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Optimal Replacement Policy for a Repairable System with Two Types of Failures modesusingAlpha Series Process

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Abstract: *This paper investigates an optimal replacement policy N for the maintenance problem with two types of failures and one repairman. In proposed model it is assumed that the system experiences with two types of failures namely repairable failures and non repairable failures. When repairable failure occurs, the repairs will be repaired once by the repairman. The repair is not as good as new. The successive working times after repair are stochastically decreasing and form a decreasing alpha series process while; the successive repair times form an increasing alpha series process. If un-repairable failures occur, the system will be replaced by a new one. When the number of repairable failures reaches N, then the system will not be repaired anymore and will be replaced immediately. With these assumptions, average cost rate (ACR). An optimal replacement policy N* is determined analytically and numerical results were given to strengthen the theoretical results.*

Key words: *Renewal Reward theorem, Renewal Process, Alpha series Process, Replacement policy N, Long-run Average cost Rate (ACR).*

I. Introduction

In the maintenance theory, optimization problem is very important problem and has been attracted by many researchers. In real world system, the factors like system configuration, degree of maintenance, maintenance cost, optimization criteria may affect the maintenance policy. Obviously, for a repairable system, the life cycle can be described by a sequence of up and down states. Initially the system operates until the first failure occurs and then it is repaired and restored to its original operating state. It will fail again after some random time of operation, get repaired again, and this process of failure and repair will repeat. Now the sequences of failure and repair times can be considered a sequence of independent and non-negative random variables constituting a renewal process. One of the main assumptions in renewal theory is that the failed components are replaced with new ones or repaired so that they are 'as good as new'.

In the beginning, Lotka [8] introduced replacement model for a repairable system under the assumption that the system after repair can't be 'as good as new'.Barlow and Hunter [1] developed a minimal repair model in which the repair activities do not alter the rate of failure of the system. Brown and Proschan [3] investigated an imperfect repair model in which the repair will be perfect repair with probability 'p' or minimal repair with probability 1-p, where $0<p<1$. For a deteriorating repairable system, the successive working time of the system after repair may become shorter and shorter, whereas the successive repair time of the system may become longer and longer. Conversely, the system may neither work nor repair any more. To model such a deteriorating system Lam [4,5] first introduced a Geometric Process Repair model. Using this model, he analyzed two kinds of replacement policies- one called policy 'T', based on the working age of the system the other, called policy 'N', based on the cumulative number of failures of the system.

Zhang [13] generalized Lam's work by a bi-variate replacement policy (T, N) under which the system is replaced at the working age T or at the time of the Nthfailure, whichever occurs first. Further, Wang and Zhang [10] considered a shock model for a repairable system with two types of failures by assuming that two kinds of shock in a sequence of random shocks will make the system failed, one based on the inter arrival time between two consecutive shocks less than given positive and the other based on the shock magnitude of single shock more than a given positive value (Ψ). Under this assumption they obtained some reliability indices of the shock model such as the system reliability and the mean working time before system failure. Also determined the replacement policy N based on the number of failures of the system by minimizing the long-run average cost per unit time. Further, it is also established through numerically.

Zhang [14] studied a deteriorating repairable system with three states including two failure states and one working state. A replacement policy N based on the failure number of the system is adopted under which the system will be replaced at the time of Nth failure and determined an optimal replacement policy N* by minimizing the average cost rate (ACR) and derived an explicit expression for the average cost rate. Lam et.al [6] developed a monotone process model for one-component degenerative system with K+1 states (K failure states and one working state) and they showed that this model is equivalent to a geometric process model of a two state one component system such that both systems have the same average cost rate and the same optimal policy. Inthe past, the maintenance problems of repairable systems are investigated with multiple failure modes. The GP model was considered for modeling repairable failure modes by Lam [7], Lam [4, 5], Wang and Zhang [11], Zhang and Wang [15] and so on.

In this direction, Wang and Zhang [12] studied the optimization problem for a system with two types of failures namely repairable failures and non repairablefailures. When repairable failure occurs, the repairs will be repaired once by the repairman. The repair is not as good as new. The successive working times after repair are stochastically decreasing and form a decreasing GP. If unrepeatable failures occur, the system will be replaced by a new one. When the number of repairable failures reaches N, then the system will not be repaired anymore and will be replaced immediately. With these assumptions, they studied two replacement models one based on the limiting availability and the other average cost rate (ACR). An optimal replacement policy N* is determined analytically and numerical results were given to strengthen the theoretical results.

However, much research work has been carried out by many researchers using the geometric processes for modeling repair time, working time and to estimate reliability of the system, Braun et.al [2] presented that both the increasing geometric process and the α -series process have a finite first moment under certain general conditions. Thus the decreasing α -series process may be more appropriate for modeling system working times while the increasing geometric process is more suitable for modeling repair times of the system.

Based on this understanding, in this paper, we study an optimal replacement policy N for the maintenance problem with two types of failures and one repairman. In the proposed model, it is assumed that the system experiences with two types of failures namely repairable failures and non repairable failure. When repairable failure occurs, the repairs will be repaired once by the repairman. The repair is not as good as new. The successive working times after repair are stochastically decreasing and form a decreasing alpha series process while; the successive repair times form an increasing alpha series process. If un-repairable failures occur, the system will be replaced by a new one. When the number of repairable failures reaches N, then the system will not be repaired anymore and will be replaced immediately. With these assumptions, average cost rate (ACR). An optimal replacement policy N* is determined analytically and numerical results were given to strengthen the theoretical results.

In modeling these deteriorating systems, the definitions according to Lam [4,5], are given below.

Definition 1: Given two random variables X and Y, if P(X>t) > P(Y>t) for all real t, then X is called stochastically larger than Y or Y is stochastically less than X. This is denoted by $X >_{st} Y$ or $Y <_{st} X$ respectively.

Definition 2: Assume that $\{X_n, n=1,2,....\}$, is a sequence of independent non-negative random variables. If the distribution function of X_n is $F_n(t) = F(k^\alpha t)$ for some $\alpha > 0$ and all n=1, 2, 3… then $\{X_n, n=1, 2…\}$ is called a α series process, α is called exponent of the process. (SeeBraun et al [2].)

Obviously:

If $\alpha >0$, then $\{X_n, n=1,2,....\}$ is stochastically decreasing, i.e, $X_n >_{st} X_{n+1}$, n=1,2,…;

If $\alpha < 0$, then $\{X_n, n=1,2,....\}$ is stochastically increasing, i.e., $X_n <_{st} X_{n+1}$, n=1,2, ….;

If $\alpha =0$, then the α series process becomes a renewal process.

II. The Model

To study an optimal replacement policy for repairable system with two types of failures, we make use the following assumptions.

Assumption 1: Assume that the system is initially new. The system has two types of failures, one repairable failure and the other is non-repairable failure.

Assumption 2: Whenever the system has repairable failure, it will repaired by the repairman.

Assumption 3: When non-repairable failure occurs, it will be replaced by a new and identical one.

Assumption 4: The system after repair is not as good as new.

Assumption 5: The successive working times of the system monotonically decreasing and form a decreasing alpha series process.

Assumption 6: The successive repair times of the system monotonically increasing and form increasing alpha series process.

Assumption 7: The successive working times and repair time are stochastically independent.

Assumption 8: Let Xn andYn,n=1,2,3,…. be the working time and repair time of the system at nth failure occurs. Further, let {X n,n=1,2,3,…} form a decreasing alpha series process with distribution function F n(nα x) and , let {Y n ,n=1,2,3,…} form an increasing alpha series process with distribution function G n(nβ x).

Assumption 9: A repairable failure occurs with probability p and non-repairable failure occurs with probability 1-p.
Assumption 10: The replacement time is negligible.
Assumption 11: Replacement policy N, under which the system is replaced whenever the number of repairable failures reaches N is adopted.
Assumption 12: The replacement cost is C and repair cost of the system is Cr and working reward is Cw.

III. ModelAnalysis

For repairable system, letlet τ_1 be the first replacement time, and let $\tau_n(n>1)$ be the time between the (n-1)-th replacement and the n-th replacement. Obviously, $\{\tau_1, \tau_2, \tau_3, . . .\}$ forms a renewal process. Let C(N) be the long-run average cost per unit time under the replacement policy N. Because $\{ \tau_1, \tau_2, \tau_3, . . .\}$ is a renewal process, the time interval between two consecutive replacements is a renewal cycle. Then according to the renewal reward theorem, Let C (N) betheaveragecost rate of the system under policy N. According to renewal reward theorem(see, e.g.,Ross[9]), then

$$C(N) = \frac{The\ \text{expected cost incurred in a renewal cycle}}{The\ \text{expected length in a renewal cycle}} \tag{3.1}$$

Let $S_k$be type of the kth failure k=1,2. And $S_k=1$ represents repairable failure occurs with $P(S_k=1)=P$, And $S_k=0$ represents repairable failure occurs with $P(S_k=0)=1-P$.

Let M be the total number of repairable failures until a non-repairable failure occur .Since Occurrences of failure are independent, we have

$$P(M=k) = P(S_1=1, S_2=1, S_3=1, ,\ldots S_{k-1}=1, S_k=0), \tag{3.2}$$

$$= P^{k-1}(1-P), k=1,2,\ldots.. \tag{3.3}$$

Under the replacement policy N, the total repair time in a renewal cycle is

$$\begin{aligned} Y(N) &= Y_1+Y_2+\ldots.+Y_{M-1} \ \ , M \le N \\ &= Y_1+Y_2+\ldots.+Y_{N-1} \ \ , M > N \end{aligned} \tag{3.4}$$

The total working time is

$$\begin{aligned} W(N) &= X_1+X_2+\ldots.+X_M \ \ , M \le N \\ &= X_1+X_2+\ldots.+X_N \ \ , M > N \end{aligned} \tag{3.5}$$

Now we evaluate the expected operating time as follows

$$E[W(N)] = \sum_{k=1}^{N} P(M=k)\sum_{j=1}^{k} E(X_j) + P(M>N)\sum_{j=1}^{N} E(X_j)$$

Now, according to Assumption (8), the expected value of working time is

$$E(X_j) = \int_0^\infty x\,dF(n^\alpha x), \tag{3.6}$$

$$E(X_j) = \frac{\lambda}{n^\alpha}, \lambda > 0, \alpha > 0 \quad . \tag{3.7}$$

$$E(Y_j) = \int_0^\infty xy\,dG(n^\beta y) \quad , \tag{3.8}$$

$$E(Y_j) = \frac{\mu}{n^\beta}, \mu > 0, \beta < 0. \tag{3.9}$$

Using equations (3.7) ,we have:

$$E[W(N)] = \sum_{k=1}^{N} P^{k-1}(1-P)\sum_{j=1}^{k}\frac{\lambda}{j^\alpha} + P^N\sum_{j=1}^{N}\frac{\lambda}{j^\alpha} \tag{3.10}$$

The expected repair time is follows

$$E[Y(N)] = \sum_{k=1}^{N} P(M = k)\sum_{j=1}^{k-1} E\left(Y_j\right) + P(M > N)\sum_{j=1}^{N-1} E\left(Y_j\right)$$

Using equation, (3.9), we have:

$$E[Y(N)] = \sum_{k=1}^{N} P^{k-1}(1-P)\sum_{j=1}^{k-1}\frac{\mu}{j^{\beta}} + P^{N}\sum_{j=1}^{N-1}\frac{\mu}{j^{\beta}} \tag{3.11}$$

Now let theexpected length of renewal cycle under policy N is

$$E(L(N)) = E[W(N)] + E[Y(N)] \tag{3.12}$$

Using equations (3.10) and (3.11), we have:

$$E[L(N)] = \sum_{k=1}^{N} P^{k-1}(1-P)\sum_{j=1}^{k}\frac{\lambda}{j^{\alpha}} + P^{N}\sum_{j=1}^{N}\frac{\lambda}{j^{\alpha}} + \sum_{k=1}^{N} P^{k-1}(1-P)\sum_{j=1}^{k-1}\frac{\mu}{j^{\beta}} + P^{N}\sum_{j=1}^{N-1}\frac{\mu}{j^{\beta}} \tag{3.13}$$

IV. The Long-Run Average Cost Rate

The main objective of the problem is to determine an expression for the long-run average cost rate (ACR) under the policy N and minimize the ACR According to renewal reward theorem(see, e.g.,Ross[9])

And according to equation (3.1), we have

$$C(N) = \frac{Cr\,E[Y(N)] + C - Cw\,E[W(N)]}{E[Y(N)] + E[W(N)]} \tag{4.1}$$

Using equations (3.10), (3.11) and (4.1), we have

$$C(N) = \frac{Cr\left(\sum_{k=1}^{N} P^{k-1}(1-P)\sum_{j=1}^{k-1}\frac{\mu}{j^{\beta}} + P^{N}\sum_{j=1}^{N-1}\frac{\mu}{j^{\beta}}\right) + C - Cw\left(\sum_{k=1}^{N} P^{k-1}(1-P)\sum_{j=1}^{k}\frac{\lambda}{j^{\alpha}} + P^{N}\sum_{j=1}^{N}\frac{\lambda}{j^{\alpha}}\right)}{\sum_{k=1}^{N} P^{k-1}(1-P)\sum_{j=1}^{k-1}\frac{\mu}{j^{\beta}} + P^{N}\sum_{j=1}^{N-1}\frac{\mu}{j^{\beta}} + \left(\sum_{k=1}^{N} P^{k-1}(1-P)\sum_{j=1}^{k}\frac{\lambda}{j^{\alpha}} + P^{N}\sum_{j=1}^{N}\frac{\lambda}{j^{\alpha}}\right)} \tag{4.2}$$

This is long-run average cost per unit time.

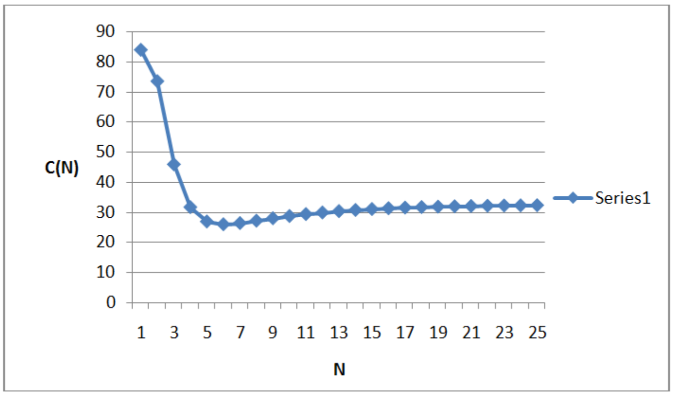
V. ResultsAnd Conclusions

For the hypothetical values C_r=40 , C=2500 , Cw=100, p=0.75, α=0.95, β=-0.95, λ =20, μ=30 it could be computed the long-run average cost per unit of time is computed as follows:

Table- 4.1:the long-run average cost per unit of time values against N

N	C(N)	N	C(N)
1	84	14	30.78261983
2	73.58611416	15	31.09462423
3	45.96808144	16	31.35253205
4	31.71588738	17	31.56544718
5	26.93002094	18	31.7410108
6	**26.01872761**	19	31.88559439
7	26.42633907	20	32.00449838
8	27.20515676	21	32.10213054
9	28.02162693	22	32.18215759
10	28.76572121	23	32.24763108
11	29.40895758	24	32.30109101
12	29.95217047	25	32.34465099
13	30.40583785		

Graph: 4.1 The plot of C(N) against N

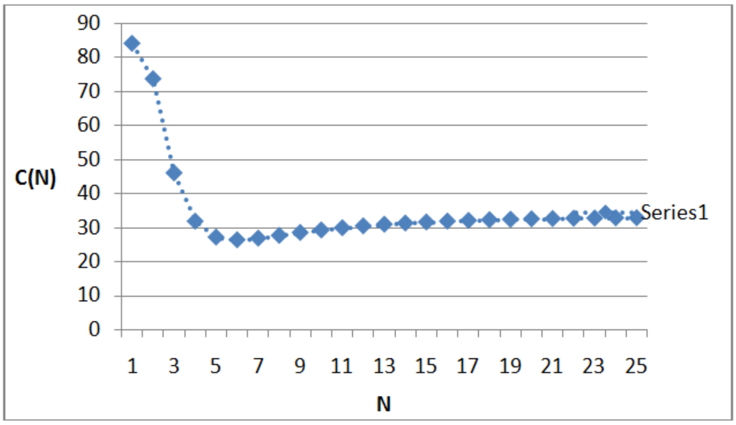


For the hypothetical values C_r=40 , C=2500 ,C_w=100, p=0.75 , α=0.95, β=-0.99,λ =20 ,μ=30 it could be computed the long-run average cost per unit of time is computed as follows:

Table:4.2 The long-run average cost per unit of time values against N

N	C(N)	N	C(N)
1	84	14	31.21485235
2	73.58611416	15	31.52688167
3	45.96318943	16	31.78443134
4	31.75998622	17	31.99683456
5	27.08444256	18	32.17186243
6	**26.27475889**	19	32.31595504
7	26.75367527	20	32.43444417
8	27.57765535	21	32.53174693
9	28.42137252	22	32.61152703
10	29.18134653	23	32.67682672
11	29.83348129	24	32.73017408
12	30.38138944	25	32.77367042
13	30.83727305		

Graph: 4.2 The plot of C (N) against N



VI. Conclusions

1. From the table 4.1 and graph 4.1, it is examined that the long-run average cost per unit time
2. C (6) =26.01872761 is minimum for the given β= -0.95, α=0.95. Thus, we should replace the system at the time of 6th failure.
3. From the table 4.2 and graph 4.2, it is observed that the long-run average cost per unit time C(5)= 26.27475889 is minimum for the given β= -0.99, α=0.95. We should replace the system at the time of 6th failure. Thus, from above conclusions in (i) and (ii), it can be concluded that the long-run average cost per unit time increases with β.
4. Similar conclusions may be drawn as 'α' decreases an increase in the number of failure, which coincides with the practical analogy and helps the decision maker for making an appropriate decision.

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