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# Connectivity in Fuzzy Soft graph and its Complement

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***Abstract:*** *The concept of connectivity plays a vital role in the theory and applications of graphs. In this paper, the concept of fuzzy sets and soft sets are applied to study uncertainty and vagueness in graphs. The concept of m-strong fuzzy soft arc and the complement of fuzzy soft graph are introduced. Connectivity in fuzzy soft graphs in comparison with their complements is discussed through various examples.*
***Keywords :*** *fuzzy graph, fuzzy soft graph, complement of fuzzy soft graph, connectivity in fuzzy soft graph, m-strong fuzzy soft arc.*

## I. Introduction

The concept of soft set theory was initiated by Molodtsov [1] for dealing with uncertainties. A Rosenfeld [2] developed the theory of fuzzy graphs in 1975 by considering fuzzy relations on fuzzy sets, which was developed by Zadeh [3] in the year 1965. Maji *et al.* [4] discussed some applications of fuzzy soft sets to decision making problems. The concept of complement of fuzzy graph was presented by M.S. Sunitha and A. Vijayakumar [5]. Some operations on fuzzy graphs were studied by Mordeson and C.S. Peng [6]. Later, Ali *et al.* [7] discussed about fuzzy sets and fuzzy soft sets induced by soft sets. Various connectedness concepts in fuzzy graphs were introduced by Yeh and Bang [8]. Connectivity of fuzzy graph and its complement was analysed by K.R. Sandeep Narayan and M.S. Sunitha [9]. Research on fuzzy systems has been very active and received much attention from researchers worldwide because of its applications in various disciplines such as pattern recognition [10], chemistry [11], neural networks [12] and decision making [13].

M Akram and S Nawaz [14] introduced fuzzy soft graphs in the year 2015. Sumit Mohinta and T K Samanta [15] also introduced fuzzy soft graphs independently. The notion of fuzzy soft graph and few properties related to it are presented in their paper. Recently, Akram and Zafar [16] introduced the notions of fuzzy soft trees, fuzzy soft cycles, fuzzy soft bridges and investigated some of their fundamental properties. In this paper, m-strong fuzzy soft arc, complement of fuzzy soft graph, connectivity of fuzzy soft graph are introduced and some of its properties are studied.

## II. Preliminaries

Some definitions which can be found in [14,15,16,17] are reviewed here.

Definition 1: A fuzzy graph is a pair $G:(\sigma,\mu)$, where $\sigma$ is a fuzzy subset of a set $V$ and $\mu$ is a fuzzy relation on $\sigma$, i.e. $\mu(x,y)\le\sigma(x)\wedge\sigma(y)\ \ \forall x,y\in V$. We assume that $V$ is finite and non-empty, $\mu$ is reflexive and symmetric. In all the examples $\sigma$ is chosen suitably. Also we denote underlying crisp graph by $G^*:(\sigma^*,\mu^*)$, where $\sigma^*=\{u\in V/\sigma(u)>0\}$ and $\mu^*=\{(u,v)\in V\times V:\mu(u,v)>0\}$.

Definition 2: A fuzzy soft graph $\tilde{G}=(G^*,\tilde{F},\tilde{K},A)$ is a 4-tuple such that

a) $G^*=(V,E)$ is a simple graph

b) A is a non-empty set of parameters

c) $(\tilde{F},A)$ is a fuzzy soft set over $V$

d) $(\tilde{K},A)$ is a fuzzy soft set over $E$

e) $\left((\tilde{F},e),(\tilde{K},e)\right)$ is a fuzzy graph of $G^*\ \forall e\in A$

i.e. $\tilde{K}(e)(xy)\le\min\{\tilde{F}(e)(x),\tilde{F}(e)(y)\}$ for all $e\in A$ and $x,y\in V$. The fuzzy graph $\left((\tilde{F},e),(\tilde{K},e)\right)$ is denoted by $\tilde{H}(e)$ for convenience.

In what follows, we will use $G^*$ for a simple graph, $\tilde{G}$ for a fuzzy soft graph and $\tilde{H}$ for a fuzzy graph.

Definition 3: $\tilde{G}_1=(G^*,\tilde{F}_1,\tilde{K}_1,A)$ is a spanning fuzzy soft subgraph of $\tilde{G}=(G^*,\tilde{F},\tilde{K},A)$ if both have the same fuzzy soft vertex set i.e. $\tilde{F}_1(e)=\tilde{F}(e)\ \ \forall e\in A.$ They differ only in the arc weights.

Definition 4: A fuzzy soft graph $\tilde{G}$ is a fuzzy soft tree if each fuzzy graph $\tilde{H}(e_i)=\left(\tilde{F}(e_i),\tilde{K}(e_i)\right)$

$\forall e_i \in A$ has a fuzzy spanning subgraph $\tilde{Q}(e_i) = \left(\tilde{F}(e_i), \tilde{T}(e_i)\right)$ which is a tree, where for all arcs $xy$ not in $\tilde{Q}(e_i)$, $\tilde{K}(e_i)(xy) < CONN_{\tilde{Q}(e_i)}(xy)$. $CONN_{\tilde{Q}(e_i)}(xy)$ denotes the strength of connectedness between two nodes x and y which is defined as the maximum of strengths of all paths between x and y.

We assume that each node has membership value 1 unless otherwise specified throughout the examples of this paper. In all odd numbered figures, **A** and **B** denotes fuzzy graphs corresponding to parameter $e_1$ and $e_2$ respectively in $\tilde{G}$ and in all even numbered figures, **A** and **B** denotes fuzzy graphs corresponding to parameter $e_1$ and $e_2$ respectively in $\tilde{G}^c$.

## III. Complement Of Fuzzy Soft Graph And M-Strong Arc In Fuzzy Soft Graph

Definition 5: The complement of a fuzzy soft graph $\tilde{G}$ is a fuzzy soft graph denoted by $\tilde{G}^c$ where the fuzzy soft set over $V$ is same in both $\tilde{G}$ and $\tilde{G}^c$ and $\tilde{K}^c(e)(xy) = \min\{\tilde{F}(e)(x), \tilde{F}(e)(y)\} - \tilde{K}(e)(xy) \quad \forall e \in A, \forall x, y \in V.$

Definition 6: An arc $(u,v)$ of $\tilde{G}$ is called m-strong if $\tilde{K}(e)(uv) = \min\{\tilde{F}(e)(u), \tilde{F}(e)(v)\}$ for some $e \in A$. If $(u,v)$ is strong, then $\tilde{K}^c(e)(uv) = 0$

Example 1) Consider a simple graph $G^* = (V, E)$ such that $V = \{a_1, a_2, a_3\}$ and $E = \{a_1a_2, a_2a_3, a_3a_1\}$. Let $A = \{e_1, e_2\}$ be a parameter set and $(\tilde{F}, A)$ be a fuzzy soft set over $V$ with its fuzzy approximate function $\tilde{F} : A \to P(V)$ defined by

$\tilde{F}(e_1) = \{(a_1, 0.3), (a_2, 0.7), (a_3, 0.5)\}$

$\tilde{F}(e_2) = \{(a_1, 0.4), (a_2, 0.3), (a_3, 0.8)\}$

Let $(\tilde{K}, A)$ be a fuzzy soft set over $E$ with its fuzzy approximate function $\tilde{K} : A \to P(E)$ defined by

$\tilde{K}(e_1) = \{(a_1a_2, 0.3), (a_2a_3, 0.4), (a_3a_1, 0.2)\}$

$\tilde{K}(e_2) = \{(a_1a_2, 0.2), (a_2a_3, 0.1), (a_3a_1, 0.3)\}$

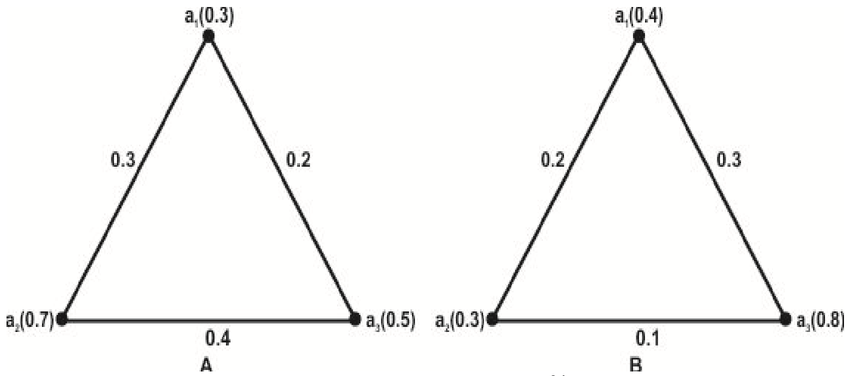


**Fig.1** Fuzzy Soft Graph $\tilde{G}$

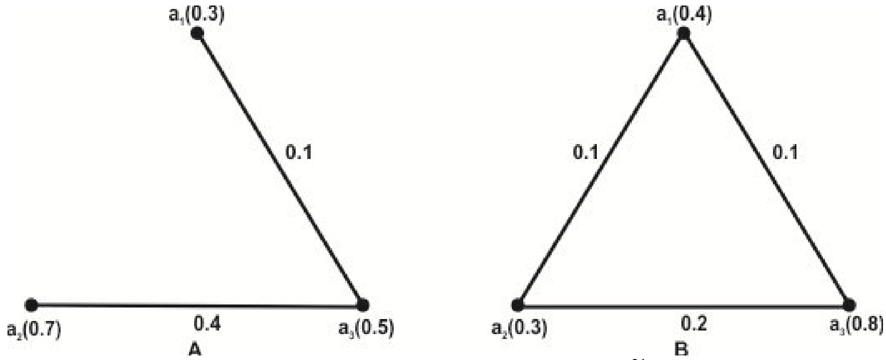


**Fig.2** Fuzzy Soft graph complement $\tilde{G}^c$

Thus, $\widetilde{H}(e_1)=(\widetilde{F}(e_1),\widetilde{K}(e_1))$, $\widetilde{H}(e_2)=(\widetilde{F}(e_2),\widetilde{K}(e_2))$ are fuzzy graphs. Hence, $\widetilde{G}=\{\widetilde{H}(e_1),\widetilde{H}(e_2)\}$ is a fuzzy soft graph as shown in Fig. 1 and its complement $\widetilde{G}^c=\{\widetilde{H}^c(e_1),\widetilde{H}^c(e_2)\}$ as shown in Fig. 2. Here, $a_1a_2$ is an m-strong arc in $\widetilde{G}$.

3.1 Connectivity in $\widetilde{G}^c$

Definition 7: Let $\widetilde{G}$ be a fuzzy soft graph of $G^*$. $\widetilde{G}$ is said to be a connected fuzzy soft graph if each $\widetilde{H}(e)$ is a connected fuzzy graph $\forall e\in A$. i.e. $CONN(x,y)>0 \quad \forall x,y\in V$ for each fuzzy graph in $\widetilde{G}$.

There may be cases in which the complement of connected fuzzy soft graphs becomes disconnected. In this paper, we propose a criterion by which a fuzzy soft graph and its complement will be connected simultaneously. We illustrate these with few examples.

Proposition: If $\widetilde{G}$ is connected fuzzy soft graph with no m-strong arcs then $\widetilde{G}^c$ is connected.

Proof: Consider a fuzzy soft graph $\widetilde{G}$ which is connected and contain no m-strong arcs. Let A be the set of parameters and $(\widetilde{F},A)$ be a fuzzy soft set over $V$. By the definition, $\widetilde{G}^c$ also contains the same fuzzy soft set over $V$ as defined in $\widetilde{G}$. Since $\widetilde{G}$ is connected, $CONN(x,y)>0 \quad \forall x,y\in V$ for every fuzzy graph in $\widetilde{G}$. Let the path of x and y be denoted by P. i.e. $P=(r_0,r_1)(r_1,r_2).......(r_{n-1},r_n)$ where $r_0=x, r_n=y$ and $\widetilde{K}(e_i)(r_{i-1},r_i)>0\ \forall i$. Since $\widetilde{G}$ contain no m-strong arcs, $\widetilde{K}^c(e_i)(r_{i-1},r_i)>0\ \forall i$ in $\widetilde{G}^c$. Hence P will also be a path in $\widetilde{G}^c$. Therefore, $\widetilde{G}^c$ is connected.

But, the converse of the above may or may not be true as shown in the following examples.

Example 2) Consider a simple graph $G^*=(V,E)$ such that $V=\{a_1,a_2,a_3\}$ and $E=\{a_1a_2,a_2a_3,a_3a_1\}$. Let $A=\{e_1,e_2\}$ be a parameter set and $(\widetilde{F},A)$ be a fuzzy soft set over $V$ with its fuzzy approximate function $\widetilde{F}:A\to P(V)$ defined by

$\widetilde{F}(e_1)=\{(a_1,1),(a_2,1),(a_3,1)\}$

$\widetilde{F}(e_2)=\{(a_1,1),(a_2,1),(a_3,1)\}$

Let $(\widetilde{K},A)$ be a fuzzy soft set over $E$ with its fuzzy approximate function $\widetilde{K}:A\to P(E)$ defined by

$\widetilde{K}(e_1)=\{(a_1a_2,1),(a_2a_3,0.3),(a_3a_1,0.2)\}$

$\widetilde{K}(e_2)=\{(a_1a_2,0.3),(a_2a_3,0.1),(a_3a_1,1)\}$

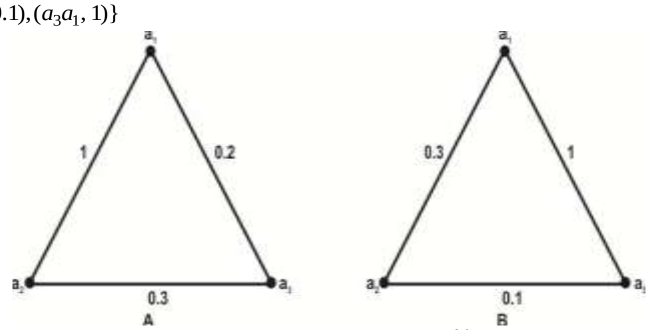


**Fig.3** Fuzzy Soft Graph $\widetilde{G}$

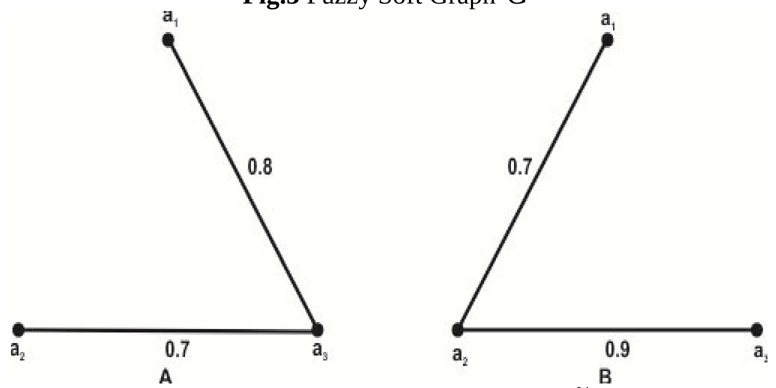


**Fig.4** Fuzzy Soft graph complement $\widetilde{G}^c$

Thus, $\tilde{H}(e_1)=(\tilde{F}(e_1),\tilde{K}(e_1))$, $\tilde{H}(e_2)=(\tilde{F}(e_2),\tilde{K}(e_2))$ are fuzzy graphs. Here, $a_1a_2$ is an m-strong arc corresponding to parameter $e_1$ in fuzzy soft graph $\tilde{G}$ and still $\tilde{G}^c$ is connected as shown in Fig. 3 and Fig. 4.

Example 3) Consider the previous example with the same set of parameter, fuzzy soft set defined over $V$ in a fuzzy soft graph $\tilde{G}$ and the fuzzy soft set over E defined by

$\tilde{K}(e_1)=\{(a_1a_2, 1),(a_2a_3, 0.3),(a_3a_1, 1)\}$

$\tilde{K}(e_2)=\{(a_1a_2, 0.3),(a_2a_3, 0.2),(a_3a_1, 0.1)\}$

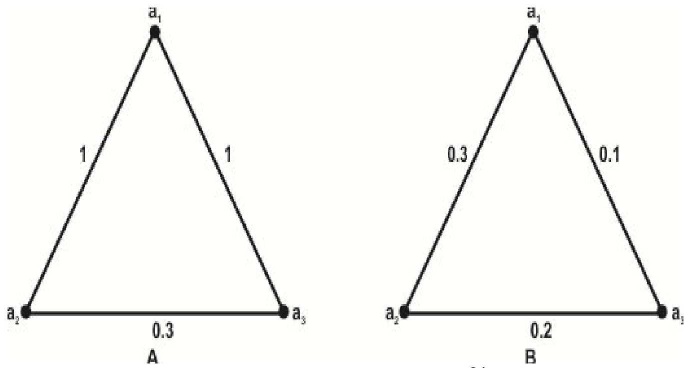


**Fig.5** Fuzzy Soft Graph $\tilde{G}$

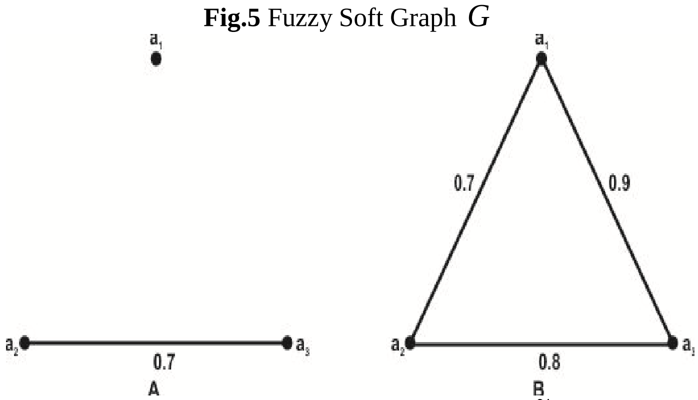


**Fig.6** Fuzzy Soft graph complement $\tilde{G}^c$

Here, $a_1a_2$ and $a_1a_3$ are two m-strong arcs corresponding to parameter $e_1$ in $\tilde{G}$ but $\tilde{G}^c$ is disconnected as shown in Fig. 5 and Fig. 6. Theorem: Let $\tilde{G}$ be a fuzzy soft graph. $\tilde{G}$ and $\tilde{G}^c$ are connected if and only if $\tilde{G}$ contains atleast one connected spanning fuzzy soft subgraph with no m-strong arcs.Proof: Let us assume that $\tilde{G}$ contains a spanning fuzzy soft subgraph $\tilde{G}_1$ that is connected, with no m-strong arcs. Using proposition, since $\tilde{G}_1$ contains no m-strong arcs and is connected, $\tilde{G}_1{}^c$ will be a connected spanning fuzzy soft subgraph of $\tilde{G}^c$ and thus $\tilde{G}^c$ is also connected.

Conversely, let us assume that $\tilde{G}$ and $\tilde{G}^c$ are connected. We have to find a connected spanning fuzzy soft subgraph of $\tilde{G}$ that contains no m-strong arcs.Let $\tilde{G}_1$ be an arbitrary connected spanning fuzzy soft subgraph of $\tilde{G}$. If $\tilde{G}_1$ contain no m-strong arcs then $\tilde{G}_1$ is the required subgraph. Suppose $\tilde{G}_1$ contain one m-strong arc say arc $(u,v)$ in a fuzzy graph $\tilde{H}(e_i)$ for some $e_i \in A$. Then arc $(u,v)$ will not be present in $\tilde{H}^c(e_i)$ in $\tilde{G}^c$. Let this path be $P_1$. Let $P_1=(u_1,u_2)(u_2,u_3)\ldots\ldots.(u_{n-1},u_n)$ where $u_1=u$ *and* $u_n=v$.

If all the arcs of $P_1$ are present in $\tilde{H}(e_i)$ in $\tilde{G}$ then $\tilde{H}(e_i)-(u,v)$ together with $P_1$ will be the required spanning subgraph. If not, then there exists atleast one arc say $(u_1,v_1)$ in $P_1$ which is not in $\tilde{H}(e_i)$ in $\tilde{G}$. Since $\tilde{G}$ is connected, replace $(u_1,v_1)$ by another $u_1-v_1$ path in $\tilde{H}(e_i)$. Let this path be denoted by $P_2$. If $P_2$ contain no m-strong arcs then $\tilde{H}(e_i)-(u,v)-(u_1,v_1)$ together with $P_1$ and $P_2$ will be the required spanning subgraph. If $P_2$ contain an m-strong arc then this arc will not be present in

$\tilde{H}^c(e_i)$ in $\tilde{G}^c$. Replace this arc by a path connecting the corresponding vertices in $\tilde{H}^c(e_i)$ in $\tilde{G}^c$ and proceed as above. Since $\tilde{G}$ contain only finite number of arcs , atlast we will get a spanning subgraph that contain no m-strong arcs.

If more than one m-strong arc is present in $\tilde{G}_1$, then the above procedure can be repeated for all other m-strong arcs of $\tilde{G}_1$ to get the required spanning subgraph of $\tilde{G}$. Corollary: Let $\tilde{G}$ be a fuzzy soft graph. $\tilde{G}$ and $\tilde{G}^c$ are connected if and only if $\tilde{G}$ contains atleast one fuzzy soft spanning tree having no m-strong arcs. Proof: From previous theorem, we get a connected fuzzy soft spanning subgraph of $\tilde{G}$ which contain no m-strong arcs. The maximum spanning fuzzy soft tree of this subgraph will be a spanning fuzzy soft tree of $\tilde{G}$ that contains no m-strong arcs.

## IV. Conclusion

In Recent Times, Fuzzy Soft Graph Is Finding Applications In The Field Of Computer Science In Decision Making Problems. In This Paper, We Propose A Criterion To Determine The Connectivity Of Fuzzy Soft Graph And Its Complement Based On M-Strong Arcs. We Have Shown That If Is A Connected Fuzzy Soft Graph With No M-Strong Arcs Then Is Connected. It Is Demonstrated With Examples That If Is A Connected Fuzzy Soft Graph With One M-Strong Arc Then Remains Connected And If Is A Connected Fuzzy Soft Graph With Two M-Strong Arcs Then Is Disconnected. We Have Also Established That And Are Connected If And Only If Contains Atleast One Connected Spanning Fuzzy Soft Subgraph With No M-Strong Arcs.

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