

## Radiation and Viscous Dissipation Effects on MHD Convective Flow of a Micro polar Fluid Past a Continuously Moving Plate with Suction/ Injection

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**Abstract:** The combined effect of thermal radiation and viscous dissipation on steady Magnetohydrodynamic heat and mass transfer flow of viscous incompressible micropolar fluid past a continuous moving plate with suction and injection is studied. A set of similarity parameters is employed to convert the governing partial differential equations into ordinary differential equations. Numerical results for the dimensionless velocity, microrotation, temperature and concentration profiles as well as the local skin-friction coefficient, the couple stress coefficient, the local Nusselt number and the local Sherwood number are displayed graphically for various physical parameters. Results show that the skin friction coefficient as well as the local Sherwood number is increases as radiation parameter  $R$  increases for both the cases suction and injection. It is also observed that the local Nusselt number decreases as radiation parameter  $R$  increases.

**Keywords:** Radiation; Viscous Dissipation; MHD; Micropolar Fluid; Suction/Injection

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### I. Introduction

In recent years, the dynamics of micropolar fluids, originated from the theory of Eringen [1, 2] has been a popular area of research. The equations governing the flow of a micropolar fluid involve a microrotation vector and a gyration parameter in addition to the classical velocity vector field. This theory may be applied to explain the flow of colloidal solutions, liquid crystals, fluids with additives, suspension solutions, animal blood and many other situations. Also, the continuous surface concept was introduced by Sakiadis [3]. Continuous surfaces are surfaces such as those of polymer sheets or filaments continuously drawn from a die. The boundary layer flow on continuous surfaces is an important type of flow occurring in a number of technical processes. These microstructural fluids include polymers, suspensions, archeological materials, etc., for which the micropolar theory is an excellent model. An excellent review of micropolar fluids and their applications were provided by Ariman et al. [4]. Through a review of the subject of micropolar fluid mechanics and its applications, Peddieson and McNitt [5] derived boundary-layer equations for a micropolar fluid, which are important in a number of technical processes, and applied these equations to the problems of steady stagnation point flow, steady flow past a semi-infinite flat plate, and flow, started by impulse, past a flat plate. On taking into account the gyration vector normal to the  $xy$  – plane and the microinertia effects, the boundary layer flow of micropolar fluids past a semi-infinite plate was studied by Ahmadi [6]. Soundalgekar and Takhar [7] analyzed the flow and heat transfer past a continuously moving plate in a micropolar fluid.

In all the previous investigation, the effect of thermal radiation on the flow and heat transfer has not been provided. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for design of reliable equipment, nuclear plants, gas turbines and various propulsion devices or aircraft, missiles, satellites and space vehicles. Based on these applications, Perdikis and Raptis [8] studied the heat transfer of a micropolar fluid in the presence of radiation. Recently, Raptis [9] studied the flow of a micropolar fluid past a continuously moving plate in the presence of radiation. Kim and Fedorov [10] analyzed transient mixed radiative convective flow of a micropolar fluid past a moving semi-infinite vertical porous plate. Arabawy [11] analyzed the problem of the effect of suction/injection on the flow of a micropolar fluid past a continuously moving plate in the presence of radiation. Rahman and Satter [12] have studied transient convective flow of micropolar fluid past a continuous moving porous plate in the presence of radiation. Abo-Eldahab and Ghonaim [13] studied radiation effects on heat transfer of a micropolar fluid through a porous medium.

However, the study of magnetohydrodynamic viscous radiate flows has important industrial, technological and geothermal applications such as high-temperature plasmas, cooling of nuclear reactors, liquid metal fluids, MHD accelerators, and power generation systems. Hossain and Takhar [14] analyzed the effect of

radiation using the Rosseland diffusion approximation which leads to non-similar solutions for the forced and free convection flow of an optically dense fluid from vertical surfaces with constant free stream velocity and surface temperature. Hossain et al. [15] studied the effect of radiation on free convection heat transfer from a porous vertical plate. The effect of variable viscosity on hydromagnetic flow and heat transfer past a continuously moving porous boundary with radiation has been studied by Seddeek [16]. Duwairi and Damseh [17, 18] obtained the radiation conduction interaction in free and mixed convection fluid flow for a vertical flat plate in the presence of a magnetic field effect. Mbeledogu and Ogulu [19] studied the heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous flat plate in the presence of radiative heat transfer. Recently, Alam et.al [20] studied the effects of variable suction and thermophoresis on steady MHD combined free-forced convective heat and mass transfer flow over a semi-infinite permeable inclined plate in the presence of thermal radiation.

In most of the studies mentioned above, viscous dissipation is neglected. The effect which bears a great importance on heat transfer is viscous dissipation. When the viscosity of the fluid is high, the dissipation term becomes important. For many cases, such as polymer processing which is operated at very high temperatures, viscous dissipation can not be neglected. Gebhart [21] has shown the importance of viscous dissipative heat in free convection flow in the case of isothermal and constant heat flux at the plate. Soundalgekar [22] analyzed viscous dissipative heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Israel Cooney et al. [23] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Recently, Alam et al. [24] presented the combined effect of viscous dissipation and joule heating on steady Magnetohydrodynamic heat and mass transfer flow of viscous incompressible fluid over an inclined radiate isothermal permeable surface in the presence of thermophoresis is studied. Ramachandra Prasad and Bhaskar reddy [25] investigated radiation and mass transfer effects on an unsteady MHD free convection flow past a heated vertical plate in a porous medium with viscous dissipation. El-Hakim [26] presented to study the effect of viscous dissipation, the thermal dispersion and Joule heating on MHD-free convection flow with a variable plate temperature in a micropolar fluid in the presence of uniform transverse magnetic field. Roja et al. [27] reported radiation and mass transfer effects on MHD free convective flow of a micropolar fluid past an infinite vertical moving porous plate embedded in a porous medium with viscous dissipation.

However, the combined effect of thermal radiation and viscous dissipation on MHD free convective micropolar fluid past a continuous moving plate has received little attention. Hence, in this chapter an attempt is made to study the effect of suction/injection on a steady magnetohydrodynamic free convective heat and mass transfer flow of a micropolar fluid past a continuous moving plate in the presence of thermal radiation and viscous dissipation. The dimensionless governing equations of the flow, heat and mass transfer are solved analytically using Runge-Kutta fourth order technique along with shooting method. Numerical results are reported in figures for various values of the physical parameters of interest.

## II. Mathematical Analysis

A steady two-dimensional, free convective heat and mass transfer flow of an electrically conducting incompressible micropolar fluid past a continuously moving plate with suction or injection in the presence of thermal radiation and viscous dissipation is considered. The  $x$ -axis is chosen along the plate and  $y$ -axis is taken normal to it. A uniform magnetic field is applied in the direction perpendicular to the plate. The fluid is assumed to be slightly conducting, and hence the magnetic Reynolds number is much less than unity and hence the induced magnetic field is negligible in comparison with the applied magnetic field. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equations. Now, under the usual Boussinesq's approximation, the governing boundary layer equations of the flow field are

Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

Linear Momentum

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g \beta_f (T - T_\infty) + g \beta_c (C - C_\infty) - \frac{\sigma B_0^2}{\rho} u + K_1 \frac{\partial N}{\partial y} \tag{2.2}$$

Angular Momentum

$$G_1 \frac{\partial^2 N}{\partial y^2} - 2N - \frac{\partial u}{\partial y} = 0 \tag{2.3}$$

Energy (Heat transfer)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{\nu}{c_p} \left( \frac{\partial u}{\partial y} \right)^2 \quad (2.4)$$

Concentration (Mass transfer)

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (2.5)$$

where  $u$  and  $v$  are the velocity components in the  $x$  and  $y$  directions respectively,  $\nu = (\mu + S)/\rho$  is the apparent kinematic viscosity,  $\mu$  is the fluid density,  $S$  a constant characteristic of the fluid,  $g$  is the acceleration due to gravity,  $\rho$  is the density of the fluid,  $\beta_f$  and  $\beta_c$  are the thermal and concentration expansion coefficients,  $T$ ,  $T_w$  and  $T_\infty$  is the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively. While  $C$ ,  $C_w$  and  $C_\infty$  are the corresponding concentration,  $c_p$  is the specific heat at constant pressure,  $B_0$  is the magnetic field strength,  $\sigma$  is the electrical conductivity of the fluid,  $G_1 (>0)$  is the microrotation constant,  $N$  is the components of microrotation or angular velocity whose rotation is in the direction of the  $xy$ -plane,  $K_1 = S/\rho (K_1 > 0)$  is the coupling constant,  $S$  is a constant characteristic of the fluid,  $k$  is the thermal conductivity,  $q_r$  is the radiative heat flux,  $D$  is the molecular diffusivity of the species of concentration. The second and third terms on right hand side of the momentum equation (2.2) denote the thermal and concentration buoyancy effects, respectively. Also, second and third terms on the right hand side of energy equation (2.3) represent the radiation and viscous dissipative heat effects, respectively.

The appropriate boundary conditions for the above model are as follows:

$$u = U_0, v = V_w, N = 0, T = T_w, C = C_w \text{ at } y = 0$$

$$u = 0, v = 0, N = 0, T = T_\infty, C = C_\infty \text{ as } y \rightarrow \infty, \quad (2.6)$$

where  $U_0$  is the uniform plate velocity in the  $x$ -direction, and  $V_w(x)$  represents the permeability of the porous surface. Here we confine our attention to the suction/injection of fluid through the porous surface and for these

we also consider that the transpiration function variable  $V_w(x)$  is of the order of  $x^{-\frac{1}{2}}$ .

By using the Rosseland approximation (Brewster [28]), the radiative heat flux in the  $y'$  direction is given by

$$q_r = -\frac{4\sigma_s}{3k_e} \frac{\partial T^4}{\partial y} \quad (2.7)$$

where  $\sigma_s$  the Stefan-Boltzmann constant and  $k_e$  the mean absorption coefficient.

It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If the temperature differences within the flow are sufficiently small, then Equation (2.6) can be linearized by expanding  $T^4$  into the Taylor series about  $T_\infty$ , which after neglecting higher order terms takes the form

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (2.8)$$

In view of equations (2.7) and (2.8), equation (2.4) reduces to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_e}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3\rho c_p k_1} \frac{\partial^2 u}{\partial y^2} + \frac{\nu}{c_p} \left( \frac{\partial u}{\partial y} \right)^2 \quad (2.9)$$

Let us introduce the following dimensionless variables:

$$u = U_0 f'(\eta), v = -\sqrt{\frac{\nu U_0}{2x}} [f(\eta) - \eta f'(\eta)], \eta = \left( \frac{U_0}{2\nu x} \right)^{1/2} y, \psi = (2\nu U_0 x)^{1/2} f(\eta),$$

$$M = \frac{\sigma B_0^2 2x}{\rho U_0}, K = \frac{K_1}{\nu}, G = \frac{G_1 U_0}{2\nu x}, Gr = \frac{g\beta_f (T_w - T_\infty) 2x}{U_0^2}, Gc = \frac{g\beta_c (C_w - C_\infty) 2x}{U_0^2}, \quad (2.10)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad N = \left(\frac{U_0}{2\nu x}\right)^{1/2} U_0 \omega(n), \quad \text{Pr} = \frac{\nu}{K}, \quad R = \frac{kk_e}{4\sigma_s T_\infty^3},$$

$$E_c = \frac{U_0^2}{c_p (T_w - T_\infty)}, \quad \text{Sc} = \frac{\nu}{D}$$

The continuity equation (2.1) is automatically satisfied by the Cauchy-Riemann equations  $u = \frac{\partial \psi}{\partial y}$  and

$$v = -\frac{\partial \psi}{\partial x}, \quad \text{where } \psi \text{ is the stream function.}$$

Substituting equation (2.10), and following the analysis of El-Arabawy [11], into the equations (2.2), (2.3), (2.5) and (2.9) yield the following dimensionless equations:

$$(1+K)f''' + ff'' - Mf' + Gr\theta + Gc\phi + Kg' = 0 \tag{2.11}$$

$$G\omega'' - 2(2\omega + f'') = 0 \tag{2.12}$$

$$(3R+4)\theta'' + 3RPr f\theta' + RPr Ec f''^2 = 0 \tag{2.13}$$

$$\phi'' + Scf\phi' = 0 \tag{2.14}$$

where the prime indicates the differentiation with respect to  $\eta$ , and  $Gr$  is the Grashof number,  $Gc$  is the modified Grashof number,  $M$  is the magnetic Parameter,  $G$  is the microrotation parameter,  $K$  is the coupling parameter,  $Pr$  is the Prandtl number,  $R$  is the radiation parameter and  $Sc$  is the Schmidt number.

The corresponding dimensionless boundary conditions are

$$\begin{aligned} f(0) = F_w, \quad f'(0) = 1, \quad \omega(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1, \\ f(\infty) = 0, \quad \omega(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \end{aligned} \tag{2.15}$$

where  $F_w = -V_w(x) \sqrt{\frac{2x}{\nu U_0}}$  is the permeability of the porous surface which is positive for suction and negative for injection.

The local skin-friction coefficient, couple stress coefficient, wall heat transfer coefficient (or local Nusselt number) and wall deposition flux (or the local Stanton number) are the important physical parameters for engineering applications. These can be obtained from the following expressions:

$$C_f = \frac{(\mu + K) \left(\frac{\partial u}{\partial y}\right)_{y=0} + K(N)_{y=0}}{(1/2)\rho U_0^2} = -2 \text{Re}_x^{-1/2} f''(0) \tag{2.16}$$

$$C_w = \frac{(\gamma/K)(\partial N/\partial y)_{y=0}}{\gamma U_0^3 (2K\nu^2)} = \text{Re}_x^{-1} \omega'(0) \tag{2.17}$$

$$Nu_x = \frac{-k \left(\frac{\partial T}{\partial y}\right)_{y=0} - \frac{4\sigma}{3k} \left(\frac{\partial T^4}{\partial y}\right)_{y=0}}{k(T_w - T_\infty)} = -\frac{1}{2} \text{Re}_x^{1/2} \left(1 + \frac{4}{3R}\right) \theta'(0) \tag{2.18}$$

$$Sh_x = \frac{-D \left(\frac{\partial C}{\partial y}\right)_{y=0}}{U_0 C_\infty} = (\text{Re}_x^{-1/2}) \phi'(0) \tag{2.19}$$

where  $\text{Re} = \frac{U_0 x}{\nu}$  is the Reynolds number

### III. Numerical Techeque

The set of coupled non-linear governing boundary layer equations (2.11) - (2.14) together with the boundary conditions (2.15) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. The computations were done by a program which uses a symbolic and computational computer language (Mathematica 4.0) on a Pentium 1 PC machine. The step size  $\Delta\eta = 0.01$  is used to obtain

the numerical solution with five decimal place accuracy as the criterion of convergence  $10^{-6}$ . From the process of numerical computation, the skin-friction coefficient, the plate couple stress, the Nusselt number and the Sherwood number, which are respectively proportional to  $f''(0)$ ,  $g'(0)$ ,  $-\theta'(0)$  and  $-\phi'(0)$  are also sorted out and their numerical values are presented graphically.

#### IV. Results And Discussion

In this chapter, the effect of suction/injection on a steady magnetohydrodynamic free convective heat and mass transfer flow of incompressible micropolar fluid past a continuous moving plate in the presence of thermal radiation and viscous dissipation has been investigated using Runge-Kutta fourth order technique along with shooting method. In order to get a physical insight of the problem, a parametric study is carried out to illustrate the effect of various thermophysical parameters  $F_w$ ,  $M$ ,  $Pr$ ,  $R$ ,  $Ec$  and  $Sc$  on the velocity, microrotation, temperature, concentration, local skin friction, couples stress, local Nusselt number and Sherwood number and are presented in figures and tables.

Figs. 1(a)-(d) represent the velocity, microrotation, temperature and concentration profiles for various values of the wall mass transfer parameter  $F_w$ , respectively. It is observed that as the wall mass transfer parameter  $F_w$  increases, there is a fall in the velocity, temperature and concentration as well as their boundary-layer thicknesses, but there is rise in the microrotation.

Figs. 2(a)-(d) display the influence of the magnetic field parameter  $M$  on the velocity, microrotation, temperature and the concentration, respectively. The presence of a magnetic field has the tendency to produce a drag-like force called the Lorentz force which acts in the opposite direction of the fluid's motion. This causes the fluid velocity and microrotation to decrease and the fluid temperature and concentration to increase as the magnetic field parameter  $M$  increases. In addition, the boundary-layer thickness decreases while the thermal boundary-layer thickness increases as  $M$  increases. It is also observed from Fig. 2(b) that with an increase in  $M$ , the microrotation first increases within the domain  $0 \leq \eta \leq 1$  and then gradually decreases for  $\eta > 1$  for both the cases of suction as well as injection. From Fig. 2(c) it is seen that  $M$  results in a decrease in the temperature. From Fig. 2(d), it is noticed that as the magnetic field parameter  $M$  increases, the concentration increases.

Figs. 3(a)-(d) depict the velocity, microrotation, temperature and concentration profiles, for variation in the coupling parameter  $K$  keeping all other parameters fixed for both suction as well as injection. From Fig. 3(a), it is seen that as  $K$  increases, the velocity increases slightly within the domain  $0 \leq \eta \leq 1.2$ , and then decreases for  $\eta > 1.2$ . Fig. 3(b) it is observed that as  $K$  increases, the microrotation increases within the domain  $0 \leq \eta \leq 3$  and decreases for  $\eta > 3$ . From Fig. 3(c), it is seen that there is no effect of  $K$  on the temperature. From Fig. 3(d) it is observed that concentration increases as the coupling parameter  $K$  increases.

Figs. 4(a)-(d) show the variations of the velocity, microrotation, temperature and concentration for different values of the Prandtl number  $Pr$  in both the cases of suction as well as injection. From Fig. 4(a) it is found that velocity decreases with the increasing values of  $Pr$ . Fig. 4(b) shows that as  $Pr$  increases first the microrotation increases within the domain  $0 \leq \eta \leq 2$  and then decreases for  $\eta > 2$ . Fig. 4(c) shows that the temperature decreases with the increase of the Prandtl number  $Pr$ . It is also noted from Fig. 4(d) that the concentration increases with the increase of the Prandtl number  $Pr$ .

Figs. 5(a)-(d), illustrate the dimensionless velocity, temperature and concentration profiles for different values of radiation parameter  $R$ , respectively. From Fig. 5(a), it is noticed that the temperature decreases as  $R$  increases. Fig. 5(b) shows that as  $R$  increases, first the microrotation increases within the domain  $0 \leq \eta \leq 2.1$  and then decreases for  $\eta > 2.1$ , for both the cases of constant fluid suction and injection. From Fig. 5(c) it is found that as the radiation parameter  $R$  increases, the temperature gradients near the porous wall increases for both the cases of constant fluid suction and injection, which increases heat transfer rates. Fig. 5(d) shows that as  $R$  increases, the concentration decreases slightly.

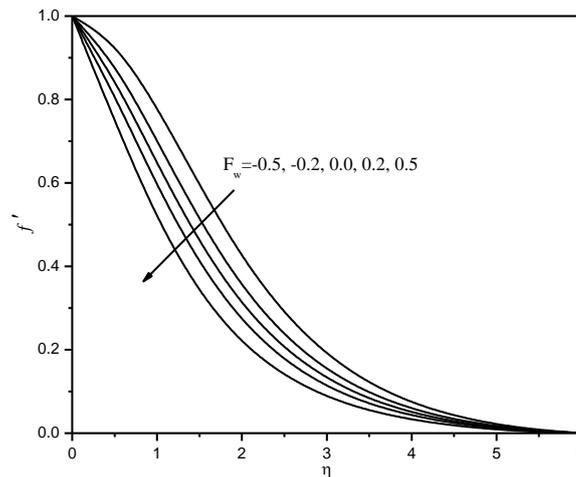
The effect of viscous dissipation parameter i.e., Eckert number  $Ec$  on the velocity, microrotation and temperature are shown in Figs. 6(a)-(c), respectively. Fig. 6(a) shows that the velocity field increases with the increase of  $Ec$  for fluid suction as well as for injection. Fig. 6(b) displays that the microrotation increases with the increase of  $Ec$  for fluid suction as well as for injection. Fig. 6(c) shows that the temperature increases with increasing values of  $Ec$  for fluid suction as well as for injection.

Figs. 7(a)-(d), illustrate the dimensionless velocity, microrotation, temperature and concentration profiles for various values of the Schmidt number  $Sc$ . From Fig. 7(a), it is noted that the velocity decreases as  $Sc$  increases for both the cases of constant fluid suction or injection. Fig. 7(b) shows that as  $Sc$  increases, first the microrotation increases within the domain  $0 \leq \eta \leq 2$  and then decreases  $\eta > 2$  for both the cases of constant fluid suction and injection. Further, from Fig. 7(c) it is seen that temperature increases as  $Sc$  increases for both

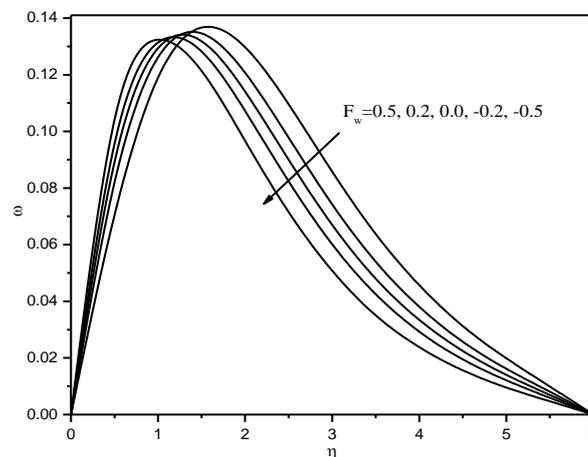
the cases of constant fluid suction and injection. From Fig. 7(d), it is noticed that the concentration decreases with the increase of  $Sc$  for both the cases of constant fluid suction and injection.

The combined effects of the magnetic field parameter  $M$  and Eckert number  $Ec$  on the local skin friction coefficient, couple stress coefficient, the local Nusselt number and the local Sherwood number are shown in Figs. 8(a)-(d) respectively. From these figures it is observed that the local Skin-friction coefficient as well as the couple stress coefficient increases for both fluid suction and injection as the viscous dissipation parameter increases, keeping  $M$  fixed. It is also seen that the local Nusselt number as well as the local Sherwood number decreases as  $Ec$  increases for fluid suction as well as for injection.

Figures 9(a)-(c) respectively, show the local skin-friction coefficient, couple stress coefficient, the local Nusselt number and the local Sherwood number against magnetic field parameter  $M$  for different values of radiation parameter  $R$ . As the radiation parameter increases, there is a fall in the local skin-friction coefficient and there is a rise in the couple stress, local Nusselt number and local Sherwood number, for the case of fluid suction and injection.



**Fig. 1 (a)** Velocity profiles for different values of  $F_w$



**Fig. 1 (b)** Micro rotation profiles for different values of  $F_w$

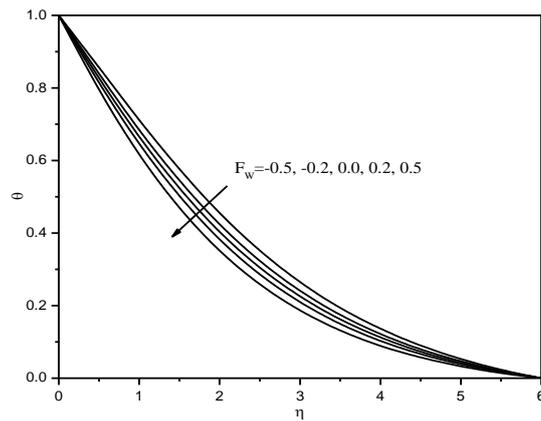


Fig. 1 (c) Temperature profiles for different values of  $F_w$

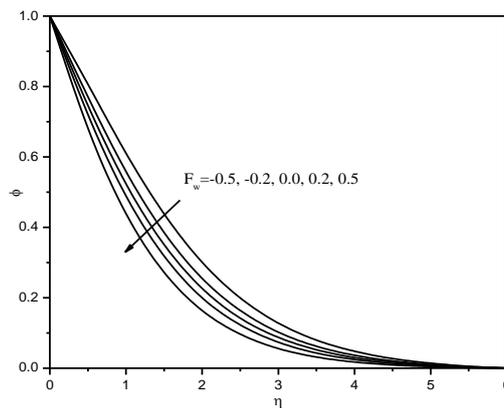


Fig. 1 (d) Concentration profiles for different values of  $F_w$ .

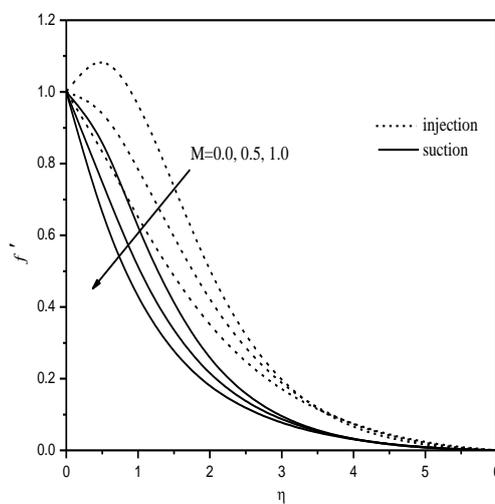


Fig. 2 (a) Velocity profiles for different values of  $M$

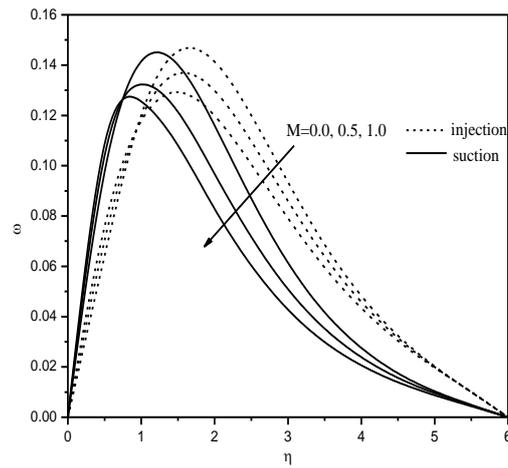


Fig.2 (b) Micro rotation profiles for different values of  $M$

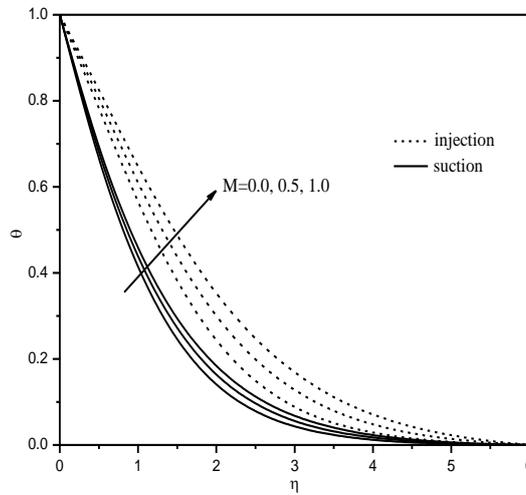


Fig. 2 (c) Temperature profiles for different values of  $M$

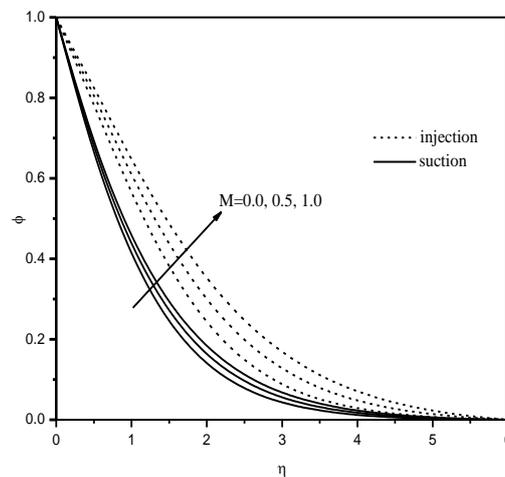


Fig. 2(d) Concentration profiles for different values of  $M$  .

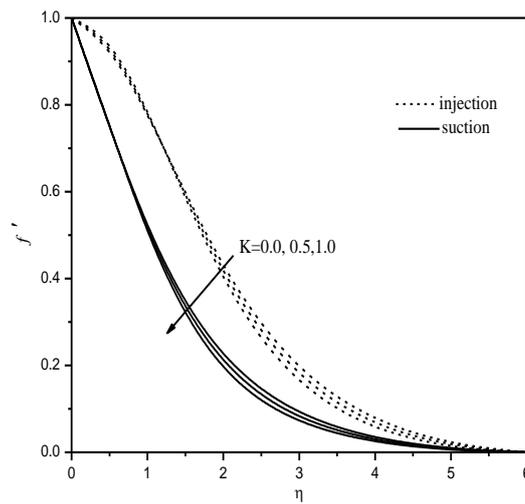


Fig. 3 (a) Velocity profiles for different values of  $K$

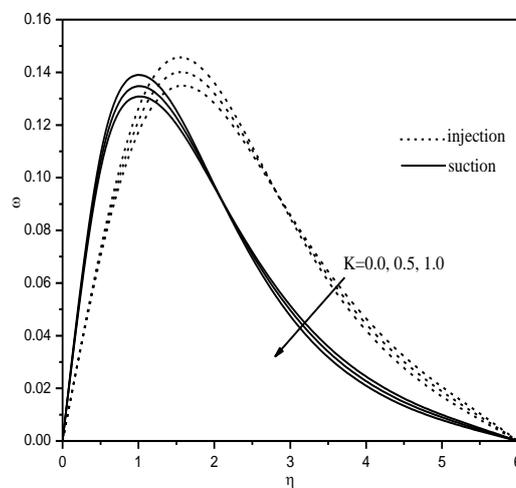


Fig. 3 (b) Micro rotation profiles for different values of  $K$

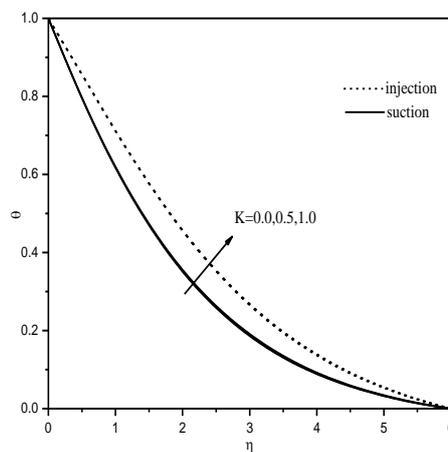


Fig. 3 (c) Temperature profiles for different values of  $K$

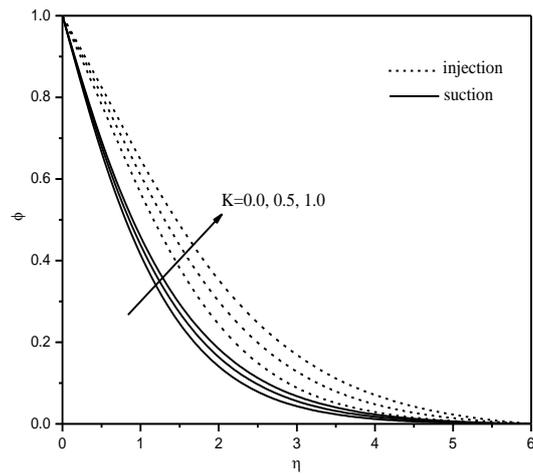


Fig. 3 (d) Concentration profiles for different values of  $K$  .

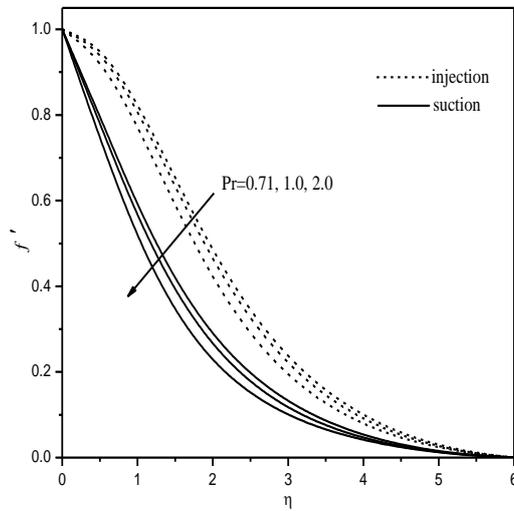


Fig. 4 (a) Velocity profiles for different values of  $Pr$

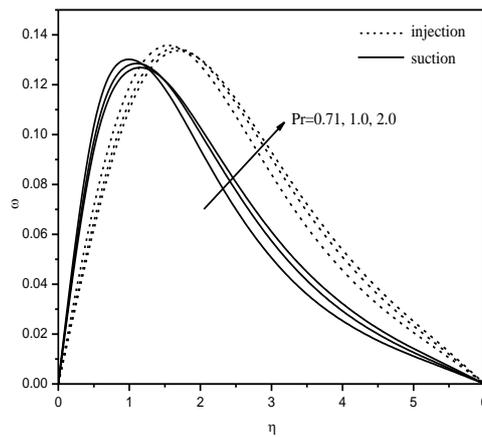


Fig. 4 (b) Micro rotation profiles for different values of  $Pr$

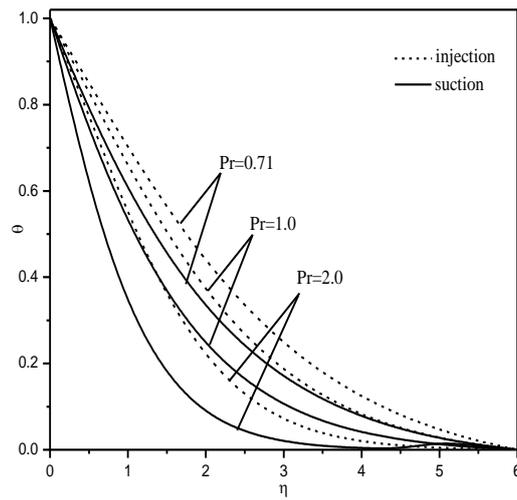


Fig. 4 (c) Temperature profiles for different values of Pr

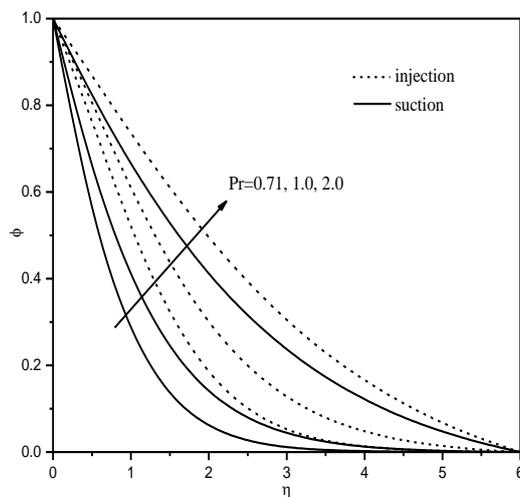


Fig. 4 (d) Concentration profiles for different values of Pr.

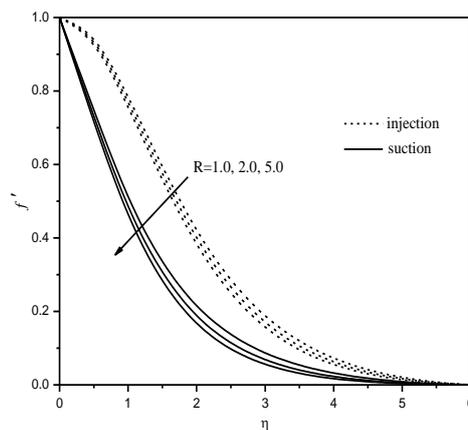


Fig. 5 (a) Velocity profiles for different values of R

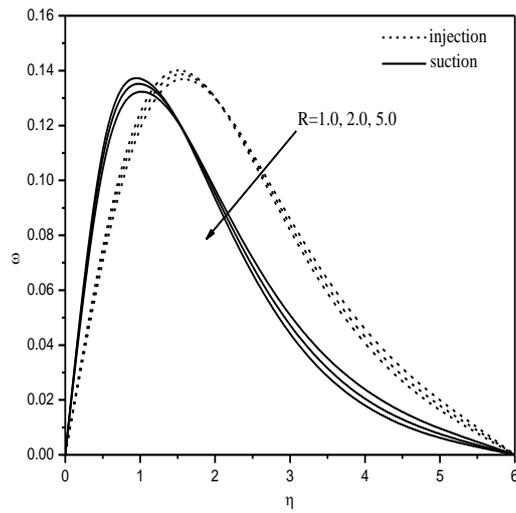


Fig. 5 (b) Micro rotation profiles for different values of  $R$

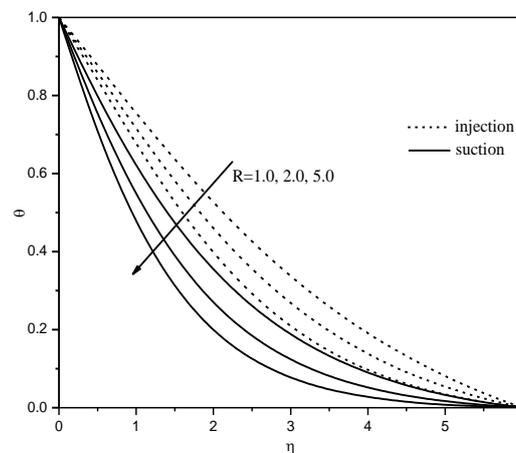


Fig. 5 (c) Temperature profiles for different values of  $R$

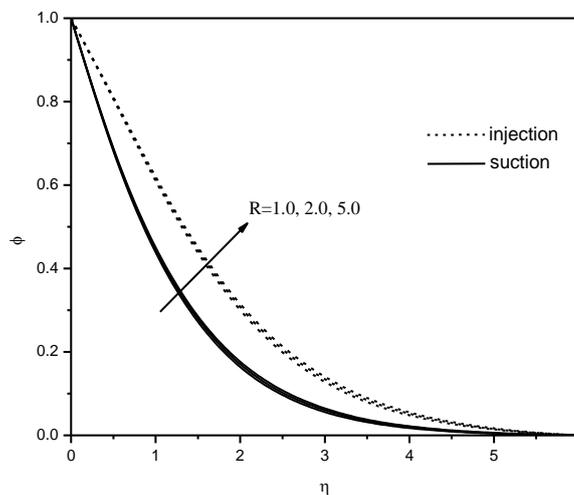


Fig.5 (d) Concentration profiles for different values of  $R$ .

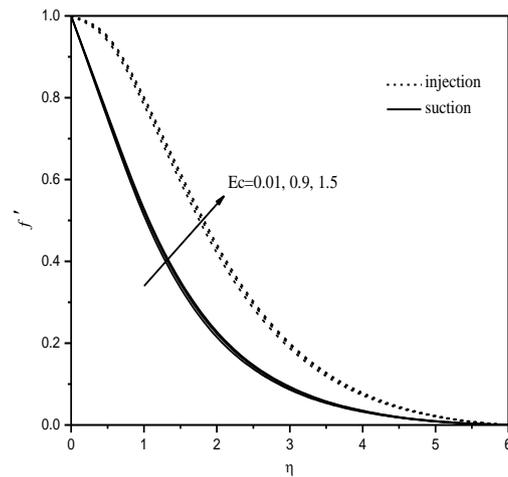


Fig. 6 (a) Velocity profiles for different values of  $Ec$

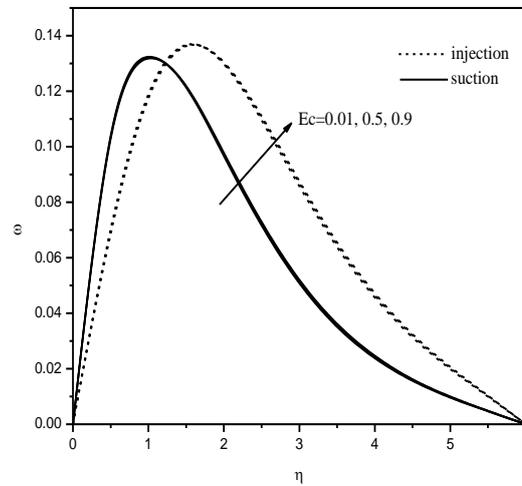


Fig. 6 (b) Micro rotation profiles for different values of  $Ec$

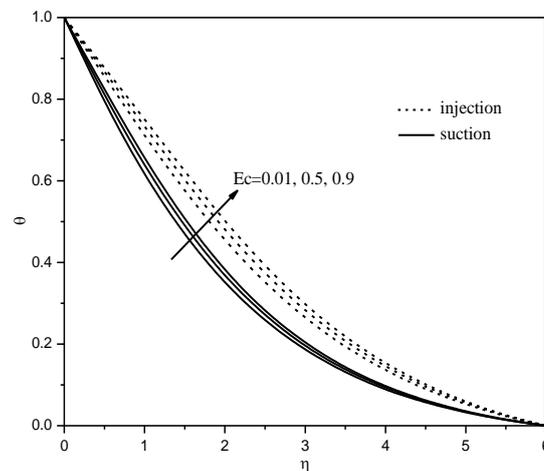


Fig. 6 (c) Temperature profiles for different values of  $Ec$  .

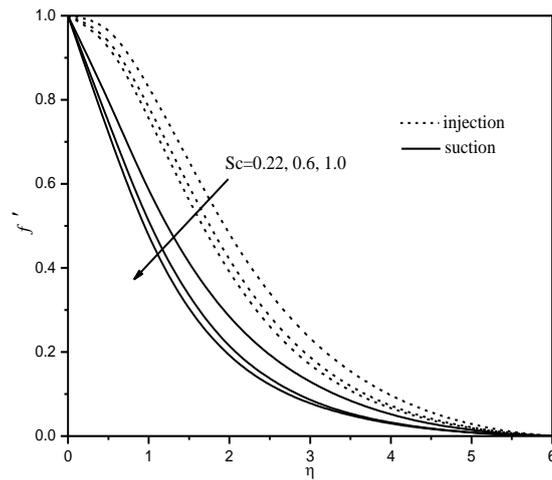


Fig. 7 (a) Velocity profiles for different values of  $Sc$

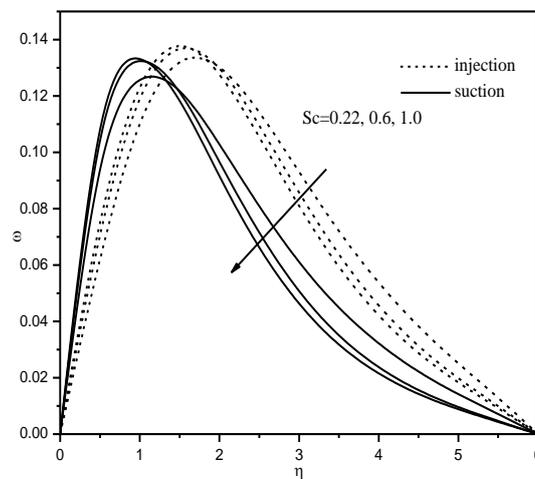


Fig. 7 (b) Micro rotation profiles for different values of  $Sc$

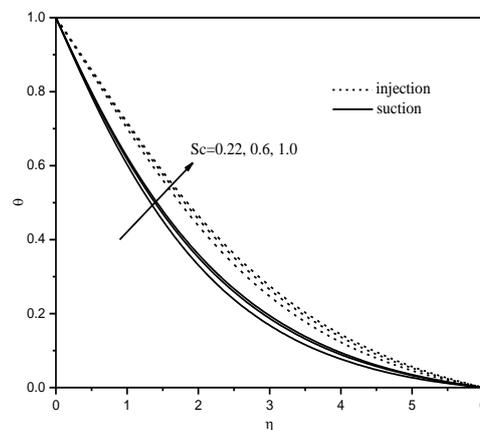


Fig. 7 (c) Temperature profiles for different values of  $Sc$

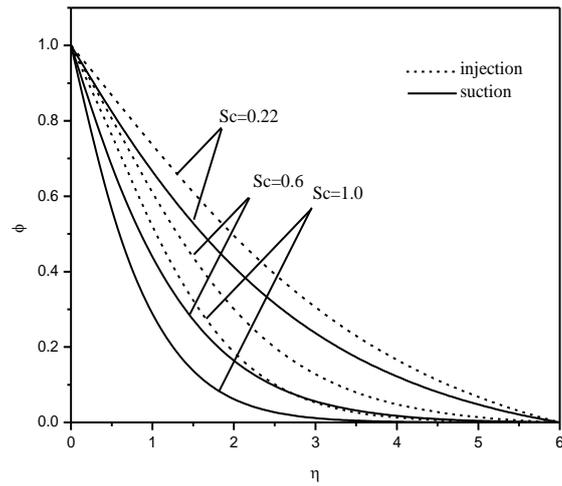


Fig. 7 (d) Concentration profiles for different values of  $Sc$ .

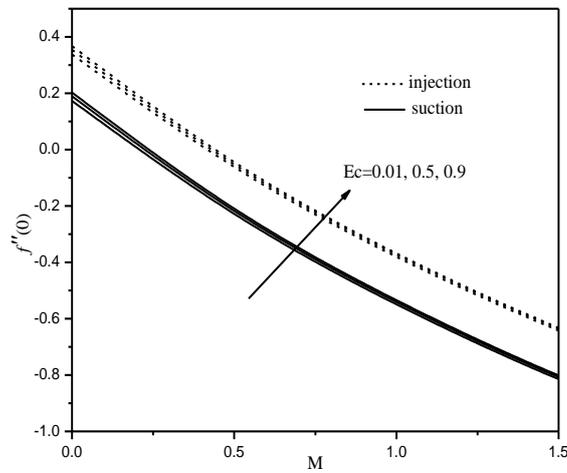


Fig. 8 (a) Local skin-friction against  $M$  for different values of  $Ec$ .

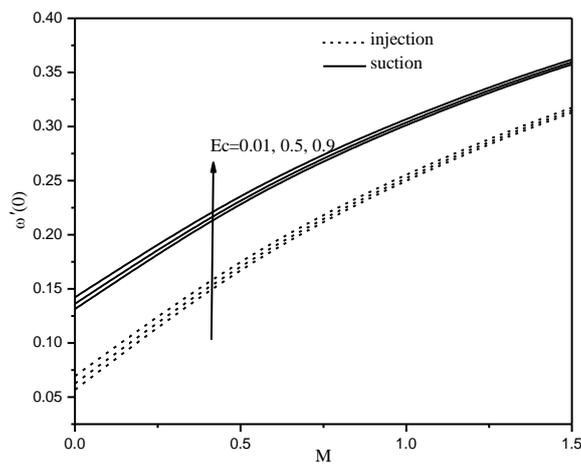


Fig. 8 (b) Couple stress against  $M$  for different values of  $Ec$ .

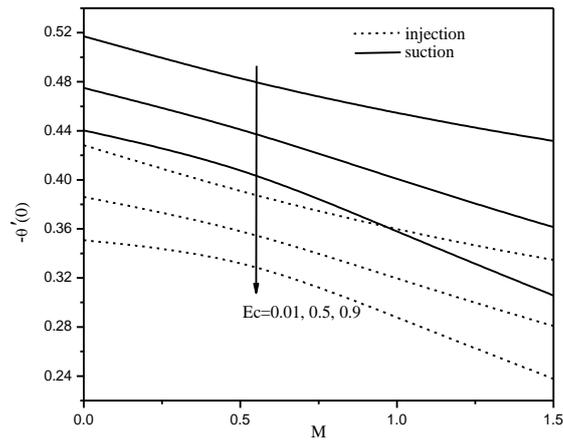


Fig. 8 (c) Local Nusselt number against  $M$  for different values of  $Ec$ .

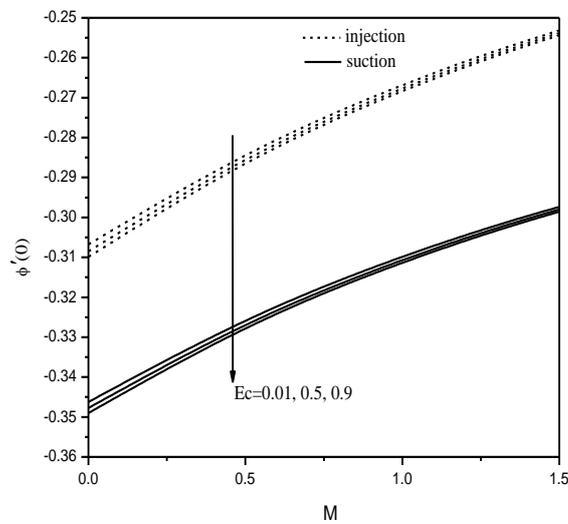


Fig. 8 (d) Local Sherwood number against  $M$  for different values of  $Ec$ .

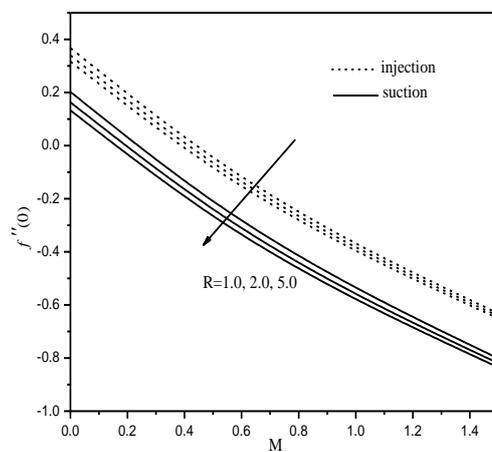


Fig. 9 (a) Local skin-friction against  $M$  for different values of  $R$ .

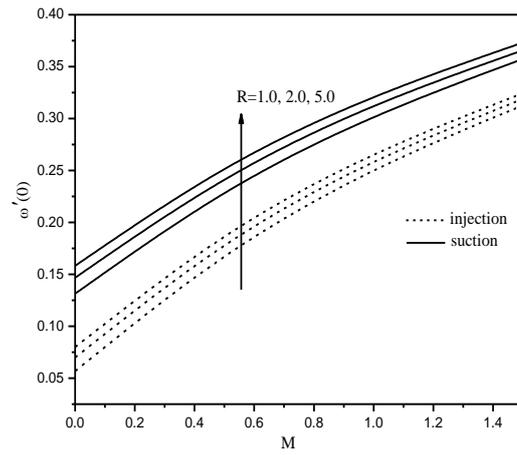


Fig. 9 (b) Couple stress against  $M$  for different values of  $R$ .

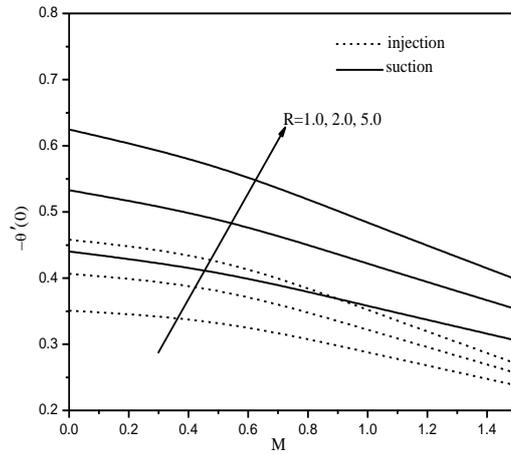


Fig. 9 (c) Local Nusselt number against  $M$  for different values of  $R$ .

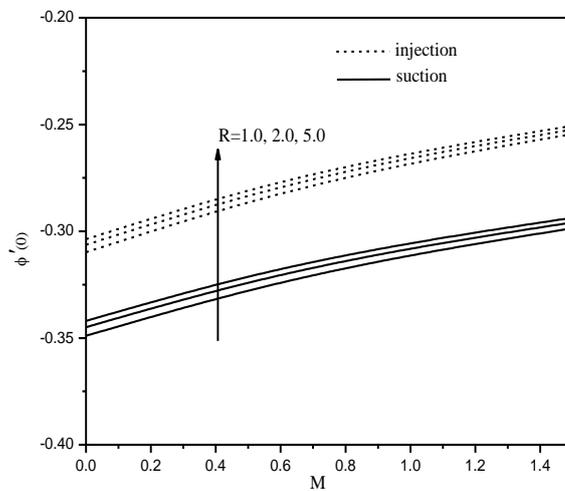


Fig. 9 (d) Local Stanton number against  $M$  for different values of  $R$ .

## V. Conclusions

In this study, we examined the effects of thermal radiation and viscous dissipation on steady MHD heat and mass transfer flow of a micropolar fluid past a continuous moving plate with suction or injection. The governing equations are solved numerically by finite difference method. We present results to illustrate the flow characteristics for the velocity, microrotation, temperature, concentration, skin – friction coefficient, couple stress coefficient, Nusselt number and Sherwood number. Based on the obtained graphical results, the following conclusions may be drawn:

- An increase in  $M$ ,  $R$  and  $Sc$  decrease the velocity distribution, while an increase in  $Ec$ , increase the velocity distribution for both fluid suction and injection.
- An increase in  $M$ ,  $R$  and  $Sc$  increases the microrotation distribution near to the plate and decreases away from plate, while a increase  $Ec$  decreases the microrotation distribution near to the plate and increases away from plate
- An increase in  $M$ ,  $Ec$  and  $Sc$  increases the temperature distribution, while an increase in  $R$ , and decrease the temperature distribution for both fluid suction and injection.
- An increase in  $M$  and  $Sc$  increases the concentration distribution, while decrease in  $R$  and  $Sc$ , decrease the concentration distribution for both fluid suction and injection.
- Increases  $R$  increases the couple stress coefficient, the heat transfer coefficient and the mass transfer coefficient whereas the skin – friction coefficient decreases for both suction and injection.
- Increases  $Ec$  increases the skin – friction coefficient and couple stress coefficient whereas the heat transfer coefficient and the mass transfer coefficient decreases for both suction and injection.

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