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Effect of Thermal Dispersion and Thermal Radiation on Boundary Payer Flow of Mhd Nanofluid With Variable Suction

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Abstract: *The present study investigates the unsteady, two dimensional, laminar flow of a nanofluid over a stretching sheet with thermal dispersion in the presence of MHD and variable suction is studied. Two water - based nanofluids containing Copper Cu, Titanium dioxide* TiO_2 *nanoparticles are considered in this study. Using the similarity transformations, the governing equations have been transformed into a system of ordinary differential equations. These differential equations are highly nonlinear which cannot be solved analytically. Therefore, Runge–Kutta Gill method together with shooting technique has been used for solving it. Numerical results are obtained for the skin-friction coefficient and the local Nusselt number as well as the velocity and temperature profiles for different values of the governing parameters, namely, unsteadiness parameter, solid volume fraction, magnetic parameter, suction parameter, thermal dispersion parameter, thermal radiation parameter and Prandtl number.*
Keywords: *Nanofluid, thermal dispersion, suction, MHD, thermal radiation.*

I. Introduction

The development of a boundary layer over a stretching sheet was first studied by Crane [1], who found an exact solution for the flow field. This problem was then extended by Gupta and Gupta [2] to a permeable surface. Salleh et al. [3] studied the boundary layer flow and heat transfer analysis past a stretching sheet in the presence of Newtonian heating using finite difference method. Variable thickness on boundary layer flow over a stretching sheet is studied by Fang et al.[4]. Khader and Megahed [5] both are studied the effects of variable thickness and slip velocity on boundary layer flow towards a stretching sheet.

Magneto-hydrodynamic (MHD) boundary layers with heat and mass transfer over flat surfaces are found in many engineering and geophysical applications such as geothermal reservoirs, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors. Many chemical engineering processes like metallurgical and polymer extrusion processes involve cooling of a molten liquid being stretched into a cooling system. The fluid mechanical properties of the penultimate product depend mainly on the cooling liquid used and the rate of stretching. Some polymer liquids like polyethylene oxide and polyisobutylene solution in cetane, having better electromagnetic properties are normally used as cooling liquid as their flow can be regulated by external magnetic fields in order to improve the quality of the final product. A comprehensive review on the subject to the above problem has been made by many researchers (Yang et al., [6]; Trevisan and Bejan., [7]; Sparrow et al., [8]; Evans, [9]). Gangadbhar et al. [10] investigated the effect if hydromagnetic and chemical reaction on heat and mass transfer through a vertical plate with convective condition. Gangadhar and Baskar Reddy [11] studied the similarity solution of magneto hydrodynamic boundary layer flow of heat and mass transfer through a moving vertical plate in a porous medium with suction. They concluded that magnetohydrodynamic effect significantly decreases the velocity boundary layer thickness. Gangadhar [12] investigated the hydro magnetic boundary layer flow of heat and mass transfer with soret and dufour effects. Rushi Kumar and Gangadhar [13] studied the effects of variable suction and heat generation effects on MHD boundary layer flow of a moving vertical plate. They concluded that magnetic parameter reduces the velocity profiles. Rushi Kumar and Gangadhar [14] investigated the free convection boundary layer flow between two parallel porous walls with MHD effect.

In recent years tremendous effort has been given to the study of nanofluids. The word nanofluid coined by Choi [15] describes a liquid suspension containing ultra fine particles (diameter less than 50 nm). Experimental studies (e.g., Masuda et al. [16], Das et al. [17], Xuan and Li [18]) showed that even with a small volumetric fraction of nanoparticles (usually less than 5%), the thermal conductivity of the base liquid is enhanced by 10-50% with a remarkable improvement in the convective heat transfer coefficient. The literature on nanofluids was reviewed by Trisaksri and Wongwises [19], Wang and Mujumdar [20] among several others.

Bachok et al. [21] investigated the dual solutions on boundary layer flow of nanofluid over a moving surface in a flowing fluid.

The study of radiation effects has important applications in engineering. Thermal radiation effect plays a significant role in controlling heat transfer process in polymer processing industry. Many studies have been reported on flow and heat transfer over a stretched surface in the presence of radiation (see El-Aziz [22], Raptis [23], Mahmoud [24]). The effect of thermal radiation and MHD boundary layer flow of the Blasius and Sakiadis flows with heat generation and viscous dissipation is studied by Gangadhar [25]. In another study, Gangadhar [26] investigated the laminar boundary layer flow of nanofluid past a vertical plate in the presence of radiation and viscous dissipation. He concluded that solid volume fraction increases the thermal boundary layer thickness. The effect of radiation and chemical reaction on MHD oscillatory flow in a channel filled with porous medium is studied by Ibrahim et al.[27] Suneetha and Gangadhar [28] studied the similarity solution for MHD boundary layer flow of a Carreau fluid with thermal radiation effect. They concluded that Nusselt number reduces with an increasing the radiation parameter. Kameswaran et al. [29] studied the effects of radiation and thermal dispersion effects on boundary layer flow of nanofluid. They considered water – based copper oxide CuO, Aluminium oxide Al_2O_3 and titanium dioxide TiO_2 nanoparticles.

The present study investigates the unsteady, two dimensional, laminar flow of a nanofluid over a stretching sheet with thermal dispersion in the presence of MHD and variable suction is studied. Two water - based nanofluids containing Copper Cu, Titanium dioxide TiO_2 nanoparticles are considered in this study. Using the similarity transformations, the governing equations have been transformed into a set of ordinary differential equations, which are nonlinear and cannot be solved analytically, therefore, Runge–Kutta Gill method together with shooting technique has been used for solving it. The results for velocity and temperature functions are carried out for the wide range of important parameters namely; namely, unsteadiness parameter, magnetic parameter, suction parameter, thermal dispersion parameter, thermal radiation parameter and Prandtl number. The skin friction coefficient and the rate of heat transfer have also been computed.

II. Mathematical Formulation

Consider the unsteady two-dimensional boundary layer flow over a stretching sheet in an electrically conducting water-based nanofluid containing different type of nanoparticles: copper (Cu) and titanium dioxide (TiO_2). The origin of the system is located at the slit from which the sheet is drawn, with x and y denoting coordinates along and normal to the sheet. The thermo-physical properties of the nanofluid are specified in Table-1.

Table 1: Thermo-physical properties of water and nanoparticles (Oztop and Abu-Nada [30])

Physical properties	Water/base fluid	Cu	TiO_2
ρ (kg/m^3)	997.1	8933	4250
c_p (J/kg K)	4179	385	686.2
k (w/m K)	0.613	401	8.9538
ϕ	0.0	0.05	0.2
σ (S/m)	5.5×10^{-6}	59.6×10^{6}	2.6×10^{6}

The sheet is stretched with velocity

$$U_w(x,t) = \frac{bx}{1-\alpha t} \tag{2.1}$$

Along the x – axis, b and α are positive constants with dimensions (time)$^{-1}$ and αt<1. The surface temperature distribution

$$T_w(x,t) = T_\infty + T_0\left[\frac{bx^{r_1}}{\upsilon_f}\right](1-\alpha t)^{-m} \tag{2.2}$$

Various both along the sheet and with time, where T_0 reference temperature, T_∞ is the ambient temperature and υ_f is the kinematic viscosity of the fluid. A uniform transverse magnetic field of strength B_0 is applied parallel to the y-axis. It is assumed that induced magnetic field produced by the fluid motion is negligible in comparison with the applied one so that we consider the magnetic field $\vec{B} = (0,0,B_0)$. This assumption is justified, since the magnetic Reynolds number is very small for metallic liquids and partially ionized fluids (Cramer and Pai [31]). Also, no external electric field is applied such that the effect of polarization of fluid is negligible (Cramer and Pai [21]), so we assume $\vec{E}=(0,0,0)$. Under these assumptions, the boundary layer equations governing the flow, heat and concentration fields can be written in a dimensional form as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.3}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_{nf}B_0^2}{\rho_{nf}}u \tag{2.4}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\left(\rho c_p\right)_{nf}}\frac{\partial q_r}{\partial y}+\frac{\partial}{\partial y}\left(\alpha_e\frac{\partial T}{\partial y}\right) \tag{2.5}$$

The suitable boundary conditions are

$$y=0:\ u=U_w(x,t), v=v_w, T=T_w$$
$$y\to\infty:\ u\to 0, T\to T_\infty \tag{2.6}$$

where u and v are the velocity components along the x and y axes, respectively, v_w is the mass transfer velocity, T is the temperature of the nanofluid, T_∞ is the ambient temperature, c_p is the specific heat at constant pressure, r_1 and m are constants, the expression for the effective thermal diffusivity taken as $\alpha_e=\alpha_m+\gamma du, \alpha_m$ is the molecular thermal diffusivity, γdu represent thermal diffusivity, γ is the mechanical thermal dispersion coefficient and d is the pore diameter, μ_{nf} is the viscosity of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid and ρ_{nf} is the density of the nanofluid, which are given by (Oztop and Abu-Nada [30]).

$$\alpha_{nf}=\frac{k_{nf}}{\left(\rho C_p\right)_{nf}}, \rho_{nf}=(1-\phi)\rho_f+\phi\rho_s, \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}$$

$$\left(\rho c_p\right)_{nf}=(1-\phi)\left(\rho c_p\right)_f+\phi\left(\rho c_p\right)_s, \frac{k_{nf}}{k_f}=\frac{\left(k_s+2k_f\right)-2\phi\left(k_f-k_s\right)}{\left(k_s+2k_f\right)+\phi\left(k_f-k_s\right)} \tag{2.7}$$

$$\sigma_{nf}=\sigma_f\left[1+\frac{3(\sigma-1)\phi}{(\sigma+2)-(\sigma-1)\phi}\right], \sigma=\frac{\sigma_s}{\sigma_f}$$

Here, ϕ is the nanoparticle volume fraction, $\left(\rho c_p\right)_{nf}$ is the heat capacity of the nanofluid, σ_f is the electrical conductivity of the base fluid, σ_s is the electrical conductivity of the nanoparticle, k_{nf} is the thermal conductivity of the nanofluid, k_f and k_s are the thermal conductivities of the fluid and of the solid fractions, respectively, and ρ_f and ρ_s are the densities of the fluid and of the solid fractions, respectively. It should be mentioned that the use of the above expression for k_{nf} is restricted to spherical nanoparticles where it does not account for other shapes of nanoparticles (Oztop and Abu-Nada [30]). Also, the viscosity of the nanofluid μ_{nf} has been approximated as viscosity of a base fluid μ_f containing dilute suspension of fine spherical particles (Brinkman [32]).

Following Rosseland's approximation, T^4 is expressed as a linear function of the temperature $T^4\cong 4T_\infty^3 T-3T_\infty^4$ and the radiative heat flux q_r is modeled as

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} \tag{2.8}$$

Where α_m is the Stefan – Boltzman constant and α_m is the mean absorption coefficient. The continuity equation (2.3) is satisfied by the Cauchy-Riemann equations

$$u=\psi_y \quad \text{and} \quad v=-\psi_x \tag{2.9}$$

where $\psi(x,y)$ is the stream function.

the following similarity transformations and dimensionless variables are introduced.

$$\psi(x,y,t)=\left(\frac{\upsilon_f b}{1-\alpha t}\right)^{1/2} xf(\eta), \eta=\left(\frac{b}{\upsilon_f(1-\alpha t)}\right)^{1/2} y, u=\left(\frac{bx}{1-\alpha t}\right) f'(\eta)$$

$$v=-\left(\frac{\upsilon_f b}{1-\alpha t}\right)^{1/2} f(\eta), \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{2.10}$$

where η is the similarity variable, υ_f is the kinematic viscosity of the fluid fraction and b is a constant.

After the substitution of these transformations (2.7) - (2.10) into the equations (2.4) - (2.6), the resulting non-linear ordinary differential equations are written as

$$f'''(\eta)+A_1\left[f(\eta)f''(\eta)-f'(\eta)^2-S\left(\frac{\eta}{2}f''(\eta)+f'(\eta)\right)\right]-(1-\Phi)^{2.5}Haf'(\eta)=0 \tag{2.11}$$

$$\theta''(\eta)\left[1+Df'(\eta)+\frac{k_R}{A_2}\left(\frac{k_{nf}}{k_f}\right)\right]+Df''(\eta)\theta'(\eta)+\Pr\left[f(\eta)\theta'(\eta)-r_1 f'(\eta)\theta(\eta)-S\left(\frac{\eta}{2}\theta'(\eta)+m\theta(\eta)\right)\right]=0 \tag{2.12}$$

where

$$k_R=1+\frac{4}{3Nr}$$

$$A_1=(1-\phi)^{2.5}\left[1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)\right]$$

$$A_2=\left[1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right]$$

$$A_3=1+\frac{3(\sigma-1)\phi}{(\sigma+2)-(\sigma-1)\phi}$$

Together with the boundary conditions

$$f(0)=f_w, f'(0)=1, \theta(0)=1$$

$$f'(\eta)\to 0, \theta(\eta)\to 0 \quad as \quad \eta\to\infty \tag{2.13}$$

Here primes denote differentiation with respect to η.

S is the unsteadiness parameter, D is the thermal dispersion parameter, Nr thermal radiation parameter, $f_w>0$ is the suction parameter and $f_w<0$ corresponds to injection, $\Pr$ is the Prandtl number, M is the magnetic parameter which are given by

$$S=\frac{\alpha}{b}, D=\frac{\gamma dU_w}{\alpha_m}, N_R=\frac{k_{nf}k*}{4\sigma*T_\infty^3}, f_w=-v_w\sqrt{\frac{b\upsilon_f}{1-\alpha t}}, \Pr=\frac{\mu_f (c_p)_f}{k_f}$$

$$M=\frac{\sigma_f B_0^2}{\rho_f b} \tag{2.14}$$

The physical quantities of interest are the skin friction coefficient C_f, the couple stress C_n and the local Nusselt number Nu_x, which are defined as

$$C_f=\frac{2\tau_w}{\rho_f u_w^2}, \quad Nu_x=\frac{xq_w}{k_f(T_\infty)} \tag{2.15}$$

where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} = -\frac{1}{(1-\Phi)^{1/2}}\rho_f \upsilon_f^{1/2}\left(\frac{b}{1-\alpha t}\right)^{3/2} x f''(0)$$
$$q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k_{nf}(T_w - T_\infty)\left(\frac{b}{\upsilon_f(1-\alpha t)}\right)^{1/2}\theta'(0) \quad (2.16)$$

with μ_{nf} and k_{nf} being the dynamic viscosity and thermal conductivity of the nanofluids, respectively.
Using the similarity variables (2.7)&(2.10), we obtain

$$C_{fx}\,\mathrm{Re}_x^{1/2}(1-\Phi)^{2.5} = -2f''(0)$$
$$\frac{Nu_x}{\mathrm{Re}_x^{1/2}}\left(\frac{k_f}{k_{nf}}\right) = -\theta'(0) \quad (2.17)$$

where $\mathrm{Re}_x = \frac{U_w x}{\upsilon_f}$ is the local Reynolds number.

III. Solution of The Problem

For solving Eqs. (2.11) – (2.13), a step by step integration method i.e. Runge–Kutta method has been applied. For carrying in the numerical integration, the equations are reduced to a set of first order differential equation. For performing this we make the following substitutions:

$$y_1 = f, y_2 = f', y_3 = f'', y_4 = h, y_5 = h', y_6 = \theta, y_7 = \theta'$$
$$y_3' = -A_1\left[y_1y_3 - y_2^2 - S\left(\frac{\eta}{2}y_3 + y_2\right)\right] + (1-\Phi)^{2.5}A_3 Hay_2$$
$$y_5' = \frac{-1}{1 + Dy_2 + \frac{k_R}{A_2}\left(\frac{k_{nf}}{k_f}\right)}\left[Dy_3y_5 + \Pr\left(y_1y_5 - r_1y_2y_4 - S\left(\frac{\eta}{2}y_5 + my_4\right)\right)\right]$$
$$y_1(0) = f_w, y_2(0) = 1, y_4(0) = 1$$
$$y_2(\infty) = 0, y_4(\infty) = 0$$

In order to carry out the step by step integration of Eqs. Refspseqn 2.11-2.13, Gills procedures as given in Ralston and Wilf [33] have been used. To start the integration it is necessary to provide all the values of y_1, y_2, y_3, y_4 at $\eta = 0$ from which point, the forward integration has been carried out but from the boundary conditions it is seen that the values of y_3, y_5 are not known. So we are to provide such values of y_3, y_5 along with the known values of the other function at $\eta = 0$ as would satisfy the boundary conditions as $\eta \to \infty(\eta = 10)$ to a prescribed accuracy after step by step integrations are performed. Since the values of y_3, y_5 which are supplied are merely rough values, some corrections have to be made in these values in order that the boundary conditions to $\eta \to \infty$ are satisfied. These corrections in the values of y_3, y_5 are taken care of by a self-iterative procedure which can for convenience be called ''Corrective procedure''. This procedure has been taken care of by the software which has been used to implement R–K method with shooting technique.

As regards the error, local error for the 4th order R–K method is $O(h^5)$; the global error would be $O(h^4)$. The method is computationally more efficient than the other methods. In our work, the step size $h = 0.01$. Therefore, the accuracy of computation and the convergence criteria are evident. By reducing the step size better result is not expected due to more computational steps vis-a` -vis accumulation of error.

IV. Results And Discussion

The governing equations (2.11) & (2.12) subject to the boundary conditions (2.13) are integrated as described in section 3. In order to get a clear insight of the physical problem, the velocity and temperature have been discussed by assigning numerical values to the parameters encountered in the problem.

The variation of velocity and temperature profiles for different values of magnetic parameter *Ha* is graphed in figures 1 and 2 for both Cu – water nanofluid and TiO_2 – water nanofluid. From the graphs, in is seen that velocity profile significantly decreases with an increase in magnetic parameter *Ha*. As magnetic parameter increases, the Lorentz force, which opposes the flow, and increases and leads to, enhanced deceleration of the flow. But the temperature profile increases with the increase in magnetic parameter. For the velocity distribution TiO_2 – water nanofluid is higher than that of Cu-water nanofluid. Under consideration of temperature distribution Cu-water nanofluid is higher than that of TiO_2 – water nanofluid because thermal conductivity of the Cu-water nanofluid is higher than that of TiO_2 – water nanofluid. Figures 3 and 4 are graphed for different values of solid volume fraction Φ on the velocity and temperature profiles for both Cu-water nanofluid and TiO_2 – water nanofluid. From these figures, it is observed that both velocity and temperature distributions increases with an increase in solid volume fraction Φ. Figures 5 and 6 depict the variation of suction parameter f_w on velocity and temperature profiles for both nanofluids. It is clearly observed that both velocity and temperature profiles significantly reduce with an increasing the values of suction parameter. Suction will leads fast cooling of the surface. Figures 7 and 8 is graphed for velocity and temperature profiles for different values unsteadiness parameter *S*. it is noticed that velocity profile decrease whereas temperature profile significantly reduces with the rising the values of unsteadiness parameter *S*. The variation in thermal dispersion parameter *D* on temeperature distribution for both Cu-water nanofluid and TiO_2 – water nanofluid is shown in figure 9. It is noticed that thermal dispersion increases, the temperature profiles increases. Figure 10 depicts the effect of thermal radiation parameter *Nr* on temperature profiles for both nanofluids. It is observed that thermal boundary layer thickness increase with the influence of thermal radiation. Figure 11 is plotted for different values of unsteadiness parameter *S*, suction parameter f_w, magnetic parameter *Ha* and solid volume fraction Φ on skin friction coefficient for both Cu – water nanofluid and TiO_2 – water nanofluid. From the figure, it is noticed that skin friction coefficient significantly increase for increase in *S*, f_w and *Ha*, whereas skin friction reduces with the rise in Φ. The local skin friction coefficient is Cu – water nanofluid higher than that of TiO_2 – water nanofluid. Figure 12 is graphed for different values of unsteadiness parameter *S*, suction parameter f_w, magnetic parameter *Ha* and solid volume fraction Φ on local Nusselt number for both Cu – water nanofluid and TiO_2 – water nanofluid. From the figure, it is noticed that local Nusselt number significantly increase for increase in f_w, whereas local Nusselt number reduces with the rise in *S*, *Ha* and Φ. The local Nusselt number is higher TiO_2 – water nanofluid than that of Cu – water nanofluid. The variation in thermal dispersion parameter *D* and thermal radiation parameter *Nr* on local Nusselt number for both nanofluids is shown figure 13. It is clearly observed that local Nusselt number reduces with an increase in both *D* and *Nr*. Table 1 shows the very good agreement with previously published results.

V. Conclusions

In the present paper, unsteady, two dimensional, laminar flow of a nanofluid over a stretching sheet with thermal dispersion in the presence of MHD and variable suction is studied. Two water - based nanofluids containing copper Cu, Titanium dioxide TiO_2 nanoparticles are considered in this study. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. It has been found that velocity profiles decrease but temperature profiles increases with the influence of magnetic field. For the velocity distribution TiO_2 – water nanofluid is higher than that of Cu-water nanofluid. Under consideration of temperature distribution Cu-water nanofluid is higher than that of TiO_2 – water nanofluid. Both velocity and temperature profiles increases with an increase in solid volume fraction. Both the thermal dispersion and thermal radiation increases the thermal boundary layer thickness. The local skin friction coefficient is Cu – water nanofluid higher than that of TiO_2 – water nanofluid. The local Nusselt number is higher TiO_2 – water nanofluid than that of Cu – water nanofluid. The local Nusselt number decreases with the influence of thermal radiation and thermal dispersion.

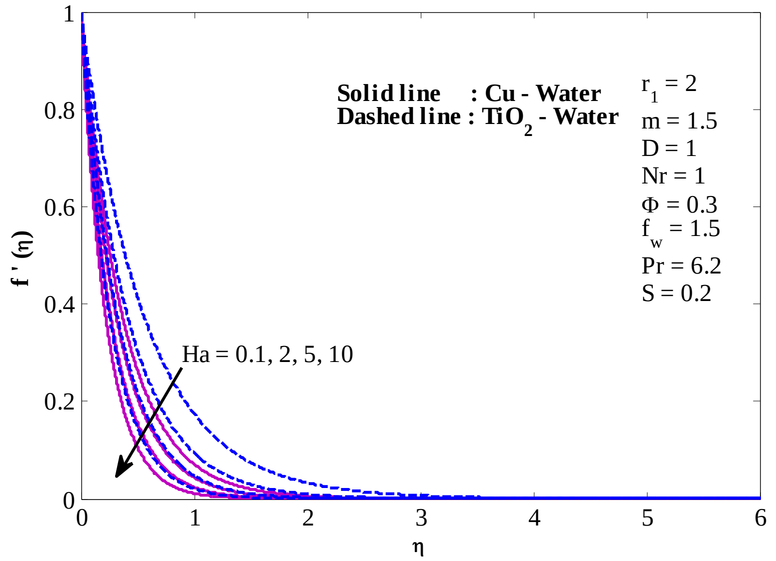

Fig.1 Velocity distribution for various values of *Ha*

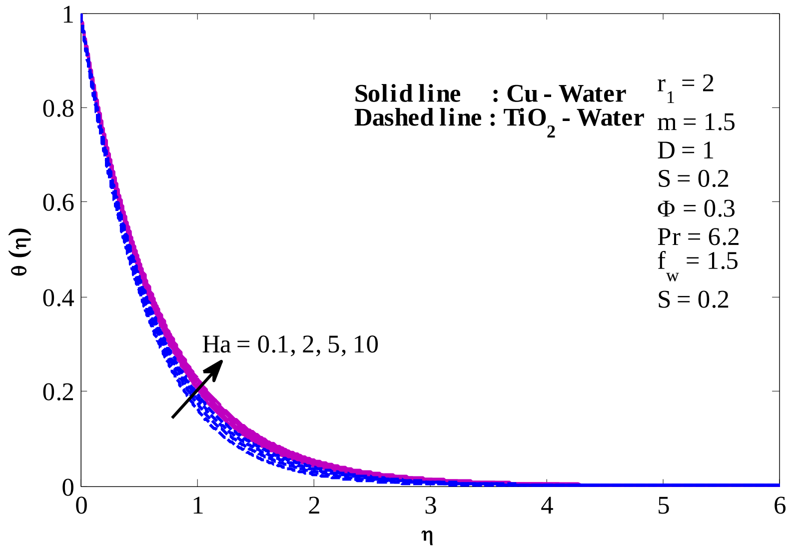

Fig.2 Temperature distribution for various values of *Ha*

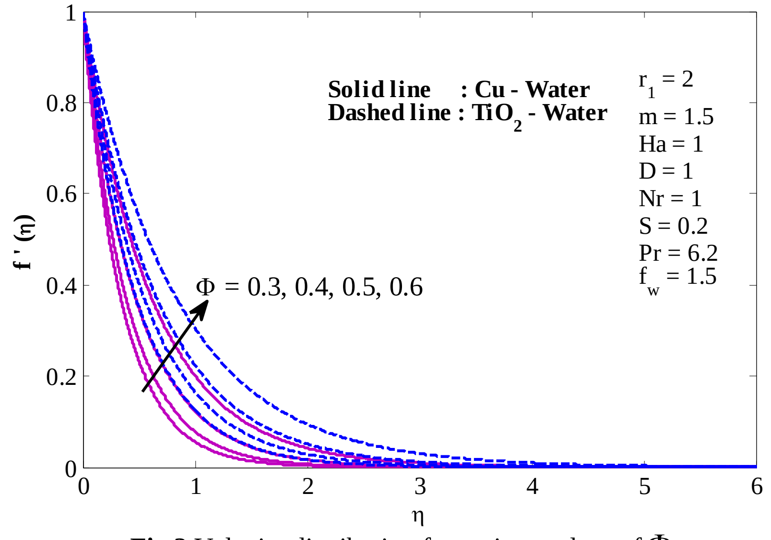

Fig.3 Velocity distribution for various values of Φ

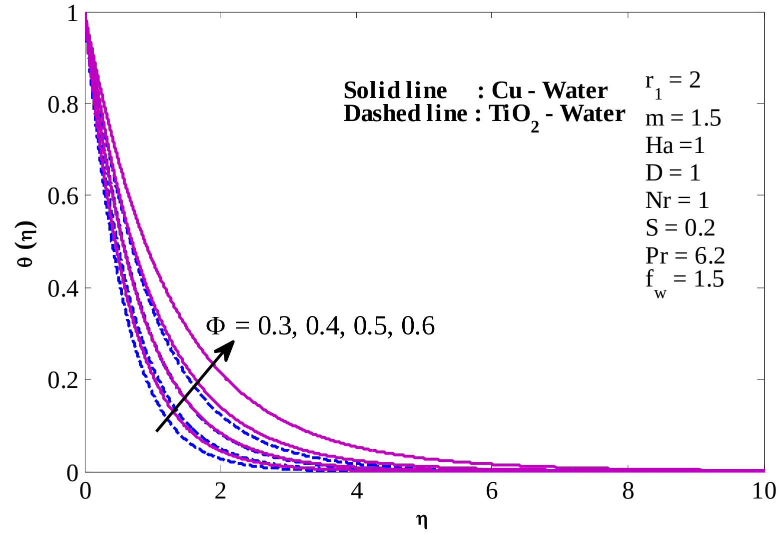

Fig.4 Temperature distribution for various values of Φ.

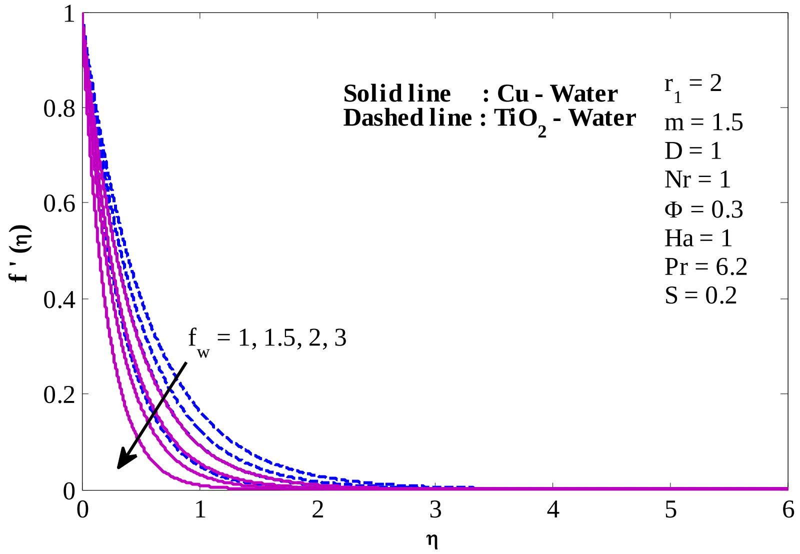

Fig.5 Velocity distribution for various values of f_w.

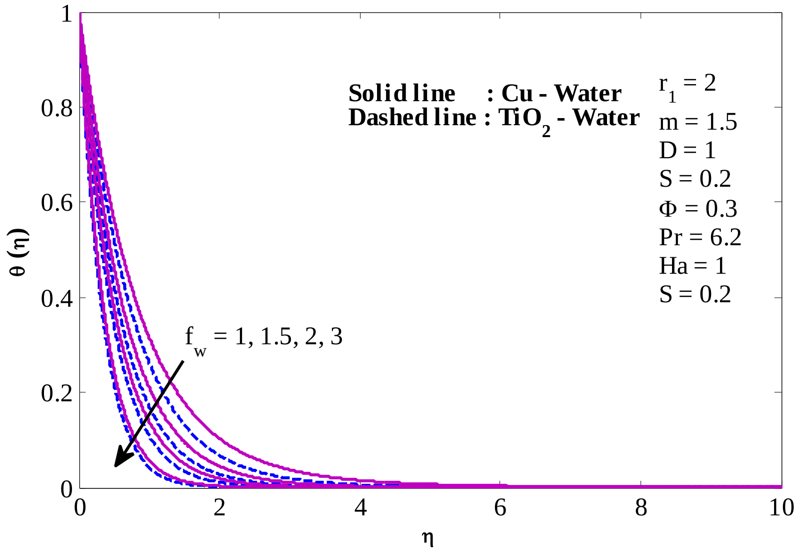

Fig.6 Temperature distribution for various values of f_w.

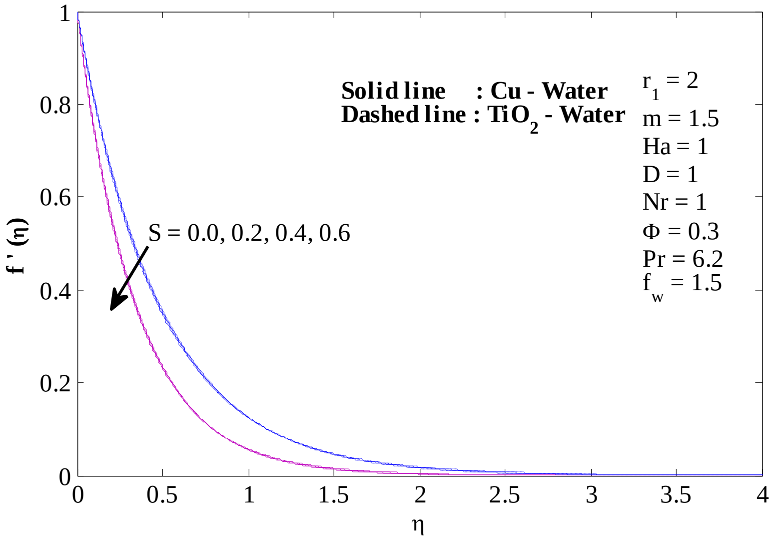

Fig.7 Velocity distribution for different values of *S*.

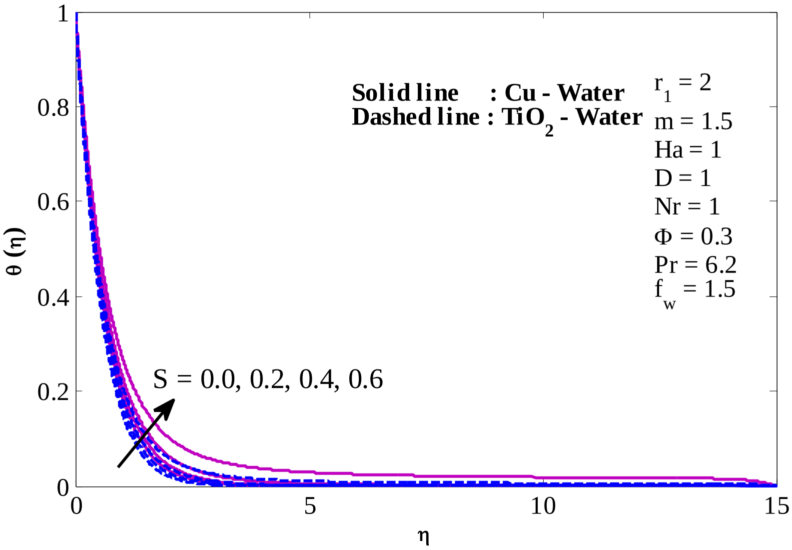

Fig.8 Temperature distribution for various values of *S*.

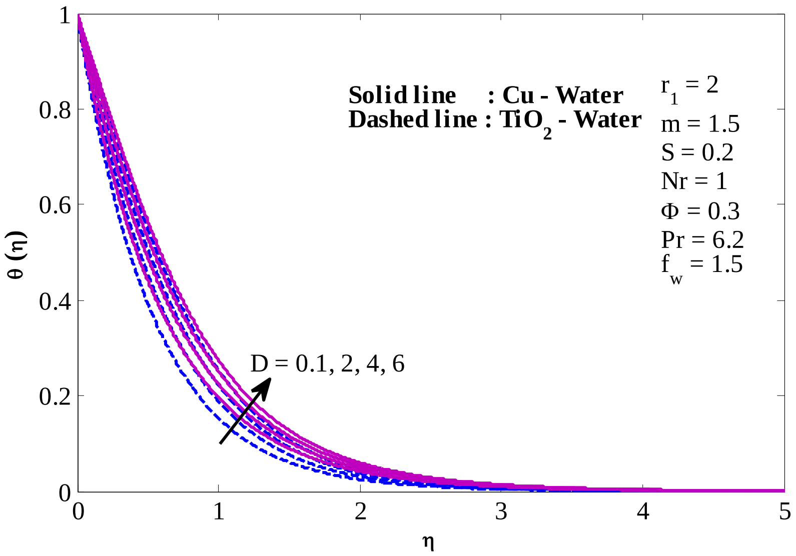

Fig.9 Temperature distribution for different values of *D*.

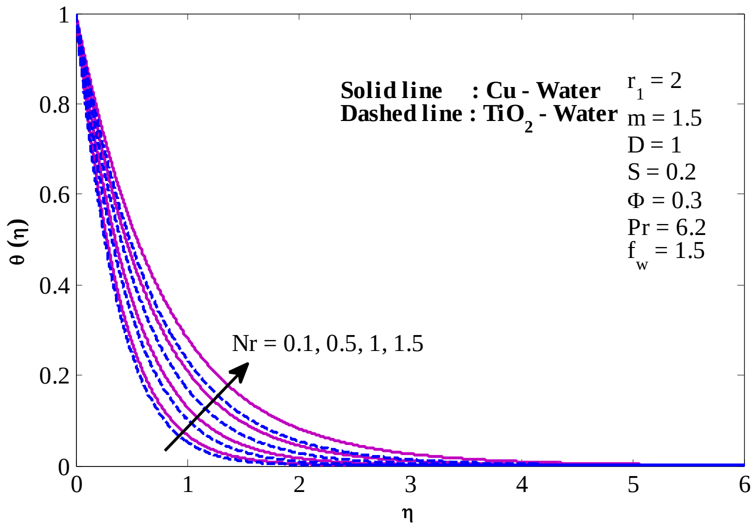

Fig.10 Temperature distribution for different values of *Nr*.

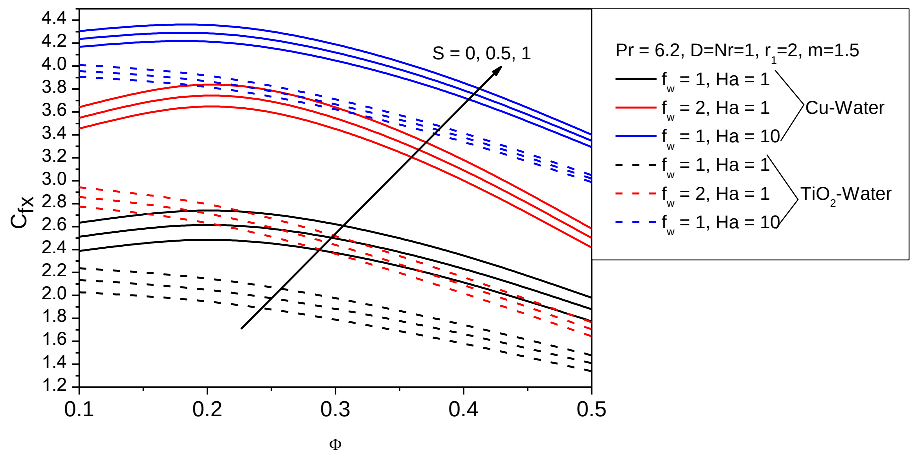

Fig.11 local Skin-friction coefficient for various values of S, f_w, Ha & Φ.

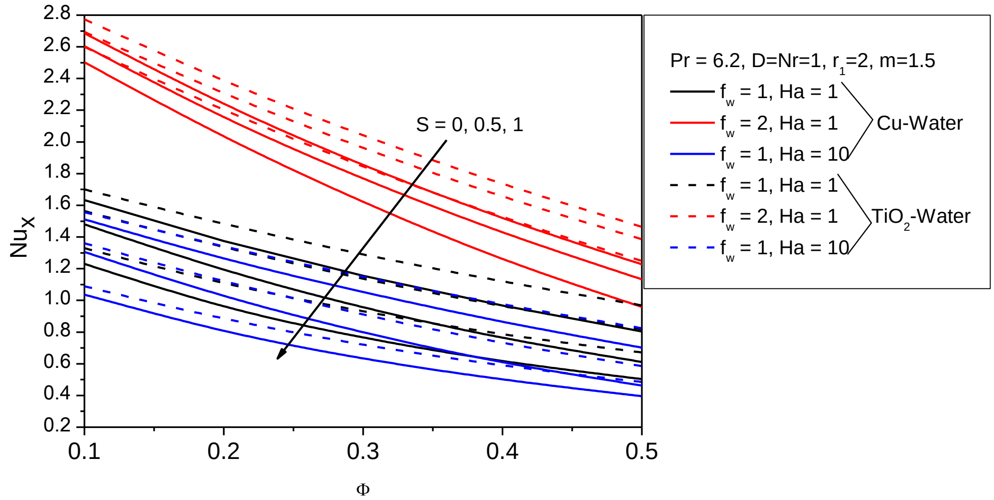

Fig.12 Local Nusselt number for different values of S, f_w, Ha & Φ.

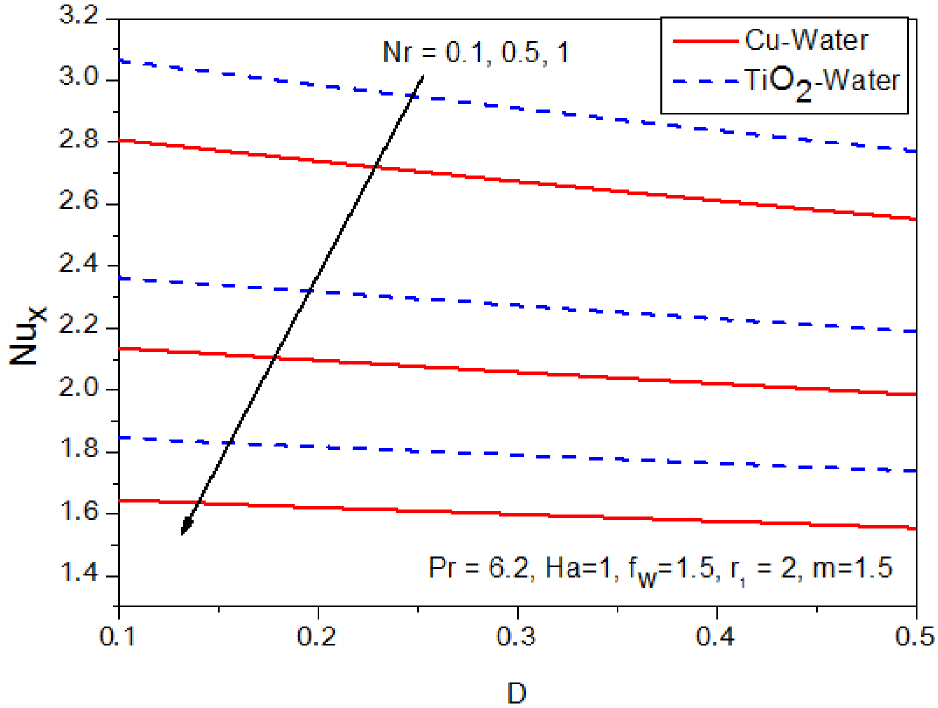

Fig.13 Local Nusselt number for different values of *Nr & D.*

Table 1. Comparison for the values of $-\theta'(0)$ for the values of *Pr*, *S* when r1 *= 2, m = 1.5, Ha = D = Nr = 0.*

Pr	*S*	$-\theta'(0)$	
		Present study	El-Aziz [33]
0.1	0.8	0.4517	0.451503
1.0	0.8	1.6728	1.672845
10	0.8	5.70503	5.705976
0.1	1.2	0.5087	0.508504
1.0	1.2	1.181	1.818005
10	1.2	6.12067	6.121024
0.1	2	0.604013	0.603518
1.0	2	2.07841	2.078413
10	2	6.88506	6.886151

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