_1 in figure 2.1, global triple connected dominating set does not exist in this graph

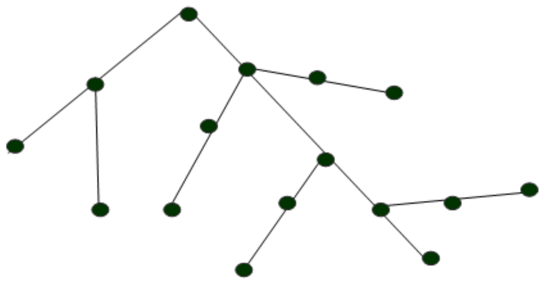


Fig 2.1

Observation 2.5 Every Global triple connected dominating set is a triple connected dominating set. But every triple connected dominating is not a global triple connected dominating set**.**

Observation 2.6 For any connected graph with G, $\gamma(G) \leq \gamma_c(G) \leq \gamma_{gc}(G) \leq \gamma_{gtc}(G) \leq p$ and the inequalities are strict.

Example 2.7 $\gamma(K_n) = 1$, $\gamma_c(K_n) = 2$, $\gamma_{gc}(K_n) = n$, $\gamma_{gtc}(K_n) = n$

Theorem 2.8 If the induced subgraph of all connected dominating set of G has more than two pendent vertices, then G does not contains a global triple connected dominating set.

Example 2.9 Triple connected domination number for some standard graphs

1. Let P be a petersen graph. Then $\gamma_{gtc}(P) = 5$.
2. For any complete graph G with P vertices $\gamma_{gtc}(G\ 0\ K_1) = P$.
3. For any path of order $P \geq 3$ $\gamma_{gtc}(P) = \begin{cases} 3\ if\ P < 5 \\ p-2\ if\ p \geq 5 \end{cases}$
4. For any cycle of length P for $P \geq 3$ $\gamma_{gtc}(C_p) = \begin{cases} 3\ \ if\ P < 5 \\ p-2\ if\ P \geq 5 \end{cases}$
5. For the complete bipartite graph $K_{m,n}$ $(m,n \geq 2)$ $\gamma_{gtc}(K_{m,n})=3$
6. For any star $K_{1,\ P-1}$ $(P \geq 3)$, $\gamma_{gtc}(K_{1,\ P-1}) = 3$
7. For any complete graph K_p, $P \geq 3$, $\gamma_{gtc}(K_P) = P$
8. For any wheel W_n , $\gamma_{gtc}(W_n) = 3$
9. For any graph Helm graph H_n (where p=2n-1, $p \geq 9$), $\gamma_{gtc}(H_n) = \frac{P+1}{2}$
10. For any friendship graph F_n, $\gamma_{gtc}(F_n) = 3$
11. The **Desargues graph** is a distance-transitive cubic graph with 20 vertices and 30 edges as shown in figure 2.2.

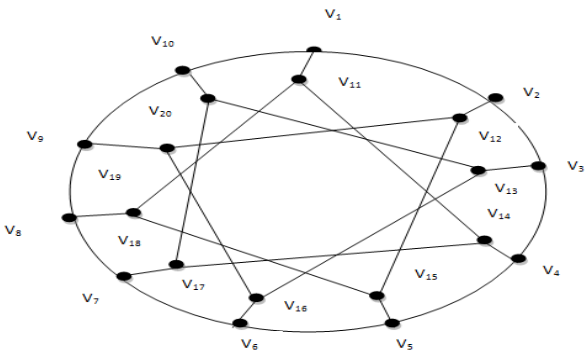


Fig 2.2

For any Desargues graph G, $\gamma_{gtc}(G) = 10$. Here S = {$v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}$} forms a global triple connected dominating set.

The **Möbius–Kantor graph** is a symmetric bipartite cubic graph with 16 vertices and 24 edges as shown in figure 2.3.

For the Mobius – Kantor graph G, $\gamma_{gtc}(G) = 8$. Here S = {$v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8$} is global triple connected dominating set.

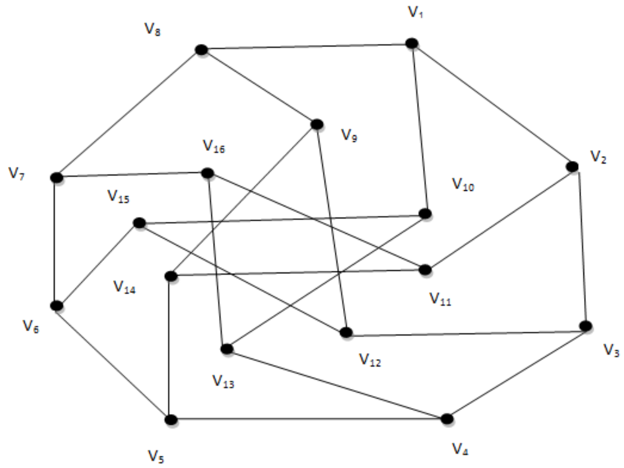


Fig 2.3

The **Chvátal graph** is an undirected graph with 12 vertices and 24 edges as shown in figure 2.4. For the Chvátal graph G, $\gamma_{gtc}(G) = 4$. Here S = {v_1, v_2, v_3, v_4} is global triple connected dominating set.

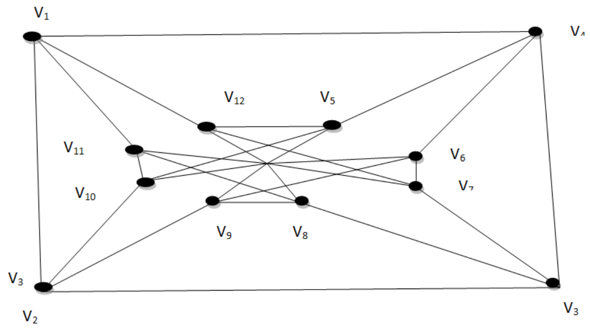


Fig 2.4

The **Dürer graph** is an undirected cubic graph with 12 vertices and 18 edges as shown below in figure 2.5.

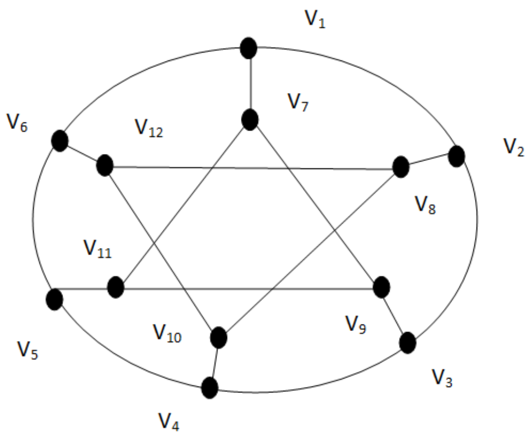


Fig 2.5

For the Dürer graph G, $\gamma_{gtc}(G) = 6$ Here S = {$v_1, v_2, v_3, v_4, v_5, v_6$} is an global triple connected dominating set.

Any path with a pendant edge attached at each vertex as shown in figure 2.6 is called **Hoffman tree** and is denoted by Pn^+ $\gamma_{gtc}(Pn^+)= n/2$

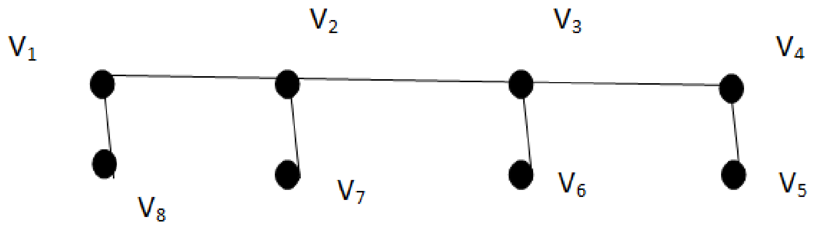


Fig 2.6

Observation 2.10 For any connected graph G with p vertices $\gamma_{gtc}(G) = p$ if and only if $G\cong P_3$ or C_3.
Observation 2.11 If a spanning subgraph H of a graph G has a global triple connected dominating set then G also has global triple connected dominating set.
Observation 2.12 For any connected graph G with p vertices $\gamma_{gtc}(G) = p$ if and only if $G\cong P_3$ or C_3.
Observation 2.13 If a spanning subgraph H of a graph G has a global triple connected dominating set then G also has global triple connected dominating set
Theorem 2.14 For any connected graph G with p≥4, we have $3\leq \gamma_{gtc}(G) \leq p$ and the bound is sharp.
Proof. The lower bound and upperbound follows from the definition 2.1 and observation 2.3. For C_4 the lowerbound is obtained and for the K_4 the upper bound is obtained.
Theorem 2.15 For any connected graph G with 4 vertices $\gamma_{gtc}(G)$ =P-1 if and only if $G \cong P_4$, C_4, $K_3(P_2)$ and K_4-{e},where e is any edge inside the cycle of K_4.
Proof Suppose G is isomorphic to P_4 , C_4, $K_3(P_2)$, K_4-{e},where e is any edge inside the cycle of K_4, then clearly $\gamma_{gtc}(G)$ =P-1. Conversely let G be a connected graph with 4 vertices and $\gamma_{gtc}(G)$= P-1. Let S = $\{v_1,v_2,v_3\}$ be a γ_{gtc} set of G. Let x be in V-S. Since S is a dominating set, there exist a vertex v_i from S such that v_i is adjacent to x. If P≥5, by taking the vertex v_i, we can construct a triple connected dominating set S with fewer elements than p-1, which is a contradiction. Hence P ≤ 4. Since $\gamma_{gtc}(C_p)$ = p-1 by using observation we have P=4 then S=$\{v_1,v_2,v_3\}$ and V-S= {x}, then $\gamma_{gtc}(G)$ = p-1.
Case (i) Let $\langle S\rangle = P_3 = v_1, v_2,v_3$
Since G is connected, x is adjacent to v_1 (or v_3) or x is adjacent to v_2. Suppose x is adjacent to v_1,then $\{x, v_1, v_2\}$ forms a $\gamma_{gtc}(G)$ set of G. Hence $G \cong P_4$ on increasing the degree $G\cong C_4$. If x is adjacent to v_2 then $\{x,v_2,v_1\}$ forms a γ_{gtc} set of G . Then $G \cong K_{1,3.}$ on increasing the degree $G\cong C_3(P_2)$
Case (ii) Let $\langle S\rangle =$ C_3 v_1,v_2,v_3,v_1. Since G is connected, x is adjacent to v_1(or v_2 or v_3). Hence $G \cong C_3(P_2)$ by increasing the degree $G \cong K_4$-e.
Theorem 2.16 For a connected graph G with 5 vertices, $\gamma_{gtc}(G)$= P-2 if and only if G is isomorphic to $G \cong P_{5,}$ C_5, $K_{2,3,}$ $C_4(P_2)$,$C_3(P_3)$, $K_3(2P_2)$, $C_3(P_2,P_2,0)$, $P_4(0,P_2,0,0)$, $C_3(2P_2)$, $K_{1,4,}$ K_3 (K_3), and G_1 to G_{10} in figure in 2.7.

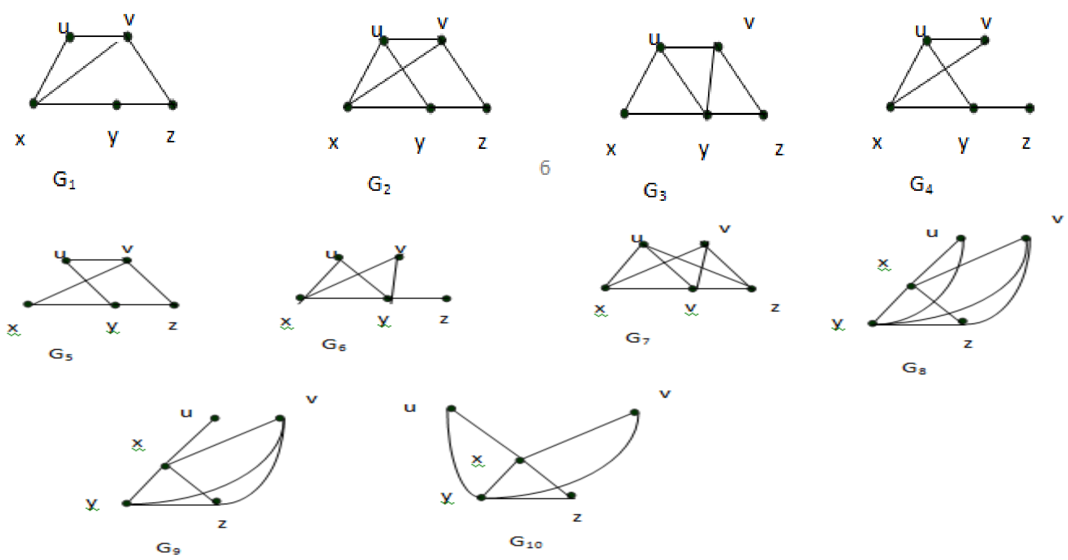


Fig 2.7

Proof Let G be a connected graph with 5 vertices and $\gamma_{gtc}(G)$ =3. Let S ={x,y,z} be a γ_{gtc} set of G ,then clearly $\langle S\rangle$ =P_3 or C_3. Let V-S= V(G) -V (S)= {u,v} then V- S=K_2, $\overline{K_2}$.

Case1 Let$\langle S\rangle$ = P_3= xyz

Subcase i $\langle V-S\rangle$ = K_2 = uv Since G is connected, there exists a vertex say (u or v) in K_2 which is adjacent to x or z in P_3.Suppose u is adjacent to x then S={u,x,y} forms a γ_{gtc} set of G. So that $\gamma_{gtc}(G)$ =P-2. If d(x) =d(y)=2, d(z)=1 then G $\cong P_5$. On increasing the degree of vertices G $\cong$ C_5, $C_3(P_2,P_2\ 0)$, $C_4(P_2)$, $G_1$to G_4.Since G is connected, there exist vertex say (u or v) in K_2 which is adjacent to y in P_3, then S= {u,y,x} forms a γ_{gtc} set of G. So that γ_{gtc}=P-2. If d(x)= d(z)=1,d(y)=3 then G$\cong$ $P_4(0,P_2,0,0)$. Now by increasing the degrees of vertices we have G$\cong$ $C_4(P_2)$ $C_3(2P_2)$, G_2 to G_5.

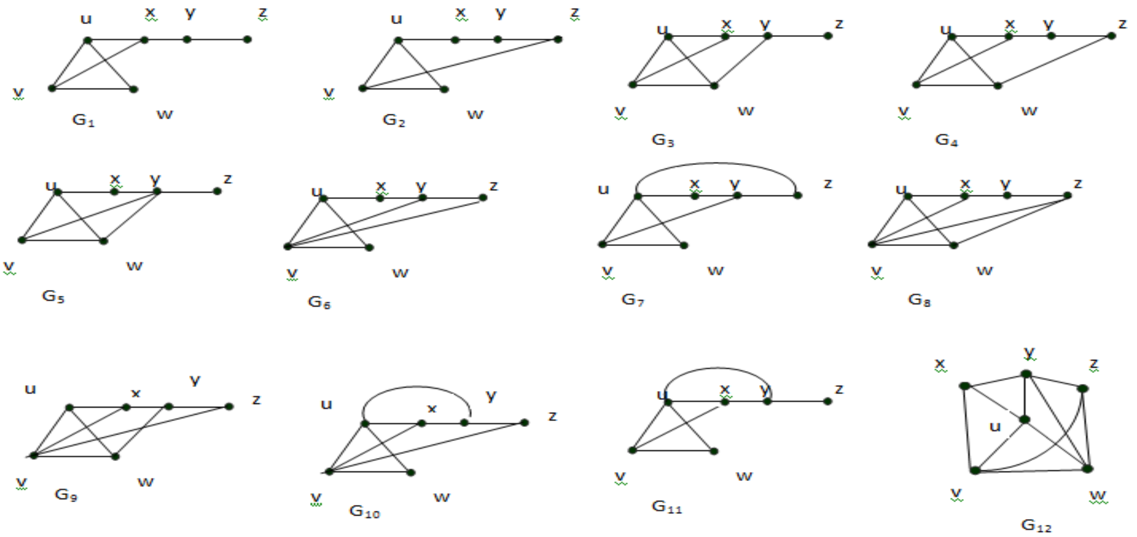
Subcase ii Let $\langle V-S\rangle$=$\overline{K_2}$. Let u and v are the vertices of $\overline{K_2}$. Since G is connected there exist a vertex u and v in $\overline{K_2}$ is adjacent to x or y or z in P_3. Suppose u and v is adjacent to x then S={u,x,y}forms a γ_{gtc} set of G. So that γ_{gtc}=P-2. If d(x)=3,d(y)=2, d(z)=1 G$\cong P_4(0,P_2,0,0)$.On increasing the degree of vertices G$\cong C_4(P_2)$, $C_3(P_2, P_2, 0)$, G_1 to G_7.Since G is connected, there exist a vertex say (u and v) in $\overline{K_2}$ is adjacent to y in P_3 then S= {x,y,u} forms a γ_{gtc} set of G. So that γ_{gtc} =P-2. If d(x)= d (z)=1, d (y)=4,then G $\cong K_{1,4}$. $K_3(0,K_3,0)$, $K_3(0,2P_2,0)$,G_6, G_7. Since G is connected there exist a vertex say u in $\overline{K_2}$ is adjacent to x and v in $\overline{K_2}$ is adjacent to y in P_3.Then S ={u,x,y} forms a γ_{gtc} set of G . So that that $\gamma_{gtc=}$ P-2. If d(x) = d (y) = d (z)=2 then G $\cong P_4$ $(0,P_2,0,0)$. On increasing the degree of vertices G_1, G_3, G_4, G_6, G_7,$C_3(P_3)$, $C_3(K_3)$, $C_3(2P_2)$, $C_4(P_2)$. Since G is connected there exist a vertex say u in $\overline{K_2}$ is adjacent to x and v in $\overline{K_2}$ is adjacent to z in P_3.Then S ={x,y,z} forms a γ_{gtc} set of G . So that that $\gamma_{gtc=}$ P-2. If d (x) = d (y)= d (z)=2 then G$\cong$ P_5 .On increasing the degree of vertices K_3 (P_3), K_3 (K_3),$C_4(P_2)$, G_1, G_5.

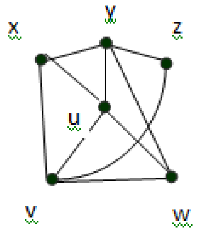
Case 2 Let $\langle S\rangle$ = C_3= xyzx

Subcase i $\langle V-S\rangle$ = K_2= uv Since G is connected, there exist a vertex say u or v in K_2 is adjacent to anyone of the vertices of C_3. If u or v in K_2 is adjacent to x in C_3 then S= {x,y,u} forms a γ_{gtc} set of G. So that $\gamma_{gtc}(G)$= P-2. If d (x)=4, d(y)=d (z)=2 then G$\cong C_3(P_3)$. On increasing the degree of vertices G$\cong$ $C_3(K_3)$,G_2, G_3, G_4, G_5 and G_7.

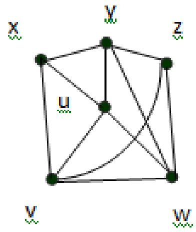
Subcase ii $\langle V-S\rangle$=$\overline{K_2}$ Since G is connected there exist a vertex u and v in $\overline{K_2}$ is adjacent to x or y or z in P_3. Suppose u and v is adjacent to x then S= {x,y,u }forms a γ_{gtc} set of G. So that γ_{gtc}=P-2, If d(x)=4,d(y)=2, d(z)=2 G $\cong K_3(2P_2)$ then G$\cong G_3$, G_6, G_8 , G_9 and G_{10}. Since G is connected there exist a vertex say u in $\overline{K_2}$ is adjacent to x and v in $\overline{K_2}$ is adjacent to y in K_3.Then S ={u,x,y} forms a γ_{gtc} set of G . So that $\gamma_{gtc=}$ P-2. If d (x) = d (y) = d (z)=2 then G$\cong K_3(0,P_2, P_2)$. On increasing the degree of vertices G$\cong G_3$, G_4, G_6, G_8 and G_{10}.

Theorem 2.17 For a connected graph G with 6 vertices $\gamma_{gtc}(G)$ = p-3 if and only if G is isomorphic to $K_3(P_4)$, $P_3(0,P_2,K_3)$, $K_3(0,P_2,K_3)$, $P_4(0,2P_2,0,0)$, C_4 $(0,2P_2,0,\ 0)$, $K_3(2P_2,P_2,0)$, $K_{1,5}$, $K_3(3P_2)$, $P_5(0,0,P_2,0,0)$, $P_5(0,P_2,0,0,0)$, $C_5(P_2,0,0,0,0)$,$C_4(P_2,\ P_2\ ,0,0)$, P_2 x P_3, K_4 $(P_2,\ P_2,\ 0,\ 0)$, $K_3(P_2,P_2,P_2)$, $K_3(P_3,\ P_2,0)$, $C_4(P_3)$, $C_4(K_3)$, $P_4(0,\ P_2,\ P_2\ ,0)$, $P_2(K_3,K_3)$, and anyone of the graphs G_1 to G_{56} in fig2.8.

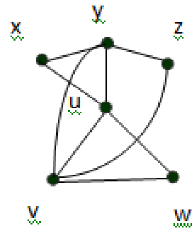




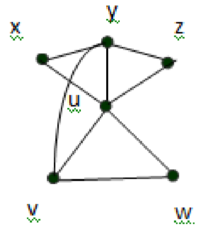
G_{13}



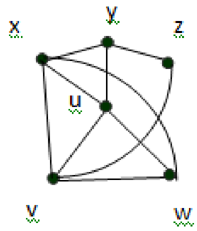
G_{14}



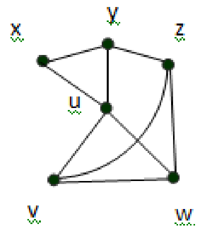
G_{15}



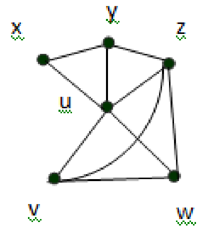
G_{16}



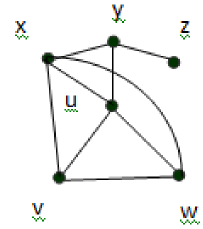
G_{17}



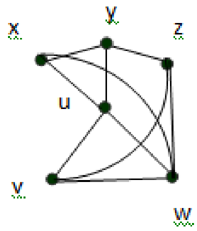
G_{18}



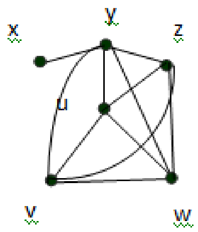
G_{19}



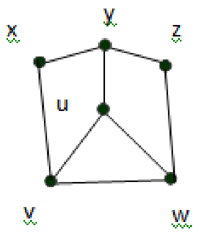
G_{20}



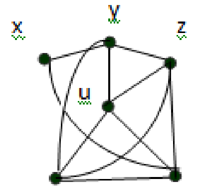
G_{21}



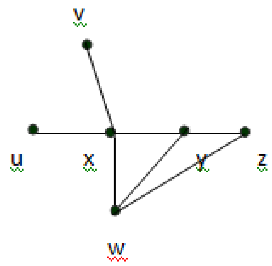
G_{22}



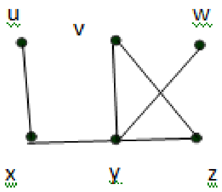
G_{23}



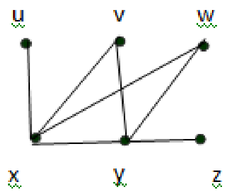
G_{24}



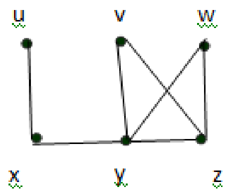
G_{25}



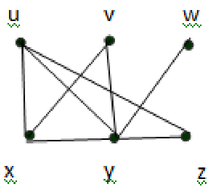
G_{26}



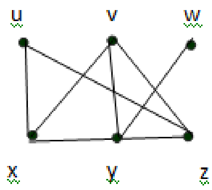
G_{27}



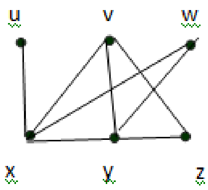
G_{28}



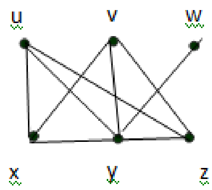
G_{29}



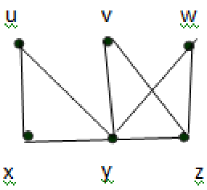
G_{30}



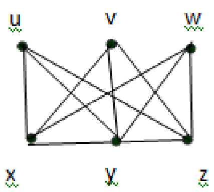
G_{31}



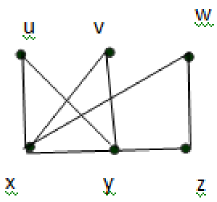
G_{32}



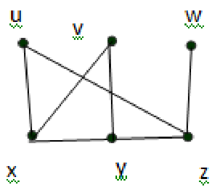
G_{33}



G_{34}



G_{35}



G_{36}

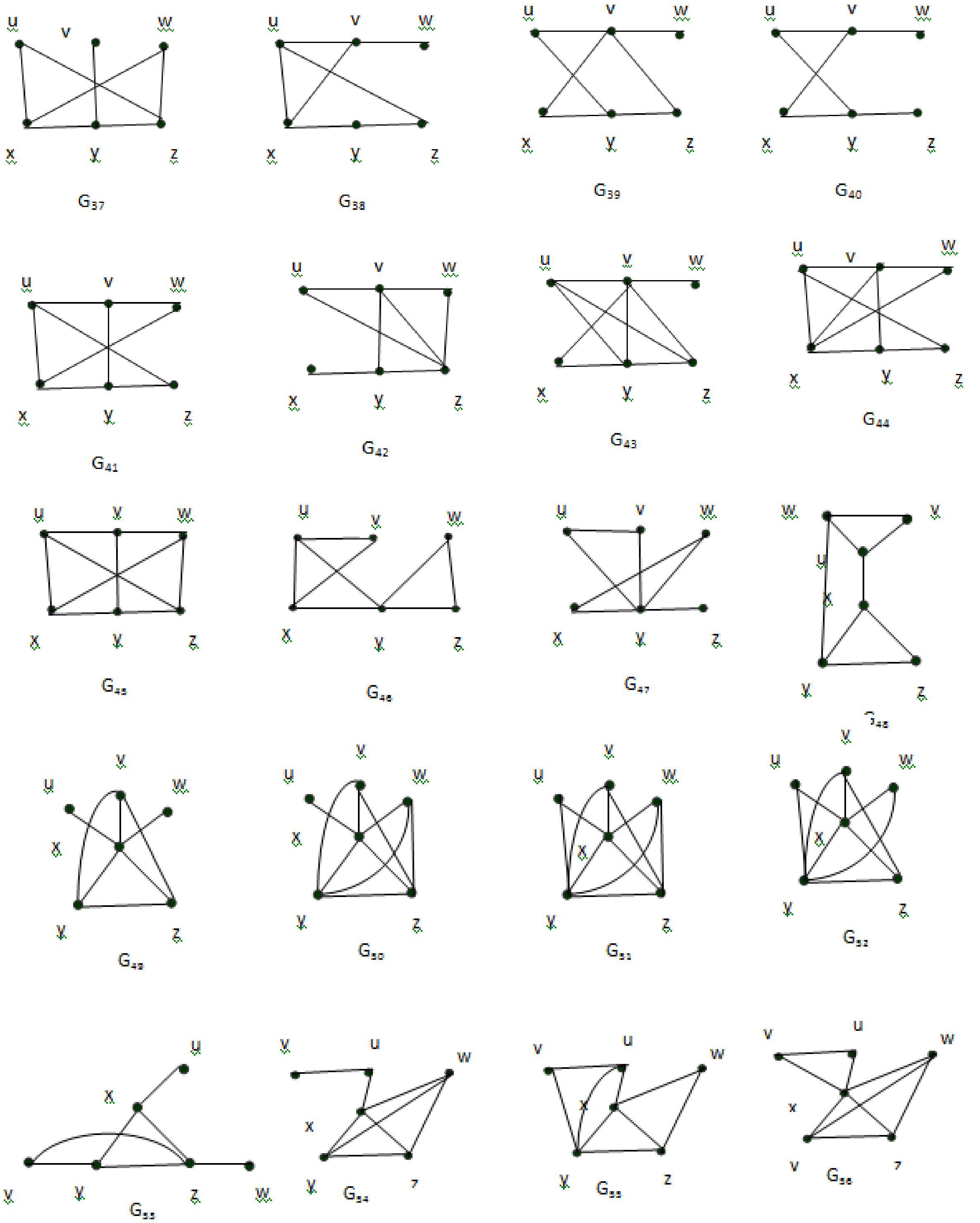


Fig 2.8

Case I Let ⟨S⟩=P_3= xyz

Subcase i ⟨$V-S$⟩ = K_3 = uvw Since G is connected, there exist a vertex say u or v in K_3 which is adjacent to anyone of the vertices of P_3 say x,y,z. If u is adjacent to x then {u, x, y} forms a γ_{gtc} set of G. If d(x) = d(y)= 2 d(z)=1 then G ≅ $K_3(P_4)$. On increasing the degree of vertices G ≅ G_1 to G_{21}. Since G is connected, there exist a vertex say u in P_3 which is adjacent to y in P_3. Then {u,y,x} forms a γ_{gtc} set of G. So that γ_{gtc}=3 Then G ≅ $P_3(0,P_2,K_3)$. On increasing the degree of vertices G≅ G_{10} to G_{24}, $K_3(0, P_2, K_3)$.

Subcase ii ⟨$V-S$⟩ = $\overline{K_3}$ Since G is connected, there exist a vertex u, v, w in $\overline{K_3}$ which is adjascent to x in P_3. Then {x, y, u} forms a γ_{gtc} set of G. So that G ≅ $P_4(0, 2P_2, 0, 0)$. On increasing the degree of vertices G ≅ C_4(0,

$2P_2$, 0, 0) , K_3 (P_2, $2P_2$,0), G_{25}.Since G is connected, there exist a vertex u, v, w in $\overline{K_3}$ which is adjascent to y in P_3. Then {u,y,x} forms a γ_{gtc} Set of G. so that γ_{gtc}=3.Then G≅ $K_{1,5}$ On increasing the degree of vertices G ≅ K_3 ($3P_2$),G_{29}, G_{32}, G_{33}, G_{34}. Since G is connected, there exist a vertex u is adjacent to x and v and w is adjacent to y. Then {u, x, y} forms a γ_{gtc} set of G. Then G ≅ P_4(0,$2P_2$,0,0).On increasing the degree of vertices G≅ K_3($2P_2$, P_2,0),C_4(0,$2P_2$,0,0), K_3($3P_2$),G_{26} to G_{34}. Since G is connected, there exist a vertex u and v adjacent to y and w is adjacent to z, then {u,y,z} forms a γ_{gtc} set of G. Then G≅ P_4(0,$2P_2$,0,0). On increasing the degree of vertices G≅ G_{33} to G_{35}. K_3($3P_2$,0,0),C_4(0,$2P_2$,0,0), Since G is connected, there exist a vertex u,v,w adjacent to x,y,z respectively. Then {x,y,z} forms a γ_{gtc} set of G. Then G≅ P_5(0,0,P_2,0,0). On increasing the degree of the vertices G≅ K_3(K_3,0,P_2),G_{33}, G_{35} to G_{37}. If {u,x,y} forms a γ_{gtc} set of G, then G≅ G_{34}.

Subcase iii $\langle V-S\rangle$ = P_3= uvw Since G is connected, there exist a vertex u is adjacent to anyone of the vertices of P_3 say {x,y,z}.If u is adjacent to x Then {u,v,x,y} forms γ_{gtc} set of G, so that γ_{gtc} (G) =4 which is a contradiction. On increasing the degree of vertices {v,x,y} forms a γ_{gtc} set of G. Hence G≅G_{38}, P_4(0,P_2,P_2,0), K_3(P_2,P_2, P_2), C_4(0,P_2,P_2,0). Since G is connected there exist a vertex v in P_3 which is adjacent to x. Then {v,x,y} forms a γ_{gtc} set of G. So that G≅ P_5(0,P_2,0,0,0). On increasing the degree of vertices G≅ K_3($2P_2$,P_2,0),C_4($2P_2$,0,0,0), C_5(P_2,0,0,0,0). G_{38} to G_{40}.Since G is connected there exist a vertex v in P_3 in $\langle V-S\rangle$ which is adjascent to y in P_3 in $\langle S\rangle$. Then {v,y,x} forms a γ_{gtc} set of. Hence γ_{gtc} (G) =3. Then G≅P_4(0, P_2, P_2,0). On increasing the degree of vertices G ≅ C_4(P_2,P_2,0,0), K_3($2P_2$,P_2,0),C_5(P_2,0,0,0) K_4(P_2,P_2,0,0), $P_2\times P_3$, G_{41} to G_{45}.

Subcase iv $\langle V-S\rangle$ = $K_2\cup K_1$ Let u and v are the vertices of K_2 and w be the vertex in K_1.Since G is connected, there exist a vertex u in K_2 and w be the vertex in K_1 which is adjacent to anyone of the vertices of P_3 say x, y, z. Let u and w is adjacent to x. Then {x,y,u} forms a γ_{gtc} set of G .Then G ≅ P_5(0,0,P_2,0,0). On increasing the degree of vertices G ≅ K_3(P_2, P_2, P_2), P_5(0,0,P_2,0,0), C_4(0,0,P_2,P_2), C_3(P_3, P_2,0), C_5(0,0,0,0,P_2). Since G is connected, the vertices u and w is adjacent to y. Then {u,y,z} forms a γ_{gtc} set of G .Then G ≅ G_{47}. Since G is connected there exist a vertex u is adjacent to x and w is adjacent to y. Then {u,x,y} forms a γ_{gtc} set of G. Then G≅ P_5(0,P_2,0,0,0).On increasing the degree of vertices G ≅ C_4($2P_2$), C_5(P_2), K_3(P_4), K_3(P_3, P_2,0), K_3($2P_2$, P_2,0),G_{46}. Since G is connected there exist a vertex v is adjacent to y and w is adjacent to z. Then {v,y,z } forms a γ_{gtc} set of G. Then G ≅ P_5(0,0,P_2,0,0). On increasing the degree of vertices G≅ C_4(P_3), C_4(K_3), K_3(P_2, P_2, P_2), K_3(P_3, P_2, 0).

Case II Let $\langle S\rangle$= K_3 = xyz

Subcase i Let $\langle V-S\rangle$=K_3. Let u,v,w be the vertices of K_3.Since G is connected, there exist a vertex u is adjacent to x. Then {v,u,x} forms a γ_{gtc} set of G. Then G ≅ P_2(K_3,K_3). On increasing the degree of vertices G≅ G_{48}.

Subcase ii Let $\langle V-S\rangle$=$\overline{K_3}$. Since G is connected there exist a vertex u,v and w in $\overline{K_3}$ which is adjacent to anyone of the vertices of K_3 say x. Hence {u,x,y} forms a γ_{gtc} set of G, so that G≅K_3($3P_2$). On increasing the degree of vertices G ≅ G_{49} to G_{52}. Since G is connected there exist a vertex u and v in $\overline{K_3}$ which is adjascent to x and w in $\overline{K_3}$ which is adjascent to y in K_3. Then {u,x,y} forms a γ_{gtc} set of G. Then G ≅K_3($2P_2$,P_2,0) . On increasing the degree of vertices G ≅ G_{50},G_{51},G_{52}. Since G is connected there exist a vertex u in $\overline{K_3}$ which is adjascent to x and v in $\overline{K_3}$ which is adjascent to y and w in $\overline{K_3}$ is adjascent to z. Then {x,y,z} forms a γ_{gtc} set of G. Then G≅K_3(P_2,P_2,P_2),On increasing the degree of vertices G≅ G_{53}.

Subcase iii Let $\langle V-S\rangle$=P_3 = uvw. Since G is connected there exist a vertex u in P_3 which is adjacent to anyone of the vertex of K_3 say x. Then {u,v,x} forms a γ_{gtc} set of G. Then G≅K_3(P_4).On increasing the degree of vertices G≅ G_1 to G_{20}. Since G is connected there exist a vertex say v in P_3 which is adjacent to anyone of the vertices of K_3 say x. Then {v,x,y} forms a γ_{gtc} set of G. Then G ≅ P_3(0,P_2,K_3).On increasing the degree of vertices G≅ G_{10} to G_{24}.

Subcase iv $\langle V-S\rangle$ = $K_2\cup K_1$ Let u and v are the vertices of K_2 and w be the vertex of K_1. Since G is connected, there exist a vertex u and w is adjacent to x, then {u,x,y} forms a γ_{gtc} set of G.Then G≅ G_{54} to G_{56}. Since G is connected there exist a vertex u is adjacent to x and w is adjacent to y. Then {u,x,y} forms a γ_{gtc} set of G. Then G≅K_3(P_3, P_2,0).On increasing the degree of vertices G≅ G_{54}, G_{56}.

Conversely if G is anyone of the graphs K_3(P_4), P_3(0,P_2,K_3), K_3(0,P_2,K_3), P_4(0,$2P_2$,0,0), C_4 (0,$2P_2$,0, 0), K_3($2P_2$,P_2,0), $K_{1,5}$, K_3($3P_2$), P_5(0,0,P_2,0,0), P_5(0,P_2,0,0,0), C_5(P_2,0,0,0,0),C_4(P_2, P_2 ,0,0), P_2 x P_3, K_4 (P_2, P_2, 0, 0), K_3(P_2,P_2,P_2), K_3(P_3, P_2,0), C_4(P_3), C_4(K_3), P_4(0, P_2, P_2 ,0), P_2(K_3,K_3), and anyone of the graphs G_1 to G_{56} in fig2.8 then it can be easily verify that γ_{gtc} = p-3.

III. Global Triple Connected Domination Number And Other Graph Theoretical Parameters

Theorem 3.1 For any connected graph with p≥5 vertices, then $\gamma_{gtc}(G)+k(G)\leq 2p-1$ and the bound is sharp if and only if $G\cong K_p$.

Proof: Let G be a connected graph with p≥5 vertices. Suppose G is isomorphic to K_p We know that $k(G)\leq p-1$ and by observation 2.3, $\gamma_{gtc}(G) \leq P$. Hence $\gamma_{gtc}(G)+k(G)\leq 2p-1$,Conversely, let $\gamma_{gtc}(G)+k(G)\leq 2p-1$. This is possible only if $\gamma_{gtc}(G) = p$ and $k(G)= p-1$. But $k(G)= p-1$, and so $G\cong K_p$ for which $\gamma_{gtc}(G)=K_p$.

Theorem 3.2 For any connected graph G with $P \geq 3$ vertices, $\gamma_{gtc}(G)+\chi(G)\leq 2p$ and the bound is sharp if and only if $G\cong K_p$.

Proof: Let G be a connected graph with P≥3 vertices. We know that $\chi(G)\leq p$ and by observation 2.3 $\gamma_{gtc}\leq p$. Hence $\gamma_{gtc}(G)+\chi(G)\leq 2p$. Suppose G is isomorphic to K_p, $\gamma_{gtc}(G)+\chi(G)\leq 2p$. Conversely $\gamma_{gtc}(G)+\chi(G)\leq 2p$. This is possible if $\gamma_{gtc}(G)= p$ and $\chi(G)=p$, So G is isomorphic to K_p for which $\gamma_{gtc}(G)= p$ so that p=3. Hence $G\cong K_p$.

Theorem 3.3. For any connected graph with p≥5 vertices, $\gamma_{gtc}(G)+\Delta(G)\leq 2p-1$ and the bound is sharp if and only if $G\cong K_p$

Proof: Let G be a connected graph with p≥5 vertices. We know that $\Delta(G)\leq p-1$ and by observation 2.3 $\gamma_{gtc}(G)\leq p$. Hence $\gamma_{gtc}(G)+\Delta(G)\leq 2p-1$. For K_p the bound is sharp.

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