IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 6 Ver. III (Nov. - Dec.2016), PP 01-03
www.iosrjournals.org

Some New Linear Generating Relations Involving A-Function of One Variable

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Abstract: *The aim of this paper is to establish some new linear generating relations involving A-function of one variable.*

I. Introduction

The A-function of one variable is defined by Gautam [1] and we will represent here in the following manner:

$$A^{m,n}_{p,q}\left[x \middle| \begin{matrix}(a_p,\alpha_p)\\(b_q,\beta_q)\end{matrix}\right] = \frac{1}{2\pi i}\int_L \theta(s)\, x^s\, ds \quad (1)$$

wherei = √(– 1) and

(i)

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(a_j + s\alpha_j)\prod_{j=1}^{n}\Gamma(1 - b_j - s\beta_j)}{\prod_{j=m+1}^{p}\Gamma(1 - a_j - s\alpha_j)\prod_{j=n+1}^{q}\Gamma(b_j + s\beta_j)} \quad (2)$$

(ii) m, n, p and q are non-negative numbers in which $m \le p$, $n \le q$.

(iii) $x \neq 0$ and parameters a_j, α_j, b_k and β_k (j = 1 to p and k = 1 to q) are all complex.

The integral in the right hand side of is convergent if

(i) $x \neq 0$, $k = 0$, $h > 0$, $|\arg(ux)| < \pi h/2$

(ii) $x > 0$, $k = 0 = h$, $(\nu - \sigma\omega) < -1$

where

$$k = \text{Im}\left(\sum_1^p \alpha_j - \sum_1^q \beta_j\right)$$

$$h = \text{Re}\left(\sum_{j=1}^{m}\alpha_j - \sum_{j=m+1}^{p}\alpha_j + \sum_{j=1}^{n}\beta_j - \sum_{j=m+1}^{q}\beta_j\right)$$

$$u = \prod_1^p \alpha_j^{\alpha_j} \prod_1^q \beta_j^{\beta_j}$$

$$\nu = \text{Re}\left(\sum_1^p a_j - \sum_1^q b_j\right) - (p - q)/2 ,$$

$$w = \text{Re}\left(\sum_1^q \beta_j - \sum_1^p \alpha_j\right)$$

and $s = \sigma + it$ is on path L when $|t| \to \infty$.

II. Formula Required

From Rainville [2]:

$$(\alpha)_n = (\alpha, n) = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}, \quad (3)$$

$$\Gamma(1 - \alpha - n) = \frac{(-1)^n\Gamma(1-\alpha)}{(\alpha)_n}, \quad (4)$$

$$(1 - z)^{-a} = \sum_{n=0}^{\infty}(a)_n \frac{z^n}{n!}, \quad (5)$$

$$(1 + z)^{-a} = \sum_{n=0}^{\infty}(a)_n \frac{(-z)^n}{n!}, \quad (6)$$

III. Generating Relations

In this section we establish the following linear generating relations:

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}A_{p+1,q}^{m+1,n}\left[x\Big|_{(b_j,\beta_j)_{1,q}}^{(\lambda+r,\alpha),(a_j,\alpha_j)_{1,p}}\right]$$
$$=(1-t)^{-\lambda}A_{p+1,q}^{m+1,n}\left[x(1-t)^{-\alpha}\Big|_{(b_j,\beta_j)_{1,q}}^{(\lambda,\alpha),(a_j,\alpha_j)_{1,p}}\right]; \qquad (7)$$

|arg (ux)| < ½ πh, where u and h are given in section 1;

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}A_{p,q+1}^{m,n+1}\left[x\Big|_{(\lambda-r,\alpha),(b_j,\beta_j)_{1,q}}^{(a_j,\alpha_j)_{1,p}}\right]$$
$$=(1-t)^{\lambda-1}A_{p,q+1}^{m,n+1}\left[x(1-t)^{\alpha}\Big|_{(\lambda,\alpha),(b_j,\beta_j)_{1,q}}^{(a_j,\alpha_j)_{1,p}}\right]; \qquad (8)$$

|arg (ux)| < ½ πh, where u and h are given in section 1;

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}A_{p+1,q}^{m,n}\left[x\Big|_{(b_j,\beta_j)_{1,q}}^{(a_j,\alpha_j)_{1,p},(\lambda+r,\alpha)}\right]$$
$$=(1+t)^{-\lambda}A_{p+1,q}^{m,n}\left[x(1+t)^{-\alpha}\Big|_{(b_j,\beta_j)_{1,q}}^{(a_j,\alpha_j)_{1,p},(\lambda,\alpha)}\right]; \qquad (9)$$

|arg (ux)| < ½ πh, where u and h are given in section 1;

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}A_{p,q+1}^{m,n}\left[x\Big|_{(b_j,\beta_j)_{1,q},(1-\lambda-r,\alpha)}^{(a_j,\alpha_j)_{1,p}}\right]$$
$$=(1+t)^{-\lambda}A_{p,q+1}^{m,n}\left[x(1+t)^{\alpha}\Big|_{(b_j,\beta_j)_{1,q},(1-\lambda,\alpha)}^{(a_j,\alpha_j)_{1,p}}\right]; \qquad (10)$$

|arg (ux)| < ½ πh, where u and h are given in section 1;

Proof:
To prove (7), consider

$$\Delta=\sum_{r=0}^{\infty}\frac{t^r}{r!}A_{p+1,q}^{m+1,n}\left[x\Big|_{(b_j,\beta_j)_{1,q}}^{(\lambda+r,\alpha),(a_j,\alpha_j)_{1,p}}\right]$$

On expressing A-function in contour integral form as given in (1), we get

$$\Delta=\sum_{r=0}^{\infty}\frac{t^r}{r!}\{\frac{1}{2\pi i}\int_L \theta(s)x^s\Gamma(\lambda+r+\alpha s)\,ds\}$$

$$=\sum_{r=0}^{\infty}\frac{(t)^r}{r!}\left\{\frac{1}{2\pi i}\int_L \theta(s)x^s(\lambda+\alpha s)_r\Gamma(\lambda+\alpha s)\,ds\right\}.$$

On changing the order of summation and integration, we have

$$\Delta=\frac{1}{2\pi i}\int_L \theta(s)x^s\Gamma(\lambda+\alpha s)\{\sum_{r=0}^{\infty}\frac{(t)^r}{r!}(\lambda+\alpha s)_r\}ds$$

$$=(1-t)^{-\lambda}\frac{1}{2\pi i}\int_L \theta(s)x^s\Gamma(\lambda+\alpha s)(1-t)^{-\alpha s}ds,$$

which in view of (1), provides (7).
Proceeding on similar lines as above, the results (8) to (10) can be derived with the help of formulae given in the section (1).

References

[1]. Gautam, G. P. and Goyel, A. N.: Ind. J. Pure and Appl. Math. 1981, 12, 1094-1105.
[2]. Rainville, E. D.: Special Functions, Macmillan, NewYork, 1960.