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A Study of Compactness and Connectedness in Rough Topological Spaces

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Abstract:*In this paper an attempt is made to study about compactness and connectedness in rough topological spaces. Also separation axioms T_1 and T_2 inrough topological spaces are discussed.*
Keywords: *Rough set, Rough Topological spaces, Rough compactness, Rough Connectedness,* T_1 *and* T_2 *separations in rough topological spaces.*

I. Introduction

The problem of imperfect knowledge has been tackled for long time by philosophers, logicians and mathematicians. Recently it became also a crucial issue for computer scientists, particularly in the area of artificial intelligence. There are many approaches to the problem of how to understand and manipulate imperfect knowledge. Rough set theory by Z.Pawalak [19] is a new mathematical approach to vagueness or imperfect knowledge. Pawalak's Rough set theory expresses vagueness by employing a boundary region of a set. If the boundary region of a set is empty it means that the set is crisp or exact, otherwise the set is rough or inexact. Nonempty boundary region of a set means that our knowledge about the set is not sufficient to define the set precisely. Alsotwo approaches are discussed in rough topological spaces. M. LellisThivagar and etal[11]first introduceda rough topology on any set using lower and upper approximations of any subset. They haveshow that this topology can be used to analyze many real life problems.Secondly, Boby P Mathew & Sunil Jacob John[3] defined a new topological structure on rough sets and studied some properties of rough topological space. In thispaper, an attempt is made to study about compactness and connectedness in rough topological spaces. The approach is as in[3].

II. Notations

The following notations are used in this paper.
$\underline{r}cl(A)$ – Lower closure of A
$\overline{r}cl(A)$ – Upper closure of A
$\underline{r}int(A)$ – Lower interior of A
$\overline{r}int(A)$ – Upper interior of A

III. Preliminaries

Let U be a non empty set of objects called the universe and an equivalence relation called indiscernibility relation R on U,then the pair (U,R) is known as the approximation space[19]. Let X be a subset of U. In order to characterize X with respect to R, we associate two crisp sets to X, called its lower and upper approximations.

3.1.Definition[19] The equivalence class of R containing an element x will be denoted by$[x]_R$and is called granules of knowledge generated by R, which represents elementary portion of knowledge we are able to perceive due to R.

3.2.Definition [19] The R-lower approximation, or positive region, is the union of equivalence classes in $[x]_R$which are contained by (i.e., are subsets of) the target set.
That is $\underline{R}X = \cup\{x/[x]_R \subseteq X\}$

3.3.Definition[19]The R-upper approximation is the union of all equivalence classes in$[x]_R$ which have non-empty intersection with the target set
That is $\overline{R}X = \cup\{x/[x]_R \cap X \neq \varphi\}$

3.4.Definition[19] The boundaryregion$BN_R(X)$, given by set difference$\overline{R}X - \underline{R}X$, consists of those objects that can neither be ruled in nor ruled out as members of the target set$X$.

3.5.Definition[19] A set is said to be a rough set, if it has a non-empty boundary region. If the boundary region is empty then the set is a crisp or exact set.

3.6.Result[19] X is an exact set if$\underline{R}X = \overline{R}X$ and X is rough set if $\underline{R}X \neq \overline{R}X$

3.7.Definition[3]Let RX=($\underline{R}X, \overline{R}X$) be rough subset of the approximation space(Ω,R). Let$\underline{\tau}$ and $\overline{\tau}$ be any two topologies which contain only exact subsets of $\underline{R}X$ and $\overline{R}X$ respectively. Then the pair τ=($\underline{\tau}$,$\overline{\tau}$) is said to be a Rough topology on the rough set RX=($\underline{R}X, \overline{R}X$) and the pair (RX,$\tau$) is known as rough topological space (RTS). Also in a rough topology τ=($\underline{\tau}$,$\overline{\tau}$),$\underline{\tau}$ is known as the lower rough topology and$\underline{\tau}$ is known as the upper rough topology on RX.

3.8.Definition[3]In any rough set RX=($\underline{R}X, \overline{R}X$) and define $\underline{\tau}$= {A⊆ $\underline{R}$X /A is definable} and $\overline{\tau}$={B⊆ $\overline{R}X$/B is definable}. Then $\underline{\tau}$ and $\overline{\tau}$ are topologies on $\underline{R}X$ and $\overline{R}X$ respectively and the rough topology τ=($\underline{\tau}$,$\overline{\tau}$) is known as the discrete rough topology on RX=($\underline{R}X, \overline{R}X$) and the topological space (X,τ) is known as the discrete rough topological space on RX.

3.9.Definition[3]In a rough set RX=($\underline{R}X, \overline{R}X$), take $\underline{\tau}$={∅,$\underline{R}X$}and$\overline{\tau} = \{\emptyset, \overline{R}X\}$, then $\underline{\tau}$ and $\overline{\tau}$ are topologies on $\underline{R}X$ and $\overline{R}X$respectively and the rough topology τ=($\underline{\tau}$,$\overline{\tau}$) on RX is known as the indiscrete rough topology on RX and (RX,τ) is known as the indiscrete rough topological space on RX.

3.10.Definition[3]If A=($\underline{A}, \overline{A}$) is any sub rough set of a rough set RX=($\underline{R}X, \overline{R}X$)**,** then$A^{\underline{C}}$=$\underline{R}X\backslash\underline{A}$iscalled the lower complement of A and $A^{\overline{C}}$=$\overline{R}X\backslash\overline{A}$ is called the upper complement of A.

3.11.Definition[3]A subset B=($\underline{B}, \overline{B}$)of the RTS(RX,$\tau$),where ($\underline{R}X, \overline{R}X$) and τ=($\underline{\tau}$,$\overline{\tau}$) is said to be lower rough closed set if $B^{\underline{C}}$=$\underline{R}X\backslash\underline{B} \in \underline{\tau}$. Also B is said to upper rough closed if $B^{\overline{C}}$=$\overline{R}X\backslash\overline{B} \in \overline{\tau}$. B is said to be rough closed if it is lower rough closed and upper rough closed. That is a subset B=($\underline{B}, \overline{B}$) of the RTS(RX,$\tau$)is rough closed subset iff its lower approximation is closed with respect to the lower topology and its upper approximation is closed with respect to the upper topology.

IV. Compactness in rough topological space

4.1.Definition. Let $RX = (\underline{R}X\,, \overline{R}X)$ be a rough set. For any open covering of$\underline{R}X$, if there is a finite subcovering, then $\underline{R}X$ is called a compact set and also the compact lower approximation of RX. Similarly, forany opencovering of$\overline{R}X$, if there is a finite subcovering, then $\overline{R}X$ is called a compact set and also the compact upper approximation of RX. Then $RX = (\underline{R}X\,, \overline{R}X)$ is called a compact rough set and also a compact rough space if both $\underline{R}X$ and $\overline{R}X$ are compact sets.

4.2.Definition. Let $A = (\underline{A}\,, \overline{A})$ be a subset of rough set$RX = (\underline{R}X\,, \overline{R}X)$. $\underline{A}$asa subset of $\underline{R}X$ is compact, if for any open covering of$\underline{A}$, ∃ a finite sub covering of $\underline{A}$. At the same time if$\overline{A}$, as a subspace of$\overline{R}X$ is also compact, then A is called a compact subset RX.

4.3.Definition. Let $RX = (\underline{R}X\,, \overline{R}X)$ be a topological rough set together with the topology$\tau = (\underline{\tau}\,, \overline{\tau})$. A subset $N_1 \subseteq \underline{R}X$ is said to be a $\underline{\tau}$- neighborhood of $x \in X$ iff there exists an open set G_1 of $\underline{R}X$ such that$x \in G_1 \subset N_1$. Similarly, a subset $N_2 \subseteq \overline{R}X$ is said to be a $\overline{\tau}$- neighborhood of $x \in X$ iff there exists an open set G_2 of $\overline{R}X$ such that$x \in G_2 \subset N_2$. At the same time if$N_1 \subseteq \underline{R}X \subseteq N_2 \subseteq \overline{R}X$, then $N = (N_1, N_2)$ is said to be a τ – neighborhood of $x \in X$.

4.4.Theorem. Let $A = (\underline{A}, \overline{A})$ be a subset of rough set $RX = (\underline{R}X\,, \overline{R}X)$satisfying$\underline{A} \subseteq \underline{R}X \subseteq \overline{A} \subseteq \overline{R}X$. Then A is open iff it is a neighborhood of each of its points.

Proof. Let $A = (\underline{A}, \overline{A})$ be an open subset of rough set$RX = (\underline{R}X\,, \overline{R}X)$. Thenfor every$x \in \underline{A}$, $x \in \underline{A} \subset \underline{A}$ and for every $y \in \overline{A}$, $y \in \overline{A} \subset \overline{A}$. Hence $\underline{A}$ and$\overline{A}$ are neighborhoods of each of the points and so is $A = (\underline{A}, \overline{A})$.On the other hand, let $A = (\underline{A}, \overline{A})$ be a neighborhood of each of is points. By assumption$\underline{A} \subseteq \underline{R}X \subseteq \overline{A} \subseteq \overline{R}X$. If A = ϕ, then it is open. If$x \in A$, then there exists an open set $G = (G_{\underline{x}}\,, G_{\overline{x}})$ in RX such that $x \in G_{\underline{x}} \subset \underline{A}$ and $x \in G_{\overline{x}} \subset \overline{A} \Longrightarrow \underline{A} = \cup \{G_{\underline{x}} | x \in \underline{A}\}$ and $\overline{A} = \cup \{G_{\overline{x}} | x \in \overline{A}\} \Longrightarrow \underline{A}$ and$\overline{A}$ are open sets ⟹ Ais an open set.

4.5.Definition. Let $RX = (\underline{R}X\,, \overline{R}X)$ and $RY = (\underline{R}Y\,, \overline{R}Y)$ be topological rough sets with topologies $\tau = (\underline{\tau}\,, \overline{\tau})$ and $\varsigma = (\underline{\varsigma}, \overline{\varsigma})$ respectively. A function $\underline{f} : \underline{R}X \to \underline{R}Y$ is said to be continuous at $x \in X$ iff to every $\underline{\varsigma}$-neighborhood M_1 of $\underline{f}(x)$ in $\underline{R}Y$ there exists a$\underline{\tau}$– neighborhood N_1 of x in $\underline{R}X$ such that $\underline{f}(N_1) \subset M_1$ and $\overline{f} : \overline{R}X \to \overline{R}Y$ is said to be continuous at $x \in X$ iff to every$\overline{\varsigma}$ -neighborhood M_2 of $\overline{f}(x)$ in $\overline{R}Y$there exists a $\overline{\tau}$ – neighborhood N_2 of x in $\overline{R}X$ such that $\overline{f}(N_2) \subset M_2$. Then the function $f = \left(\underline{f}, \overline{f}\right) : RX \to$RY is said to be a continuous function at x if both $\underline{f}$ and$\overline{f}$ are continuous functions at x. f is said to be τ-ςcontinuous or simply continuous if it is continuous at each point of RX.

4.6.Definition.Let $RX = (\underline{R}X\,, \overline{R}X)$ and $RY = (\underline{R}Y\,, \overline{R}Y)$ be topological rough sets with topologies $\tau = (\underline{\tau}\,, \overline{\tau}\,)$ and $\varsigma = (\,\underline{\varsigma}, \overline{\varsigma}\,)$ respectively. The product topology η_1 on $\underline{R}X \times \underline{R}Y$ is the topology having as basis the collection

$\mathcal{B}_1$ of open sets of the form $\underline{U} \times \underline{V}$, where$\underline{U}$ is a $\underline{\tau}$ – open set and $\underline{V}$is a $\underline{\varsigma}$ – open set. Similarly the product topology η_2 on $\overline{R}X \times \overline{R}Y$ is the topology having as basis the collection $\mathcal{B}_2$ of open sets of the form $\overline{U} \times \overline{V}$, where$\overline{U}$ is a $\overline{\tau}$ – open set and$\overline{V}$ is a $\overline{\varsigma}$ – open set. Then $\eta = (\eta_1, \eta_2)$ is said to be a product topology on$RX \times RY = (\underline{R}X \times \underline{R}Y, \overline{R}X \times \overline{R}Y)$.

4.7.Definition.Let $RX = (\underline{R}X\,, \overline{R}X)$ and $RY = (\underline{R}Y\,, \overline{R}Y)$ be topological rough sets with topologies $\tau = (\underline{\tau}\,, \overline{\tau})$ and $\varsigma = (\underline{\varsigma}, \overline{\varsigma})$ respectively. The mappings $\Pi_{\underline{x}} : \underline{R}X \times \underline{R}Y \to \underline{R}X$ and $\Pi_{\overline{x}} : \overline{R}X \times \overline{R}Y \to \overline{R}X$ defined by $\Pi_{\underline{x}}\big((x,y)\big) = x, \forall (x,y) \in \underline{R}X \times \underline{R}Y$ and $\Pi_{\overline{x}}\big((x,y)\big) = x, \forall (x,y) \in \overline{R}X \times \overline{R}Y$ respectively are called projection mappings. Then $\Pi_x = (\Pi_{\underline{x}}, \Pi_{\overline{x}})$ is called a projection mapping from $RX \times RY \to RX$.Similarly, the projection mapping $\Pi_y = (\Pi_{\underline{y}}, \Pi_{\overline{y}})$ is defined from $RX \times RY \to RY$.

4.8.Theorem.Let (RX, τ) and (RY, ς) be topological rough sets and $f = \left(\underline{f}, \overline{f}\right): RX \to R\mathrm{Y}$. For every ς – open set $H = \left(\underline{H}, \overline{H}\right), \underline{f}^{-1}\left(\underline{H}\right) \subseteq \underline{R}X \subseteq \overline{f}^{-1}\left(\overline{H}\right) \subseteq \overline{R}X$.Then$f$ is continuous iff the inverse image of every open set in RY under f is open in RX.

Proof. Let $f = (\underline{f}, \overline{f}) : RX = \left(\underline{R}X\,, \overline{R}X\right) \to RY = (\underline{R}Y\,, \overline{R}Y)$ be a continuous function and $H = \left(\underline{H}, \overline{H}\right)$ be an open set in RY. To prove $f^{-1}(H) = \left(\underline{f}^{-1}\left(\underline{H}\right), \overline{f}^{-1}\left(\overline{H}\right)\right)$ is an open set inRX.If $\underline{f}^{-1}\left(\underline{H}\right)$ and$\overline{f}^{-1}\left(\overline{H}\right)$ are empty, then there is nothing to prove.

Case(i): Assume $x \in \underline{f}^{-1}\left(\underline{H}\right) \Longrightarrow x \in \overline{f}^{-1}\left(\overline{H}\right) \Longrightarrow \underline{f}(x) \in \underline{H}$ and $\overline{f}(x) \in \overline{H}$. By continuity of $\underline{f}$ there exists a neighborhood $N_1 of x$ such that$\underline{f}(N_1) \subset \underline{H} \Longrightarrow x \in N_1 = \underline{f}^{-1}\ (\underline{f}(N_1)) \subseteq \underline{f}^{-1}(\underline{H}) \Longrightarrow \underline{f}^{-1}\left(\underline{H}\right)$ is a neighborhood of $x \in X$ and hence is open (since x is arbitrary). Similarly$\overline{f}^{-1}\left(\overline{H}\right)$ is also open. Given$\underline{f}^{-1}\left(\underline{H}\right) \subseteq \underline{R}X \subseteq \overline{f}^{-1}\left(\overline{H}\right) \subseteq \overline{R}X$. So $f^{-1}(H)$ is an open set.

Case(ii): Assume $y \in \overline{f}^{-1}\left(\overline{H}\right) but y \notin \underline{f}^{-1}\left(\underline{H}\right). So let x \in \underline{f}^{-1}\left(\underline{H}\right) \Longrightarrow \underline{f}(x) \in \underline{H}$ and $\overline{f}(y) \in \overline{H}$. By continuity of$\underline{f}$ there exists a neighborhood $N_1 of x$ such that$\underline{f}(N_1) \subset \underline{H} \Longrightarrow x \in N_1 = \underline{f}^{-1}\ (\underline{f}(N_1))\ \subseteq \underline{f}^{-1}\left(\underline{H}\right) \Longrightarrow \underline{f}^{-1}\left(\underline{H}\right)$ is a neighborhood of $x \in X$ and hence is open(since x is arbitrary). Similarly$\overline{f}^{-1}\left(\overline{H}\right)$ is open. Given$\underline{f}^{-1}\left(\underline{H}\right) \subseteq \underline{R}X \subseteq \overline{f}^{-1}\left(\overline{H}\right) \subseteq \overline{R}X$. So $f^{-1}(H)$ is an open set.

Conversely, let$f^{-1}(H)$ be an open set in RX for every open set H in RY. To prove f is a continuous function.

Case(i):Let $\underline{f}(x) \in \underline{H} \Longrightarrow \overline{f}(x) \in \overline{H}$, (by hypothesis) $\Longrightarrow x \in \underline{f}^{-1}\left(\underline{H}\right) and x \in \overline{f}^{-1}\left(\overline{H}\right) \Longrightarrow \underline{f}\left(\underline{f}^{-1}\left(\underline{H}\right)\right) \subset \underline{H}and\overline{f}(\overline{f}^{-1}\left(\overline{H}\right)) \subset \overline{H} \Longrightarrow \underline{f}$ and$\overline{f}$ arecontinuous at x. Since x is arbitrary, $\underline{f}$and $\overline{f}$ are continuous everywhere. So f is continuous.

Case(ii):Let $\overline{f}(y) \in \overline{H} but \underline{f}(y) \notin \underline{H}$ .So let $\underline{f}(x) \in \underline{H}$ for $x, y \in X \Longrightarrow x \in \underline{f}^{-1}\left(\underline{H}\right)$ and $y \in \overline{f}^{-1}\left(\overline{H}\right) \Longrightarrow \underline{f}\left(\underline{f}^{-1}\left(\underline{H}\right)\right) \subset \underline{H}$and $\overline{f}(\overline{f}^{-1}\left(\overline{H}\right)) \subset \overline{H} \Longrightarrow \underline{f}$ and$\overline{f}$are continuous at x and y respectively. Since$x$ and y are arbitrary, so $\underline{f}$ and $\overline{f}$are continuous in RX. Hence$f$ is continuous.

4.9.Theorem. Continuous image of compact topological rough set is compact and so, compactness in topological rough sets is a topological property.

Proof. Let $RX = (\underline{R}X\,, \overline{R}X)$be a compact rough set and $f = (\underline{f}, \overline{f}) : \left(\underline{R}X\ , \overline{R}X\right) \to\ (\underline{R}Y\,, \overline{R}Y)$ be a continuous mappings. Then$\underline{f}: \underline{R}X \to \underline{R}Yand\overline{f}: \overline{R}X \to \overline{R}Y$ are continuous mappings. Let $\mu_1 = \{v_\lambda | \lambda \epsilon \Lambda\}$ be an open covering of $\underline{f}\left(\underline{R}X\right) = \ \underline{R}Y$. Then$\underline{f}^{-1}(\mu_1) = \{\ \underline{f}^{-1}(v_\lambda) | \lambda \epsilon \Lambda\}$ is an open covering of $\underline{R}X$. Since $\underline{R}X$ is compact so it has a finite subcovering $\{\ \underline{f}^{-1}\left(V_{\lambda_i}\right) |$ i = 1, 2….., n$\} \Longrightarrow \{V_{\lambda_i} | i = 1,2 \ldots.., n\}$ is a finite open subcovering of$\underline{f}\left(\underline{R}X\right) = \underline{R}Y$. Hence$\underline{R}Y$ is compact. Similarly$\overline{R}Y$ is compact and consequently $Y = (\underline{R}Y\,, \overline{R}Y)$ is a compact rough set.

V. Connectedness in Rough Topological Space

5.1.Definition. Let$RX = (\underline{R}X\,, \overline{R}X)$ be a rough topological space. A separation of $\underline{R}X$ is a pair of disjoint nonempty lower open sets $\underline{U}$and $\underline{V}$in $\underline{R}X$ such that$\underline{U} \cup \underline{V} = \underline{R}X$. Also, a separation of $\overline{R}X$ is pair of disjoint nonempty upper open sets $\overline{U}$ and $\overline{V}$ in$\overline{R}X$ such that$\overline{U} \cup \overline{V} = \overline{R}X$. A rough topological space $RX = (\underline{R}X\,, \overline{R}X)$ is said to be rough connected if there does not exist a separation of$RX = (\underline{R}X\,, \overline{R}X)$

5.2.Example. Consider the following sample information system

Object	P_1	P_2	P_3	P_4	P_5
O_1	1	2	0	1	1
O_2	1	2	0	1	1
O_3	2	0	0	1	0
O_4	0	0	1	2	1
O_5	2	1	0	2	1
O_6	0	0	1	2	2
O_7	2	0	0	1	0
O_8	0	1	2	2	1
O_9	2	1	0	2	2
O_{10}	2	0	0	1	0

Let X $=\{O_1, O_2, O_3, O_4\}$ and let attribute subset R $=\{P_1, P_2, P_3, P_4, P_5\}$
$\underline{R}X = \{O_1, O_2\} \cup \{O_4\}$, $\overline{R}X = \{O_1, O_2\} \cup \{O_4\} \cup \{O_3, O_7, O_{10}\}$

$$RX =< (\underline{R}X , \overline{R}X)$$

$=< \{O_1, O_2, O_4\}, \{O_1, O_2, O_3, O_4, O_7, O_{10}\} >$and
$\underline{\tau} = \{ \underline{R}X, \varphi, \{O_1, O_2\}, \{O_2\}, \{O_4\}\}$,
$\overline{\tau} = \{\overline{R}X, \varphi, \{O_1, O_2, O_3\}, \{O_2\}, \{O_2, O_4, O_7\}, \{O_4, O_7, O_{10}\}, \{O_4, O_7\}$
Since $\underline{\tau}$satisfies all the conditions ofthetopology ,$\underline{\tau}$ is a lower rough topology on$\underline{R}X$.Also since $\overline{\tau}$satisfies all the conditions of the topology,$\overline{\tau}$ is an upper rough topology on$\overline{R}X$.

(i) Consider lower open sets ,$\underline{U} = \{O_1, O_2\}$, $\underline{V} = \{O_4\}$

$$\underline{U} \cup \underline{V} = \{O_1, O_2, O_4\} = \underline{R}X$$

$\underline{U} \cap \underline{V} = \emptyset$, Therefore $\underline{R}X$ is not lower rough connected.

(ii) Consider upper open sets $\overline{U} = \{O_1, O_2, O_3\}$, $\overline{V} = \{O_4, O_7, O_{10}\}$
$\overline{U} \cup \overline{V} = \{O_1, O_2, O_3, O_4, O_7, O_{10}\} = \overline{R}X$,$\overline{U} \cap \overline{V} = \varphi$
Therefore $\overline{R}X$ is not upper rough connected
Hereboth are separable

Remark. Even if one of $\underline{R}X$ and $\overline{R}X$ is separable, then rough set is separable

5.3.Result:-Any rough set with discrete rough topology is neednot be rough connectedness.

5.4.Theorem. A space $RX = (\underline{R}X , \overline{R}X)$ is rough connected if and only if the empty set and $RX = (\underline{R}X , \overline{R}X)$ are only subsets of $RX = (\underline{R}X , \overline{R}X)$ that are both rough open and rough closed in$RX = (\underline{R}X , \overline{R}X)$.

Proof. Let $U = (\underline{U}, \overline{U})$ be a non empty proper subset of $RX = (\underline{R}X , \overline{R}X)$ which is both rough open and rough closed in$RX = (\underline{R}X , \overline{R}X)$. Let $\underline{V} = \underline{R}X - \underline{U}$,$\overline{V} = \overline{R}X - \overline{U}$. Since $\underline{U}$ is lower closed,$\underline{R}X - \underline{U}$ is lower open and also $\overline{U}$ is upper closed,$\overline{R}X - \overline{U}$ is upper open. Thus $V = (\underline{V}, \overline{V})$ is rough open. Here $U = (\underline{U}, \overline{U})$ and $V = (\underline{V}, \overline{V})$ are disjoint rough open subsets of $(\underline{R}X , \overline{R}X)$ such that$\underline{U} \cup \underline{V} = \underline{R}X$ and $\overline{U} \cup \overline{V} = \overline{R}X$ which is a contradiction to the factthat $RX = (\underline{R}X , \overline{R}X)$ is rough connected. Thus $U = (\underline{U}, \overline{U})$ cannot be rough open and rough closed. Hence the only both rough closedand rough open subsets of $RX = (\underline{R}X , \overline{R}X)$ are empty set and $RX = (\underline{R}X , \overline{R}X)$ itself.

5.5.Example. Any rough set with an indiscrete rough topology is rough connected
Let $RX = (\underline{R}X , \overline{R}X)$,In an indiscrete topology,$\underline{\tau}$ $=\{\emptyset, \underline{R}X\}and\overline{\tau} = \{\varphi, \overline{R}X\}$.
So the empty set $\varphi = (\varphi, \varphi)$,RX are only subsets of $RX = (\underline{R}X , \overline{R}X)$ that are both rough open and rough closed in $RX = (\underline{R}X , \overline{R}X)$.Therefore $RX = (\underline{R}X , \overline{R}X)$ is rough connected.

5.6.Theorem. If the sets C and D form a separation of $RX = (\underline{R}X , \overline{R}X)$ and if$RY = (\underline{R}Y, \overline{R}Y)$is a rough connected subspace ofRX, then $RY = (\underline{R}Y, \overline{R}Y)$ lies entirely within either C or D.

Proof. Since $\underline{C}$ and $\underline{D}$ are both lower open in$\underline{R}X$, the sets $\underline{C} \cap \underline{R}Y$ and$\underline{D} \cap \underline{R}Y$ are lower open in $\underline{R}Y$.These two sets are disjoint and their union is $\underline{R}Y$; if they were both non emptythey would constitute a separation of $\underline{R}Y$.Therefore one of them is empty .Hence$\underline{R}Y$ must lie entirely in $\underline{C}$ or $\underline{D}$. Also, since $\overline{C}$ and $\overline{D}$ are both upper open in $\overline{R}Y$.These two sets are disjoint and their union is$\overline{R}Y$; if they were both nonempty, they would constitute a separation of $\overline{R}Y$.Therefore,one of them is empty. Hence $\overline{R}Y$ must lie entirely in $\overline{C}$ or in$\overline{D}$.Therefore if the sets C and D form a separation of $RX = (\underline{R}X , \overline{R}X)$ and if $RY = (\underline{R}Y, \overline{R}Y)$ is aroughconnected subspace ofRX , then $RY = (\underline{R}Y, \overline{R}Y)$ lies entirely within either C or D. Hence the theorem is proved.

5.7.Theorem. The union of a collection of rough connected subspaces of $RX = (\underline{R}X , \overline{R}X)$ that have a point in common is rough connected.

Proof. Let $\{A_{\alpha}\}$ be a collection of lower connected subspaces of a space $\underline{R}X$; let 'p' be the point of $\cap A_{\alpha}$ .We prove that the space $\underline{R}Y = \cup A_{\alpha}$ is lower connected. Suppose that $\underline{R}Y = \underline{C} \cup \underline{D}$ is a separation of $\underline{R}Y$. The point 'p' is in one of the sets $\underline{C}$ or $\underline{D}$; suppose p$\in \underline{C}$ .Since A_{α} is lower connected,it must lie entirely in either $\underline{C}$ or $\underline{D}$ and it cannot lie in $\underline{D}$ because it contains a point 'p' of $\underline{C}$. Hence $A_{\alpha} \subset \underline{C}$ forevery α, so that $\cup A_{\alpha} \subset \underline{C}$, contradicting the fact that $\underline{D}$ isnon empty. Similarly, union of a collection of upper subspaces of $\overline{R}X$ that have a point in common is upper connected. Therefore the union of a collection of rough subspaces of $RX = (\underline{R}X, \overline{R}X)$ that have a point in common is rough connected. Hence the theorem is proved.

5.8.Theorem. Let A be a rough connected subspace of $RX = (\underline{R}X, \overline{R}X)$. If A$\subset B \subset cl(A)$,then B is also rough connected.

Proof. Let A be lower connected and $A \subset B \subset cl(A)$.Suppose that $B = \underline{C} \cup \underline{D}$ where $\underline{C}$ and $\underline{D}$ lower open sets, is a separation of B. By theorem5.6, the set A must lie entirely in $\underline{C}$ or in $\underline{D}$;suppose that $A \subset \underline{C}$.Then $cl(A) \subset rcl(\underline{C})$;since $cl(\underline{C})$ and $\underline{D}$ are disjoint, B cannot intersect $\underline{D}$.This contradicts the fact that $\underline{D}$ is a nonempty subset of B. Also, let A be upper connected and $A \subset B \subset cl(A)$.Suppose that $B = \overline{C} \cup \overline{D}$ where $\overline{C}$ and $\overline{D}$ upper open sets, is a separation of B. By theorem 5.6,the set A must lie entirely in $\overline{C}$ or in $\overline{D}$;suppose that $A \subset \overline{C}$ Then $cl(A) \subset rcl(\overline{C})$;since $cl(\overline{C})$ and $\overline{D}$ are disjoint, B cannot intersect $\overline{D}$. This contradicts the fact that $\overline{D}$ is a nonempty subset of B. Therefore, if A be a rough connected subspace of $RX = (\underline{R}X, \overline{R}X)$ and if A$\subset B \subset cl(A)$, the B is also rough connected.

5.9.Theorem. Let $f = (\underline{f}, \overline{f}) = (\underline{R}X, \overline{R}X) \to (\underline{R}Y, \overline{R}Y)$ be a continuous surjection. Then if $RX = (\underline{R}X, \overline{R}X)$ is rough connectedso is $RY = (\underline{R}Y, \overline{R}Y)$

Proof. Firstly,suppose $\underline{R}Y$ is not lower connected. Then we write $\underline{R}Y = \underline{U} \cup \underline{V}$ where $\underline{U}$ and $\underline{V}$ are disjoint, nonempty and lower open subsets of $\underline{R}Y$. Then $\underline{R}X = \underline{f}^{-1}(\underline{R}Y) = \underline{f}^{-1}(\underline{U}) \cup \underline{f}^{-1}(\underline{V})$ .The sets $\underline{f}^{-1}(\underline{U})$ and $\underline{f}^{-1}(\underline{V})$ are both lower open in $\underline{R}X$ and disjoint also since $\underline{f}$ is continuous and each is nonempty since $\underline{f}$ is onto and hence $\underline{R}X$ is not lower connected which is a contradiction. Therefore $\underline{R}Y$ is lowerconnected. Also, suppose $\overline{R}Y$ is not upper connected. Then we can write $\overline{R}Y = \overline{U} \cup \overline{V}$ where $\overline{U}$ and $\overline{V}$ are disjoint,nonempty and upper open subsets of $\overline{R}Y$.Then $\overline{R}X = \overline{f}^{-1}(\overline{R}Y) = \overline{f}^{-1}(\overline{U}) \cup \overline{f}^{-1}(\overline{V})$. The sets $\overline{f}^{-1}(\overline{U})$ and $\overline{f}^{-1}(\overline{V})$ are both upper open in $\overline{R}X$ and disjoint also since $\overline{f}$ is continuous and each is nonempty since $\overline{f}$ is onto and hence $\overline{R}X$ is not upper connected which is a contradiction. Therefore $\overline{R}Y$ is upper connected.Therefore, if RX is rough connected, RY is rough connected. Hence the theorem is proved.

VI. Separation axioms(T_1 and T_2 in rough topology)

6.1. Definition. Let $RX = (\underline{R}X, \overline{R}X)$ be a topological space with thetopology $\tau = (\underline{\tau}, \overline{\tau})$. Then $\underline{R}X$ is said to be lower $T_1 -$ space if for every pair of distinct points x, y in $\underline{R}X$, there exists two lower open sets $\underline{U}$ and $\underline{V}$ such that $x \in \underline{U}, y \notin \underline{U}$ and $y \in \underline{V}, x \notin \underline{V}$ .Also, $\overline{R}X$ said to be upper $T_1 -$ space if for every pair of distinct points x, y in $\overline{R}X$, there exist two upper open sets $\overline{U}$ and $\overline{V}$ such that $x \in \overline{U}, y \notin \overline{U}$ and $y \in \overline{V}, x \notin \overline{V}$. Then $RX = (\underline{R}X, \overline{R}X)$ is said to be rough $T_1 -$ space

6.2. Example.In the example5.2. $\underline{R}X = \{O_1, O_2\} \cup \{O_4\}$, $\overline{R}X = \{O_1, O_2\} \cup \{O_4\} \cup \{O_3, O_7, O_{10}\}$ and $\underline{\tau} = \{\underline{R}X, \varphi, \{O_1, O_2\}, \{O_2\}, \{\ , O_4\}\}$,
$\overline{\tau} = \{\overline{R}X, \varphi, \{O_1, O_2, O_3\}, \{O_2\}, \{O_2, O_4 O_7\}, \{O_4, O_7\}\}$.Since $\underline{\tau}$ satisfies all the conditions of the topology, $\underline{\tau}$ is a lower rough topology on $\underline{R}X$. Also since $\overline{\tau}$ satisfies all the conditions of the topology, $\overline{\tau}$ is a upper rough topology on $\overline{R}X$. Take $x = O_1, y = O_2$ and lower open sets $\underline{U} = \{O_1, O_2\}, \underline{V} = \{O_2\}$.Here the distinct points O_1 and O_2 there does not exist two lower open sets $\underline{U}$ and $\underline{V}$ satisfying $x \in \underline{U}, y \notin \underline{U}$ and $y \in \underline{V}, x \notin \underline{V}$ .This is not lower $T_1 -$ space. Also take $x = O_1, y = O_2$ and upper open sets $\overline{U} = \{O_1, O_2, O_3\}, \overline{V} = \{O_2\}$. Here the distinct points O_1 and O_2 there does not exist two upper open sets $\overline{U}$ and $\overline{V}$ satisfying $x \in \overline{U}, y \notin \overline{U}$ and $y \in \overline{V}, x \notin \overline{V}$. This is not upper $T_1 -$ space. Therefore this is not rough $T_1 -$ space.

6.3.Theorem.A topological space $RX = (\underline{R}X, \overline{R}X)$ is T_1 if and only if points in $RX = (\underline{R}X, \overline{R}X)$ are closed sets.

Proof. Assume $\underline{R}X$ is T_1. Take two different points x and y in $\underline{R}X$.Now, take the complement of y. By the T_1 definition, we can express y^c as the union of all lower open sets $O_{\underline{x}} \in \underline{R}X$ in which $y \notin O_{\underline{x}}$.But the arbitrary union of lower open sets is lower open, so the complement of y is lower open and y is lower closed. Now assume points are lower closed setsin$\underline{R}X$. Take a pair of distinct points $x, y \in \underline{R}X$ and consider the complement of y. This set is lower open and contains x. We have thus found a lower open set which contains x, but does not contain y, so $\underline{R}X$is T_1. Similarly, also a topological space $\overline{R}X$ is T_1 if and only if points in $\overline{R}X$ are upper closed sets. Therefore, a topological space $RX = (\underline{R}X, \overline{R}X)$is T_1 if and only if points in $RX = (\underline{R}X, \overline{R}X)$ are closed sets.

6.4.Definition. Let $RX = (\underline{R}X, \overline{R}X)$ be a topological space with the topology $\tau = (\underline{\tau}, \overline{\tau})$.Then$\underline{R}X$ is said to be lower T_2 - space if for every pair of distinct points $x, y \in \underline{R}X$, there exist two lower open sets $\underline{U}$ and $\underline{V}$ such that $x \in \underline{U}, y \in \underline{V}$ and $\underline{U} \cap \underline{V} = \varphi$. Also,$\overline{R}X$ is said to be upper T_2 - space if for every pair of distinct points $x, y \in \overline{R}X$,there exist two upper open sets $\overline{U}$and $\overline{V}$ such that $x \in \overline{U}, y \in \overline{V}$ and $\overline{U} \cap \overline{V} = \varphi$.Therefore the set $RX = (\underline{R}X, \overline{R}X)$ is rough T_2 -space in rough topology

6.5.Theorem. The product space of two topological spaces $RX \times RY$ where $RX = \left(\underline{R}X, \overline{R}X\right)$ and $RY = \left(\underline{R}Y, \overline{R}Y\right)$is Hausdorff if and only if both $RX = (\underline{R}X, \overline{R}X)$ and $RY = (\underline{R}Y, \overline{R}Y)$ are Hausdorff.

Proof. Assume $\underline{R}X \times \underline{R}Y$ is Hausdorff. We know the product topology equips us with two continuous projection functions $\pi_{\underline{x}}$ and $\pi_{\underline{y}}$which maps open sets from $\underline{R}X \times \underline{R}Y$ to $\underline{R}X$ and$\underline{R}Y$ respectively. If we take two distinct points $(x_1, y_1)$and $(x_2, y_2) \in \underline{R}X \times \underline{R}Y$. We have two disjoint lower open sets$O_{\underline{x_1}} \times O_{\underline{y_1}}andO_{\underline{x_2}} \times O_{\underline{y_2}}$. Now ,take $\pi_{\underline{x}}\left(O_{\underline{x_1}} \times O_{\underline{y_1}}\right) = O_{\underline{x_1}}$ and $\pi_{\underline{x}}\left(O_{\underline{x_2}} \times O_{\underline{y_2}}\right) = O_{\underline{x_2}}$ .We can always do this, because these lower open sets are just the union of the base sets$\underline{U} \times \underline{V}$ of $\underline{R}X \times \underline{R}Y$. By our construction $O_{\underline{x_1}}$ and $O_{\underline{x_2}}$ are disjoint, so $\underline{R}X$ is Hausdorff. The argument for $O_{\underline{y}}$ is identical, so $\underline{R}Y$ is Hausdorff. Therefore $\underline{R}X$ and $\underline{R}Y$ are Hausdorff. Similarly,$\overline{R}X$ and $\overline{R}Y$ are Hausdorff. Now assume $\underline{R}X$ and $\underline{R}Y$ are Hausdorff. Take two distinct points x_1 and x_2 in $\underline{R}X$ and the open sets surrounding them, $O_{\underline{x_1}}$and$O_{\underline{x_2}}$. We can form disjoint open sets in$\underline{R}X \times \underline{R}Y$, just by taking $O_{\underline{x_1}} \times \underline{R}Y$ and $O_{\underline{x_2}} \times \underline{R}Y$.We can do the same thing for $\underline{R}Y$, and create disjoint, open sets containing the points (x, y). Thus $\underline{R}X \times \underline{R}Y$is Hausdorff. Similarly, also $\overline{R}X \times \overline{R}Y$ is Hausdorff. Therefore the product space of two topological spaces$RX \times RY$ where $RX = (\underline{R}X, \overline{R}X)$ and$RY = (\underline{R}Y, \overline{R}Y)$is Hausdorff if and only if both $RX = (\underline{R}X, \overline{R}X)$ and $RY = (\underline{R}Y, \overline{R}Y)$ are Hausdorff.

VII. Conclusion

Rough set theory is a mathematical tool to deal with vagueness or imperfect knowledge using boundary region approach. In this paper, compactness and connectedness in rough topological spaces is studied. Then some topological properties of the resulting Rough Topological Space,are studied. This is just a beginning of a new area and using the ideas presented in this paper, several topological properties of the rough topological spaces can be studied. So there are lots of research scopes in this area.

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