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Prime Labeling For Some Octopus Related Graphs

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Abstract: *In this paper we investigate prime labeling for some graphs related to an octopus graph. We discuss prime labeling in the context of some graph operations namely duplication, fusion, switching in an octopus graph O_n.*
Keywords: *Prime Labeling, Prime Graph, Octopus Graph, Duplication, Fusion, Switching, Coloring.*

I. Introduction

In this paper, we consider only simple, finite, undirected and non – trivial graph $G = (V(G), E(G))$ with the vertex set $V(G)$ and the edge set $E(G)$. For notations and terminology we refer to Bondy and Murthy[1]. The notion of a prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout. A(1982 P 365 – 368)[8]. Many researchers have studied prime graph for example in Fu. H(1994 P 181 – 186)[4] have proved that the path P_n on *n* vertices is a prime graph. In Deretsky. T(1991 P 359 – 369)[3] have proved that the Cycle C_n on *n* vertices is a prime graph. Lee. S(1998 P 59 – 67)[6] have proved that Wheel W_n is a prime graph iff. *n* is even. In [7] S. Meena and K. Vaithilingam have proved the prime labeling for some Fan related graphs. For latest survey on graph labeling we refer to [5] (Gallian. J. A., 2009). Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in past four decades. In this paper, we investigate the prime labeling for some an octopus graph and its some new graph operations.

II. Preliminary Definitions

Definition [7]
Let $G = (V(G), E(G))$ be a graph with *p* vertices. A bijection $f : V(G) \to \{1, 2, \ldots, p\}$ is called a *prime labeling* if for each edge $e = uv, \gcd\{f(u), f(v)\} = 1$. A graph which admits prime labeling is called a *prime graph*.
Definition [7]
Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex $v_k{}'$ with $N(v_k{}') = N(v_k)$. In other words a vertex $v_k{}'$ is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to $v_k{}'$ also.
Definition [7]
Let *u* and *v* be two distinct vertices of a graph G. A new graph G_1 is constructed by *identifying(fusing)* two vertices *u* and *v* by a single vertex *x* is such that every edge which was incident with either *u* or *v* in G is now incident with *x* in G_1.
Definition [7]
A *vertex switching* G_v of a graph G is obtained by taking a vertex *v* of G, removing all the entire edges incident with *v* and adding edges joining *v* to every vertex which are not adjacent to *v* in G.
Definition [2]
A k – coloring of a graph $G = (V, E)$ is a function $c : V \to C$, where $|c| = k$. (Most often we use $c = [k]$). Vertices of the same color form a color class. A coloring is *proper* if adjacent vertices have different colors. A graph is *k – colorable* if there is a proper *k* – coloring. The chromatic number $\chi(G)$ of a graph G is the minimum *k* such that G is *k* – colorable.

III. Prime Labeling For Some An Octopus Related Graphs

3.1. Octopus Graph

An *Octopus graph* $O_n, (n \geq 2)$ can be constructed by a fan graph $F_n, (n \geq 2)$ joining a star graph $K_{1,n}$ with sharing a common vertex, where *n* is any positive integer. i.e., $O_n = F_n + K_{1,n}$.

Example 3.2.

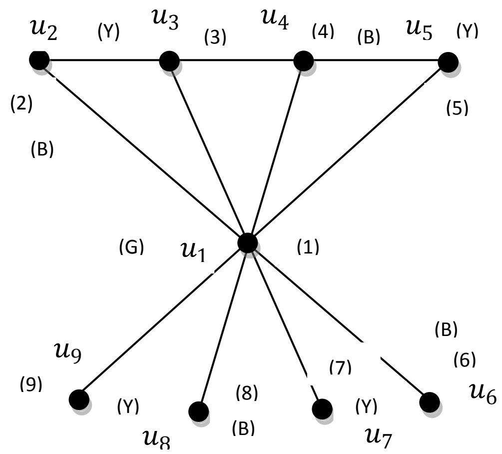

Figure 3.1. An octopus graph O_4.

Theorem 3.3. An octopus graph O_n admits prime graph, where *n* is any positive integer.
Proof. Let G be an octopus graph O_n. Let $\{u_1, u_2, \ldots, u_{2n+1}\}$ be the vertices of O_n. Let $E(O_n)$ be the edges of an octopus graph where $E(O_n) = \{u_1u_i/1 \le i \le 2n+1\} \cup \{u_iu_{i+1}/2 \le i \le n\}$. Here $|V(O_n)| = 2n+1$, where *n* is any positive integer.
Define a labeling $f : V(O_n) \to \{1, 2, \ldots, 2n+1\}$ as follows.

$$f(u_i) = i \quad \text{for } 1 \le i \le 2n+1$$

Clearly vertex labels are distinct. Then for any edge $e = u_1u_i \in O_n$, $\gcd\big(f(u_1), f(u_i)\big) = \gcd\big(1, f(u_i)\big) = 1$ for $i = 1, 2, \ldots, n, n+1, \ldots, 2n+1$ and for any edge $e = u_iu_{i+1} \in O_n$, $\gcd\big(f(u_i), f(u_{i+1})\big) = 1$ for $2 \le i \le n$. Since it is consecutive positive integers. Then f admits prime labeling. Thus O_n is a prime graph.

Example 3.4.

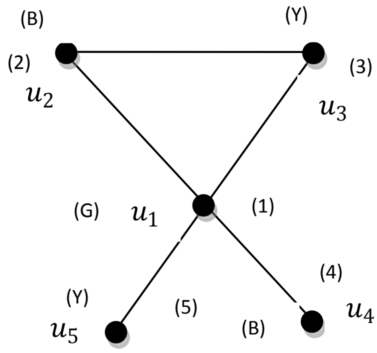

Figure 3.2. Prime labeling for O_2 .

Theorem 3.5. The graph obtained by duplicating a vertex u_k to u_k' of an octopus graph O_n is a prime graph, where *n* is any positive integer.
Proof. Let G be an octopus graph O_n. Let u_k be the vertex of an octopus graph O_n, u_k' be its duplicated vertex and G_k be the graph resulted due to duplication of the vertex u_k in O_n, where *n* is any positive integer. Let u_k' be the duplication of u_k in G_k. Then $|V(G_k)| = 2n+2$. We define a labeling $f : V(G_k) \to \{1, 2, \ldots, 2n+2\}$ as follows.

$$f(u_i) = i \quad \text{for } 1 \le i \le n+1$$
$$f(u_i) = i+1 \quad \text{for } n+2 \le i \le 2n+1$$
$$f(u_3') = 5$$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_k is a prime graph.

Example 3.6.

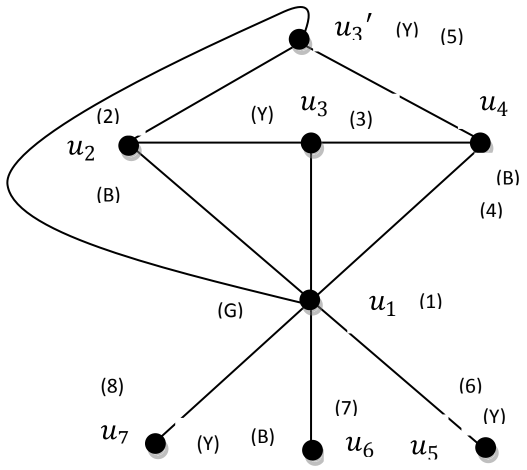

Figure 3.3. Duplication of u_3 in O_3.

Theorem 3.7. The graph obtained by duplication of a pendant vertex u_k to $u_k{'}$ of an octopus graph O_n is a prime graph, where *n* is any positive integer.

Proof. Let G be an octopus graph O_n. Let u_k be the pendant vertex of an octopus graph O_n, $u_k{'}$ be its duplicated pendant vertex and G_k be the graph resulted due to duplication of the pendant vertex u_k in O_n, where *n* is any positive integer. Let $u_k{'}$ be the duplication of u_k in G_k. Then $|V(G_k)| = 2n + 2$. We define a labeling $f : V(G_k) \to \{1, 2, \dots, 2n + 2\}$ as follows.

$$f(u_i) = i \quad \text{for } 1 \le i \le 2n + 1$$
$$f(u_k{'}) = 2n + 2$$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_k is a prime graph.

Example 3.8.

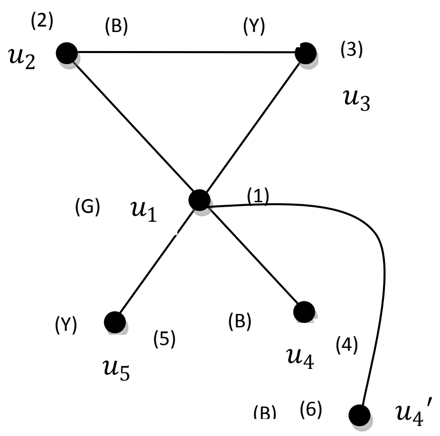

Figure 3.4. Duplication of a pendant vertex u_4 in O_2.

Theorem 3.9. The graph obtained by duplicating of an apex vertex u_1 to $u_1{'}$ in an octopus graph O_n is a prime graph, where *n* is any positive integer.

Proof. Let G be an octopus graph O_n. Let u_k be an apex vertex of an octopus graph O_n, $u_k{'}$ be its duplicated of an apex vertex and G_k be the graph resulted due to duplication of an apex vertex u_k in O_n, where *n* is any positive integer. Let G_k be the graph obtained by duplicating an apex vertex u_1 in O_n, where *n* is any positive integer. Let $u_1{'}$ be the duplication of an apex vertex u_1 in G_k. Then $|V(G_k)| = 2n + 2$.

We define a labeling $f : V(G_k) \to \{1, 2, \dots, 2n + 2\}$ as follows.

$$f(u_i) = i \quad \text{for } 1 \le i \le 2n - 2$$
$$f(u_i) = i + 1 \quad \text{for } 2n - 1 \le i \le 2n + 1$$
$$f(u_1{'}) = 7$$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_k is a prime graph.

Example 3.10.

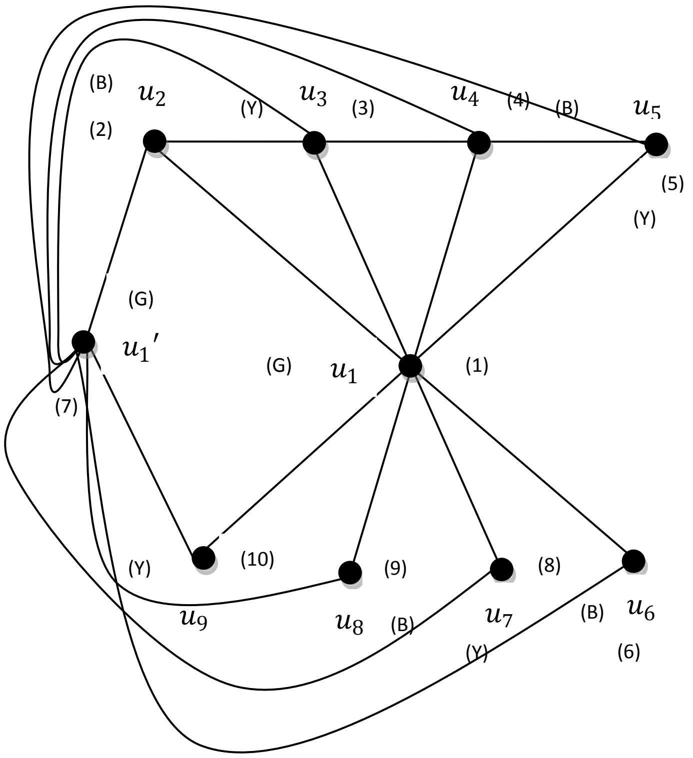

Figure 3.5. Duplication of an apex vertex u_1 in O_4.

Theorem 3.11. The graph obtained by fusing the vertex u_i with u_k (where $d(u_i, u_k) \geq 3$) in an octopus graph O_n is a prime graph, where *n* is any positive integer.

Proof. Let $V(O_n) = \{u_1, u_2, \ldots, u_{2n+1}\}$. $E(O_n) = \{u_1u_i / 2 \leq i \leq 2n+1\} \cup \{u_iu_{i+1} / 2 \leq i \leq n\}$. Let G_k be the graph obtained by fusing the vertex u_i with u_k in O_n. Here $|V(G_k)| = 2n$. Define a labeling $f: V(G_k) \rightarrow \{1, 2, \ldots, 2n\}$ as follows

$$f(u_i) = i \quad \text{for } 1 \leq i \leq n-1$$
$$f(u_i) = i - 1 \quad \text{for } n+2 \leq i \leq 2n+1$$
$$f(u_6) = 6 = f(u_7)$$

Then f admits prime labeling. According to this pattern the vertices are labeled such that for any edge $e = u_iu_k \in G_k$, $\gcd(f(u_i), f(u_k)) = 1$. Clearly vertex labels are distinct. Thus we proved that the graph under consideration admits prime labeling. That is, the graph obtained by fusing (identifying) any two vertices u_i and u_k (where $d(u_i, u_k) \geq 3$) of an octopus graph O_n is a prime graph.

Example 3.12.

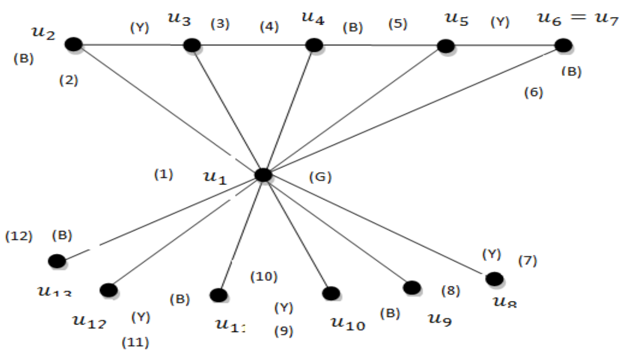

Figure 3.6. Fusion of u_6 and u_7 in O_6.

Theorem 3.13. The graph obtained by identifying any two pendant vertices u_i and u_k (where $d(u_i, u_k) \geq 3$) of an octopus graph O_n is a prime graph, where n is any positive integer.

Proof. Let $V(O_n) = \{u_1, u_2, \ldots, u_{2n+1}\}$. $E(O_n) = \{u_1 u_i / 2 \leq i \leq 2n+1\} \cup \{u_i u_{i+1} / 2 \leq i \leq n\}$. Let G_k be the graph obtained by identifying a pendant vertices u_i and u_k in an octopus graph O_n. Here $|V(G_k)| = 2n$. For n and k are both odd or even.
Define a labeling $f: V(G_k) \rightarrow \{1,2, \ldots, 2n\}$ as follows

$$f(u_i) = i \quad \text{for } 1 \leq i \leq n+3$$
$$f(u_6) = 6 = f(u_8)$$
$$f(u_i) = i - 1 \quad \text{for } i = 2n+1$$

Then f admits prime labeling. According to this pattern the vertices are labeled such that for any edge $e = u_i u_k \in G_k$, $\gcd(f(u_i), f(u_k)) = 1$. Clearly vertex labels are distinct. Thus we proved that the graph under consideration admits prime labeling. That is, the graph obtained by fusing (identifying) any two pendant vertices u_i and u_k (where $d(u_i, u_k) \geq 3$) of an octopus graph O_n is a prime graph.

Example 3.14.

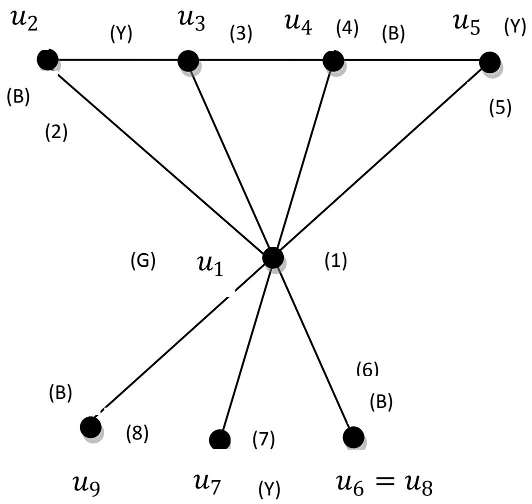

Figure 3.7. Fusion of the pendant vertices u_6 and u_8 in O_4.

Theorem 3.15. The graph obtained by identifying an apex vertex u_1 and u_k (where $d(u_1, u_k) \geq 3$) in an octopus graph O_n is a prime graph, where n is any positive integer.

Proof. Let $V(O_n) = \{u_1, u_2, \ldots, u_{2n+1}\}$. $E(O_n) = \{u_1 u_i / 2 \leq i \leq 2n+1\} \cup \{u_i u_{i+1} / 2 \leq i \leq n\}$. Let G_k be the graph obtained by identifying an apex vertex u_1 and u_k in an octopus graph O_n. Here $|V(G_k)| = 2n$. For n and k are both odd or even.
Define a labeling $f: V(G_k) \rightarrow \{1,2, \ldots, 2n\}$ as follows

$$f(u_1) = 1 = f(u_5)$$
$$f(u_i) = i \quad \text{for } 2 \leq i \leq n$$
$$f(u_i) = i - 1 \quad \text{for } n+2 \leq i \leq 2n+1$$

Then f admits prime labeling. According to this pattern the vertices are labeled such that for any edge $e = u_1 u_k \in G_k$, $\gcd(f(u_1), f(u_k)) = 1$. Clearly vertex labels are distinct. Thus we proved that the graph under consideration admits prime labeling. That is, the graph obtained by fusing (identifying) an apex vertex and any vertices u_1 with u_k (where $d(u_1, u_k) \geq 3$) of an octopus graph O_n is a prime graph.

Example 3.16.

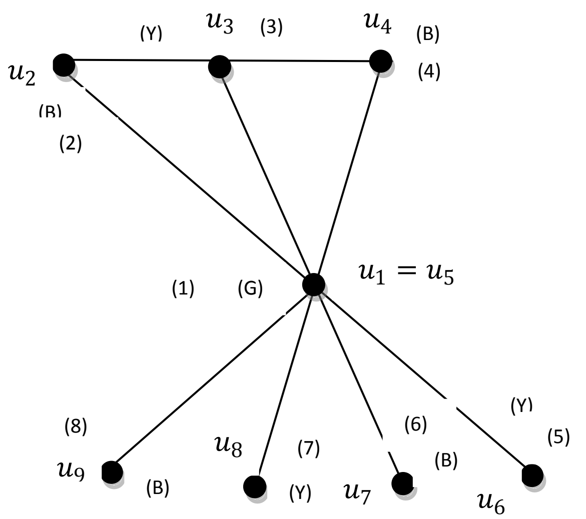

Figure 3.8. Fusion of u_1 and u_5 in O_4.

Theorem 3.17. The switching of any vertex u_k in an octopus graph O_n produces a Prime graph, where *n* is any positive integer.

Proof. Let $V(O_n) = \{u_1, u_2, \ldots, u_{2n+1}\}$. $E(O_n) = \{u_1u_i / 2 \le i \le 2n+1\} \cup \{u_iu_{i+1} / 2 \le i \le n\}$. Let G_u be the graph obtained by switching any vertex u_k in O_n. Here $|V(G_u)| = 2n+1$. Define a labeling $f: V(G_u) \to \{1, 2, \ldots, 2n+1\}$ as follows

$$f(u_1) = 1$$
$$f(u_2) = 5$$
$$f(u_i) = i - 1 \text{ for } 3 \le i \le n+1$$
$$f(u_i) = i \quad \text{ for } 6 \le i \le 2n+1$$

Then for any edge $e = u_iu_{i+1} \in G_u$, $\gcd(f(u_i), f(u_{i+1})) = 1$ and for any edge $e = u_1u_i \in G_u$, $\gcd(f(u_1), f(u_i)) = \gcd(1, f(u_i)) = 1$. Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_u is a prime graph.

Example 3.18.

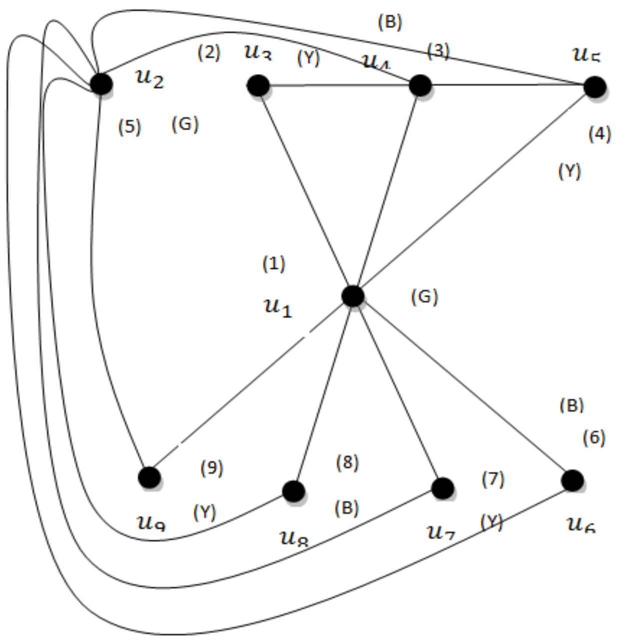

Figure 3.9. Switching the vertex u_2 in O_4.

Theorem 3.19. The switching of any pendant vertex u_k in an octopus graph O_n produces a Prime graph, where *n* is any positive integer.

Proof. Let $V(O_n) = \{u_1, u_2, \dots, u_{2n+1}\}$. $E(O_n) = \{u_1 u_i / 2 \leq i \leq 2n+1\} \cup \{u_i u_{i+1} / 2 \leq i \leq n\}$. Let G_u be the graph obtained by switching any pendant vertex u_k in O_n. Here $|V(G_u)| = 2n+1$. Define a labeling $f: V(G_u) \rightarrow \{1,2,\dots,2n+1\}$ as follows

$$f(u_i) = i \quad \text{for } 1 \leq i \leq 2n+1$$

Then for any edge $e = u_i u_{i+1} \in G_u$, $\gcd(f(u_i), f(u_{i+1})) = 1$ and for any edge $e = u_1 u_i \in G_u$, $\gcd(f(u_1), f(u_i)) = \gcd(1, f(u_i)) = 1$. Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_u is a prime graph.

Example 3.20.

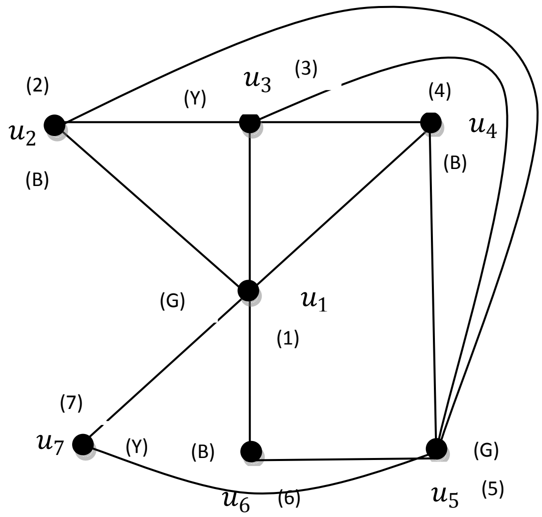

Figure 3.10. Switching the pendant vertex u_5 in O_3.

Theorem 3.21. The switching of an apex vertex u_1 in an octopus graph O_n produces a Prime graph, where *n* is any positive integer.

Proof. Let $V(O_n) = \{u_1, u_2, \dots, u_{2n+1}\}$. $E(O_n) = \{u_1 u_i / 2 \leq i \leq 2n+1\} \cup \{u_i u_{i+1} / 2 \leq i \leq n\}$. Let G_u be the graph obtained by switching an apex vertex u_1 in O_n. Here $|V(G_u)| = 2n+1$. Define a labeling $f: V(G_u) \rightarrow \{1,2,\dots,2n+1\}$ as follows

$$f(u_i) = i \quad \text{for } 1 \leq i \leq 2n+1$$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus the resulting graph G_u is a prime graph and it is a disconnected graph.

Example 3.22.

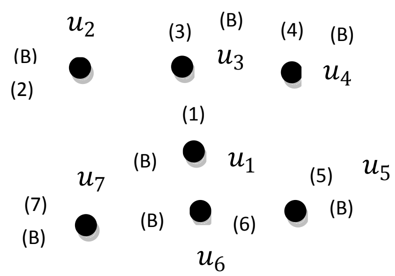

Figure 3.11. Switching an apex vertex u_1 in O_3.

IV. Conclusion

In this paper we proved that an octopus graph O_n , duplication of an octopus graph O_n, fusing of an octopus graph O_n, switching of an octopus graph O_n are prime graphs. There may be many interesting prime graphs can be constructed in future.

References

[1]. Bondy.J.A and Murthy.U.S.R, "Graph Theory and Applications" (North-Holland), Newyork, 1976.
[2]. Brooks R. L., "On colouring the nodes of a network". Proc. Cambridge Phil. Soc. 37:194–197, 1941.
[3]. Dretskyetal.T "on Vertex Prime labeling of graphs in graph theory", Combinatories and applications vol.1 Alari.J (Wiley. N.Y.) 299-359, 1991.
[4]. Fu.H.C and Huany.K.C "on Prime labeling Discrete Math",127, 181-186, 1994.
[5]. Gallian J.A, "A dynamic survey of graph labeling". The Electronic Journal of Combinations 16 # DS6, 2009.
[6]. Lee.S.M, Wui.L and Yen.J "on the amalgamation of Prime graphs Bull", Malaysian Math.Soc.(Second Series) 11, 59-67, 1988.
[7]. Meena .S and Vaithilingam .K "Prime labeling for some fan related graphs", International Journal of Engineering Research & Technology(IJERT) Vol. 1, Issue 9, 2012.
[8]. Tout.A Dabboucy.A.N and Howalla.K "Prime labeling of graphs". Nat.Acad.Sci letters 11, 365-368, 1982.