

# An Exact Derivation of The Quark Coupling Constant Without QCD

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**Abstract:** In this note we show that the quark coupling constant  $q=0.06583$  yields the lattice of  $E_6$  which has been shown to map the Standard Model. Thus there is no appeal to QCD for binding the quarks. AMS Classification Code: 14J247, 14K25, 14M25, 22E70, 81V05.

**Keywords:** Equiharmonics, Jacobi Theta Function, QCD, Coupling Constant,  $E_6$  lattice, Standard Model.

## I. Introduction

Fig.1 is the lattice of  $E_6$  with quarks assigned to the vertices, based on a model by Slansky [9], which has been shown to agree with the Standard Model in several papers summarised in [6]. The vertices are also labeled by  $0, \mu, \nu$  according to anotation adopted by Coxeter [4], Section 12.3, where  $0; \mu, \nu$  can assume the values  $0, 2, 3$  indicating rotations  $\omega$  through 120 and 240 degrees. In this way the vertices of each equilateral triangle are a rotation of 120 degrees so nucleons are rebound by a rotation of quarks according to  $su_3$  color symmetry with no appeal to a Strong Force. Fig.1 is not the same as that given in Ref[4] but is taken from an earlier reference [3] which is a torus with the leptons  $\tau^\pm, \nu_\tau$  situated in the center dictated by the infinitesimal structure of a cubic or elliptic surface.

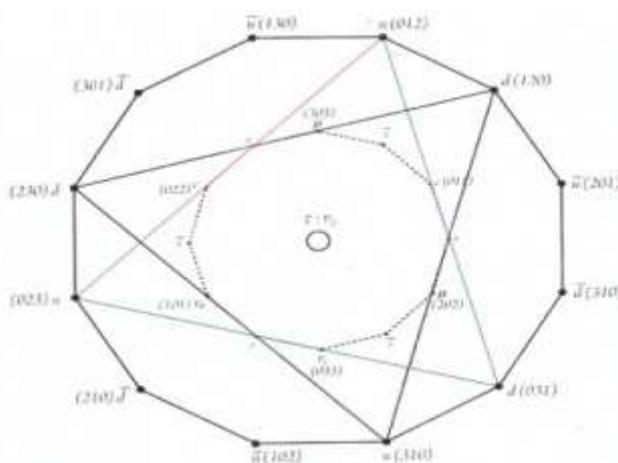
In this note we will see how the lattice of Fig.1 is governed by a quark Coupling Constant  $q=0.06583$  which is close to the constant 0.118 found by Davies et al. [5] where a smaller rectangular QCD matrix is employed.

Specifically  $E_6 = CP^3$ , the complex projective 3-space, has 3 real and 3 complex dimensions so we must consider rotations which are Jacobi Theta Functions with a nome  $q = \exp(-i\pi K/K')$ , where  $K$  and  $iK'$  are quarter periods on the real and imaginary axes. If these are equiharmonic, or multiples of a fundamental frequency  $f$ , then  $q=0.06583$  [1], which is shown in [8] to yield  $iK/K' = \sqrt{3}/2 = \sin 120 = \sin \omega$  or  $\sin 60$  that is precisely the angle in Fig.1 of the tritangent that maps the quarks and anti-quarks in an equiharmonic lattice. In this way the  $E_6$  lattice carries the coupling constant  $q$  uniting the up and down quarks and the fundamental frequency  $f$  could well be electromagnetic occupying all of space.

For example the Jacobi Theta Function given by [2] Ch.4 is

$$\theta_{E_6+[1]} = 27q^{4/3} + 216q^{10/3} + 459q^{16/3} + \dots \tag{1}$$

when the origin is moved to a deep hole, i.e. a translation to include the leptons  $\tau^\pm, \nu_\tau$ . Here 27 is the number of quarks and leptons of the Standard Model (also the number of vertices in Fig.1 together with the 3 leptons in the center) and 216 is the order of the subalgebra  $(su_3)_{\text{rotation}} + (su_3)_{\text{isospin}} + (su_3)_{\text{color}}$  of  $E_6$  (cf. [6]).



**Figure 1:**  $E_6$  Polytope

## II. The Equiharmonic Lattice

Here we will provide details of the calculation of  $\omega = 120$  degrees from the nome  $q = \exp(-i\pi K/K')$  found in [1]. Writing  $iK/K' = \sqrt{3}/2$  we have the identity

$$\ln(q^{-1}) = \pi\sqrt{3}/2 = 2.7207 \Rightarrow q = 0.06583 \quad (2)$$

for the quark coupling constant without any appeal to QCD.

## III. Conclusion

The equiharmonic lattice of Fig.1 may also result from  $iK/K' = \sqrt{3}$  when  $\omega = 60$  degrees in which case we find a possible nuclear coupling constant of 0.00433 which is the same order of magnitude as that suggested by Rees [9], Ch.4. E6 is also the orbifold of Type II String Theory [6].

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