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Hall Effects on Steady MHD Flow of A Couple Stress Fluids through a Porous Medium

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Abstract: *In this paper, we have discussed the steady MHD flow of a couple stress fluid in a rotating parallel plate channel in a porous medium under the influence of a uniform transverse magnetic field taking into hall current. The perturbations are created by a constant pressure gradient along the plates. The equations for the couple stress fluid flow in the porous medium are based on Brinkman's model. The exact solution of the velocity and shear stresses on the boundaries are analytically derived, its behaviour computationally discussed with reference to the various governing parameters.*

Keywords: *Hall currents, steady flows, Rotating channels, Couple stress fluids, Brinkman's model.*

I. Introduction

The flow between parallel plates is a classical problem that has important applications in magneto hydro dynamic (MHD) power generators and pumps, accelerators, aerodynamic heating, electrostatic precipitation polymer technology, petroleum industry, purification of crude oil and fluid droplets, sprays, designing cooling systems with liquid metal, centrifugal separation of matter from fluid and flow meters. Hartman and Lazarus [3] studied the influence of a transverse uniform magnetic field on the flow of a viscous incompressible electrically conducting fluid between two infinite parallel stationary and insulating plates. Then the problem was extended in numerous ways. Closed form solutions for the velocity fields were obtained [1, 2 &11] under the different physical effects. In the above mentioned cases the Hall term was ignored in applying Ohm's Law as it has no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magneto- hydrodynamics is to words a strong magnetic field, so that the influence of electro- magnetic force is noticeable by Cramer et al [2]. Under these conditions, the Hall current is important and it has a marked effect on the magnitude and direction of the current density and consequently on the magnetic force. Pop [5], Sato [9], Yamanishi [13], Sherman and Sutton [10] have discussed the Hall effects on the steady hydro magnetic flow between two parallel plates. Tani [12] studied the Hall Effect on the steady motion of electrically conducting and viscous fluids in channels. Linga Raju . T and Ramana Rao. V.V. [4] studied steady viscous incompressible fluid flow between two parallel walls in the presence of a uniform magnetic field applied transversely to the flow and when rotated at an angular velocity about an axis perpendicular to the walls, taking Hall current into account. Rao and Krishna [6] studied Hall effects on the non-torsionally generated unsteady hydro magnetic flow in semi-infinite expansion of an electrically conducting viscous rotating fluid. Krishna and Rao [7 & 8] discussed the Stokes and Ekman problems in magneto-hydrodynamics taking Hall effects into account. The effects of Hall current on the hydrodynamic boundary layers and shear stress are discussed. In this paper, we discuss the Hall effects on steady hydro magnetic flow of couple stress fluid through a porous medium in a rotating parallel plate channel.

II. Formulation and Solution of the Problem

We consider an incompressible viscous and electrically conducting couple stress fluid in a rotating parallel plate channel bounded by porous medium. Both the fluid and the plates are in state of rigid rotation with uniform angular velocity Ω about z-axis normal to the plates. The entire flow is subjected to strong uniform transverse magnetic field normal to the plate in its own plane. In the equation of motion along x-direction, the x-component current density $\mu_e J_y H_o$ and the y-component current density $-\mu_e J_x H_o$.

We choose a Cartesian system 0(x,y,z) such that the boundary walls are at z=0 and z=l . z-axis being the axis of rotation of the plates. The Brinkman equations for the steady hydro magnetic flow governing the couple stress fluid through porous medium under the influence of a transverse magnetic field with reference to a frame rotating with a constant angular velocity Ω are

$$-2\Omega\, v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{d^2 u}{dz^2} + \mu_e J_y H_0 u - \frac{\nu}{k}u - \frac{\eta}{\rho}\frac{d^4 u}{dz^4} \qquad (2.1)$$

$$2\Omega\, u = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{d^2 v}{dz^2} - \mu_e J_x H_0 v - \frac{\nu}{k}v - \frac{\eta}{\rho}\frac{d^4 v}{dz^4} \tag{2.2}$$

Where, (u, v) is the velocity component along O(x, y) direction. ρ is the density of the fluid, μ_e is the magnetic permeability, ν is the coefficient of kinematic viscosity, k is the permeability of the medium, H_o is the applied magnetic field. When the strength of the magnetic field is very large, the generalized Ohm's law is modified to include the Hall current, so that

$$J + \frac{\omega_e \tau_e}{H_0} J \times H = \sigma\,(E + \mu_e\, q \times H) \tag{2.3}$$

Where, q is the velocity vector, H is the magnetic field intensity vector, E is the electric field , J is the current density vector, ω_e is the cyclotron frequency, τ_e is the electron collision time, σ is the fluid conductivity and, μ_e is the magnetic permeability. In equation (2.3) the electron pressure gradient, the ion-slip and thermo-electric effects are neglected. We also assume that the electric field $E=0$ under assumptions reduces to

$$J_x + m\, J_y = \sigma\mu_e H_0 v \tag{2.4}$$

$$J_y + m\, J_x = -\sigma\mu_e H_0 u \tag{2.5}$$

Where $m = \omega_e \tau_e$ is the Hall parameter

On solving equations (2.4) and (2.5) we obtain

$$J_x = \frac{\sigma\mu_e H_0}{1+m^2}(v + mu) \tag{2.6}$$

$$J_y = \frac{\sigma\mu_e H_0}{1+m^2}(mv - u) \tag{2.7}$$

Using the equations (2.6) and (2.7), the equations of the motion with reference to rotating frame are given by

$$-2\Omega\, v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{d^2 u}{dz^2} + \frac{\sigma\mu_e^{\,2} H_0^{\,2}}{\rho(1+m^2)}(mv - u) - \frac{\nu}{k}u - \frac{\eta}{\rho}\frac{d^4 u}{dz^4} \tag{2.8}$$

$$2\Omega\, u = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{d^2 v}{dz^2} - \frac{\mu_e^{\,2} H_0^{\,2}}{\rho(1+m^2)}(v + mu) - \frac{\nu}{k}v - \frac{\eta}{\rho}\frac{d^4 v}{dz^4} \tag{2.9}$$

Let $q = u + iv\,,\quad \xi = x - iy$

Now combining the equations (2.8) and (2.9), we obtain

$$\frac{\eta}{\rho}\frac{d^4 q}{dz^4} - 2i\Omega\, q = -\frac{1}{\rho}\frac{\partial p}{\partial \xi} + \nu\frac{d^2 q}{dz^2} + \frac{\sigma\mu_e^{\,2} H_0^{\,2}}{\rho(1+m^2)}(1 + im)q - \frac{\nu}{k}q \tag{2.10}$$

The boundary conditions are

$$\left.\begin{aligned} q(0) &= 0 = q^{11}(0) \\ q(l) &= 0 = q^{11}(l) \end{aligned}\right\} \qquad (2.11)$$

We introduce the non-dimensional variables

$$z^* = \frac{z}{l}\ ,\quad q^* = \frac{ql}{\nu}\ ,\quad q_p{}^* = \frac{q_p\, l}{\nu},$$
$$P^* = \frac{Pl^2}{\rho \nu^2},\quad h^* = \frac{h}{l}\ ,\quad \xi^* = \frac{\xi}{l}.$$

Using the non-dimensional variables, the governing non-dimensional equation is (dropping asterisks)

$$S\ \frac{d^4 q}{dz^4} - \frac{d^2 q}{dz^2} + \left[2iE^{-1} + \frac{M^2(1+im)}{1+m^2} + D^{-1} \right]\ q = P \qquad (2.12)$$

where, $M^2 = \dfrac{\sigma \mu_e{}^2 H_0{}^2 l^2}{\rho \nu}$ is the Hartmann number,

$m = \omega_e \tau_e$ is the Hall Parameter,

$D^{-1} = \dfrac{l^2}{k}$ is the Inverse Darcy parameter,

$E = \dfrac{\nu}{\Omega l^2}$ is the Eckman number,

$S = \dfrac{\eta}{\rho l^2 \nu}$ is the Couple stress parameter ,

$P = -\dfrac{\partial p}{\partial \xi}$ is the constant pressure gradient.

Corresponding boundary conditions are

$$\left.\begin{aligned} q(0) &= 0 = q^{11}(0) \\ q(1) &= 0 = q^{11}(1) \end{aligned}\right\} \qquad (2.13)$$

Solving the equation (2.12) and making use of boundary conditions (2.13) we obtain

$$q = \left\{\ (Ae^{d_1 z} + Ce^{-d_1 z})Cos\, e_1 z + (Be^{d_2 z} + De^{-d_2 z})\, Cos\, e_2 z \right\}$$
$$+ i\left\{\ (Ae^{-d_1 z} - Ce^{d_1 z})\ Sin\, b_1 z + (Be^{-d_2 z} - De^{d_2 z})\, Sin\, e_2 z\ \right\} \qquad (2.14)$$

The shear stresses on the upper plate and lower plate are given by

$$\tau_U = \left(\frac{dq}{dz}\right)_{z=1} \quad and \quad \tau_L = \left(\frac{dq}{dz}\right)_{z=0}$$

Where, the constants A, B, C and D are mentioned in appendix.

III. Results And Discussion

The flow governed by the non-dimensional parameters E the Ekman number, M the Hartmann number, D^{-1} the inverse Darcy parameter, S the couple stress parameter and m the hall parameter. Figures (1-5) represent the velocity profiles for u while figures (6-10) correspond to the velocity profiles for v. We observe that the velocity components both u and v increases with increase in E and the resultant velocity enhance with increase in E (fig 1&6). The velocity components u increases and v decreases with increase in M and D^{-1}. However the resultant velocity reduces with increase the intensity of the magnetic field M, likewise both the velocity components reduce with increasing the inverse Darcy parameter D^{-1} (fig 2, 3, 7 & 8). Also an increase in S enhances u and decreases v. The resultant velocity enhances with increase in S (fig 4&9). The velocity components u decreases and v increases with increase in Hall parameter m. However the resultant velocity enhances with increase in m. The shear stresses for τ_x and τ_y is evaluated for variations in governing parameters on the upper and lower plate and is tabulated in the tables (1-2). On the upper plate we notice that τ_x increases and τ_y reduces in magnitude with increase in the couple stress parameter S, E, M, D^{-1} and m, while on the lower plate both τ_x and τ_y are enhances with increase for all governing parameters S, E, M, D^{-1} and m (tables are not mentioned paucity of the space).

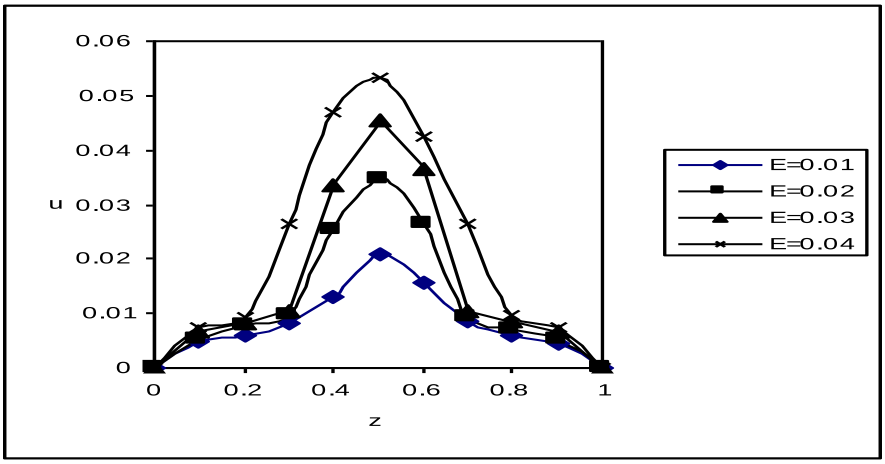

Fig 1: The velocity profile for u with E
M=2, D^{-1} =3000, m=1, S=1.

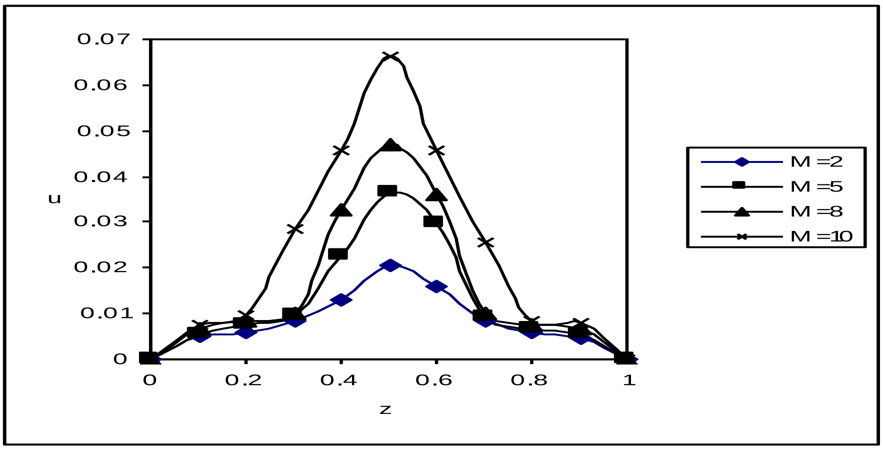

Fig 2: The velocity profile for u with M.
E=0.01, D^{-1} =3000, m=1, S=1

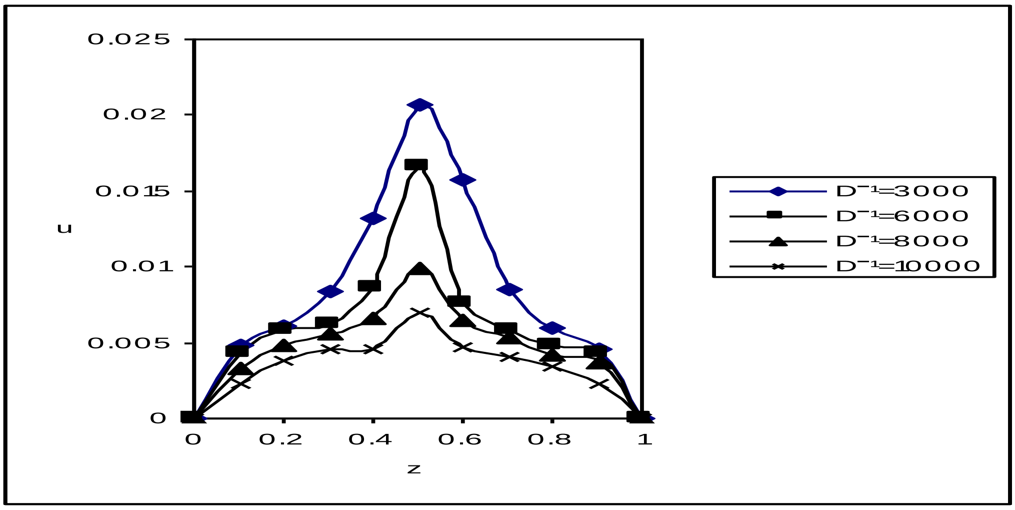

Fig 3: The velocity profile for u with D^{-1}
E=0.01, M=2, m=1, S=1

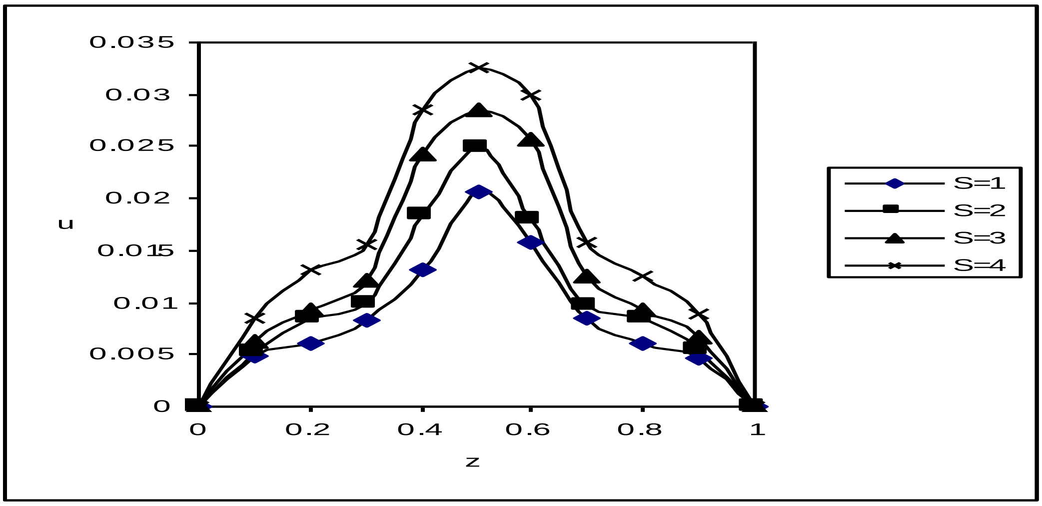

Fig 4: The velocity profile for u with S.
M=2, D^{-1}=3000, m=1, E=0.01.

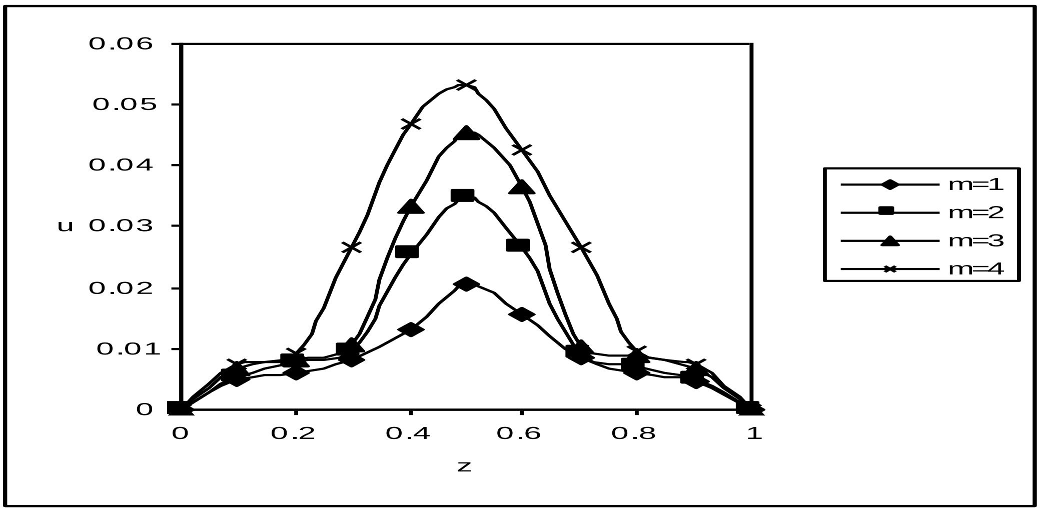

Fig 5: The velocity profile for u with m.
E=0.01, D^{-1} =3000, M=2, S=1

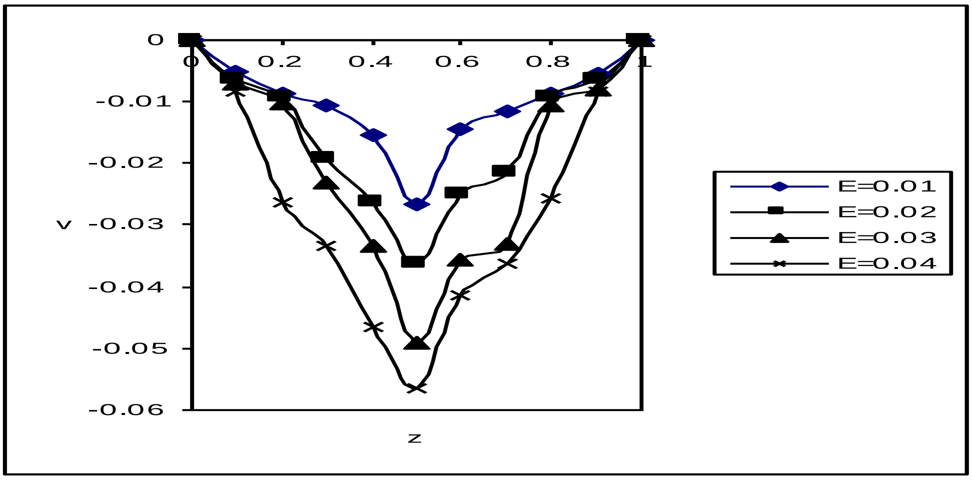

Fig 6: The velocity profile for v with E.
M=2, D^{-1} =3000, m=1, S=1.

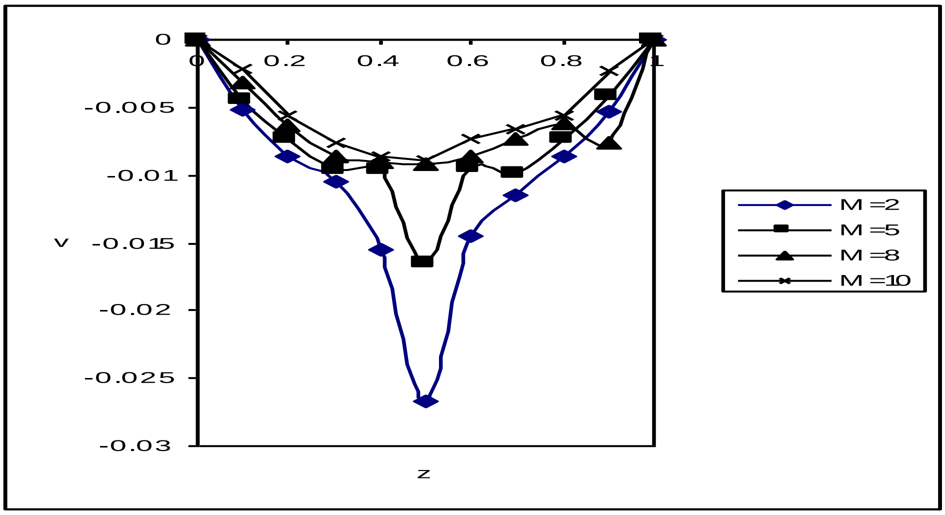

Fig 7: The velocity profile for v with M.
E=0.01, D^{-1} =3000, m=1, S=1

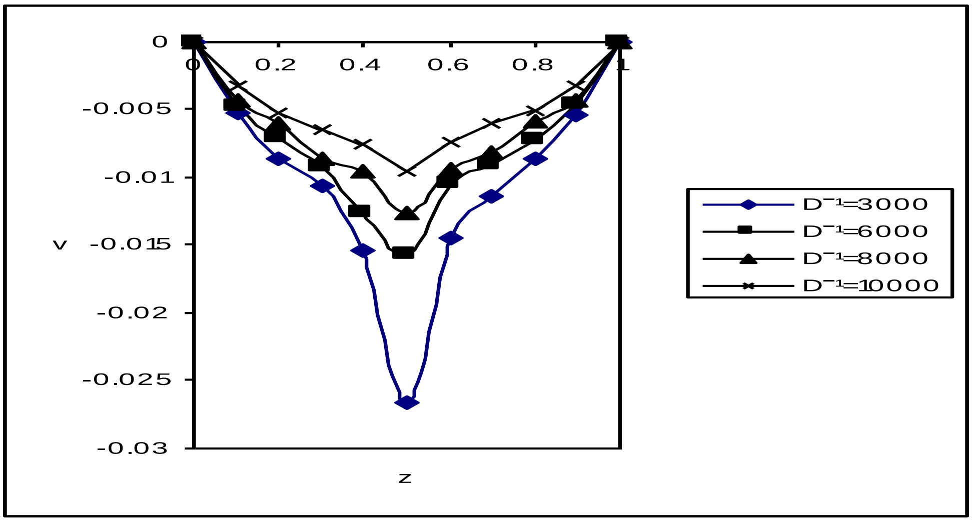

Fig 8: The velocity profile for v with D^{-1}
E=0.01, M=2, m=1, S=1.

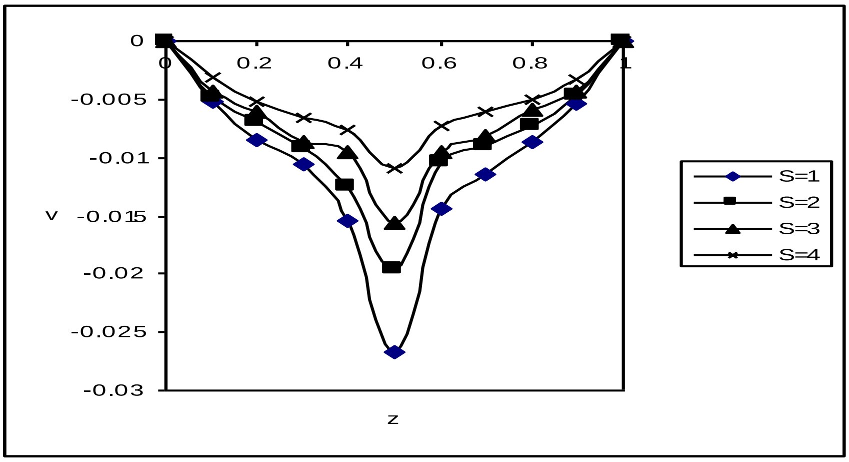

Fig 9:The velocity profile for v with S
M=2, D^{-1} =3000, m=1, E=0.01

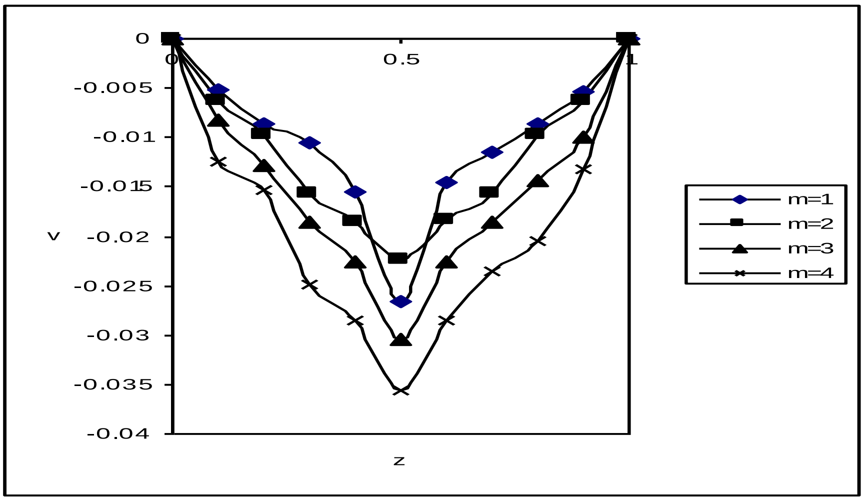

Fig 10: The velocity profile for v with m.
E=0.01, D^{-1} =3000, M=2, S=1

Table 1: The shear stress (τ_x) on the upper plate

M	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**
2	0.445688	0.524312	0.641838	0.522443	0.586433	-0.32163	-0.22431
5	0.485266	0.634678	0.684403	0.584435	0.626799	-0.40520	-0.30765

	I	**II**	**III**	**IV**	**V**	**VI**	**VII**
E	0.01	0.02	0.01	0.01	0.01	0.01	0.01
S	1	1	2	1	1	1	1
D^{-1}	3000	3000	3000	6000	8000	1000	1000
M	1	1	1	1	1	2	3

Table 2: The shear stress (τ_y) on the upper plate

M	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**
2	-0.005278	-0.00326	-0.00473	-0.00085	-0.00032	-0.01434	-0.02668
5	-0.003834	-0.00145	-0.00287	-0.00058	-0.00028	-0.00952	-0.01452

	I	II	III	IV	V	VI	VII
E	0.01	0.02	0.01	0.01	0.01	0.01	0.01
S	1	1	2	1	1	1	1
D^{-1}	3000	3000	3000	6000	8000	1000	1000
M	1	1	1	1	1	2	3

IV. Conclusions

1) The resultant velocities for the flow field enhances with increase in governing parameters E, M, S and m while it reduces with increases the inverse Darcy parameter D^{-1}.
2) On the upper plate, τ_x increases and τ_y reduces in magnitude with increase in the couple stress parameter S, E , M and D^{-1} and τ_x decreases τ_y enhances with increase hall parameter m.

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