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Regular Weakly Generalized Locally Closed Sets in Ideal Topological Spaces

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Abstract: *In this paper we have introduced the concept of regular weakly generalized locally closed sets in ideal topological spaces. Properties and characterizations are discussed.*
Keywords: *I_{rwg}-lc set, $I_{rwg}lc^*$ set, $I_{rwg}lc^{**}$ set.*

I. Introduction

A nonempty collection I of subsets on a topological space (X, τ) is called a topological ideal [3] if it satisfies the following two conditions:
(i) If A∈ I and B⊂ A implies B∈ I (heredity)
(ii) If A ∈ I and B∈I, then A∪ B∈I (finite additivity)

Local function in topological spaces using ideals was introduced by Kuratowski [3]. Donchev [2] introduced the concept of I-locally closed sets. After that Navaneetha Krishnan and Sivaraj [4] introduced I - locally *- closed sets and I_g-locally *-closed sets.

II. Preliminaries

Definition 2.1.: A subset A of a topological space (X, τ, I) is called
(i) I-locally *-closed [4] if there exist an open set U and a *-closed set F such that A = U ⋂ F,
(ii) I_g-locally *-closed [4] if there exist an I_g-open set U and a *-closed set F such that A = U ⋂ F.
Definition 2.2: For a subset A of a topological space (X, τ) is said to be
(i) A ⊆ $GLC^*(X, \tau)$[1] if there exist a g-open set U and a closed set F of (X, τ) such that A = U ⋂ F,
(ii) A ⊆ $GLC^{**}(X, \tau)$[1] if there exist a open set U and a g-closed set F of (X, τ) such that A = U ⋂ F.
Definition 2.3: A subset A of an ideal topological space (X, τ, I) is called a
(i) rpsIlc-set [5] if there exists a rpsI-open set U and a rpsI-closed set F of (X, τ, I) such that A = U ⋂ F,
(ii) rpsIlc*-set [5] if there exists a rpsI-open set U and a closed set F of (X, τ, I) such that A = U ⋂ F,
(iii) rpsIlc**-set [5] if there exists a open set U and a rpsI-closed set F of (X, τ, I) such that A = U ⋂ F.

III. I_{rwg}LC SETS AND $I_{rwg}LC^*$ SETS

In this section, regular weakly generalized locally closed sets are and introduced.
Definition 3.1: A subset A of an ideal topological spaces (X, τ, I) is said to be a regular weakly generalized locally closed (I_{rwg}lc) set if A = U ∩ F where U is I_{rwg}-open and F is I_{rwg}-closed in X.
Definition 3.2: A subset A of an ideal topological space (X, τ, I) is said to be $I_{rwg}lc^*$ if there exist an I_{rwg}-open set U and a closed set F of X such that A = U ∩ F .
Definition 3.3: A subset A of an ideal topological spaces (X ,τ , I) is said to be $I_{rwg}lc^{**}$ if there exist a open set U and a I_{rwg}-closed set F of X such that A = U ∩ F.
The collection of all I_{rwg}lc- sets (resp. $I_{rwg}lc^*$ and $I_{rwg}lc^{**}$) of (X , τ, I) is denoted by I_{rwg} LC in (X ,τ) (resp. I_{rwg} LC^* (X ,τ) and $I_{rwg}LC^{**}$(X ,τ).
Theorem 3.4: For a ideal topological space (X, τ, I) the following implications hold.
(i) ILC (X ,τ) ⊆ IRWGLC*(X ,τ)⊆ IRWGLC (X ,τ)
(ii) ILC (X ,τ) ⊆ IRWGLC**(X ,τ)⊆ IRWGLC (X ,τ)
The reverse implications need not be true as seen from the following example.
Example 3.5: Let X = { a,b,c}, τ = { ∅, {a},{b},{a,b}, X}, I = { ∅, {a}}, then $I_g$lc closed sets are **{**∅ , {a}, {b}, {c}, {a,b}, {b,c}, X} and the I_{rwg}lc sets are {∅, {a}, {b}, {c}, {a,b},{b,c},{a,c}, X}. Hence {a,c} is an I_{rwg}lc sets but not Ilc set.
Example 3.6: Let X = { a, b, c, d}, τ = { ∅, {a},{b},{a, b}, X}, I = { ∅, {a}}, then I_g lc closed sets are **{** ∅, {a},{b},{c},{d},{a, b},{b, c},{c, d},{a, b, c},{b, c, d}, X} and the $I_{rwg}lc^*$ sets are {∅, {a},{b},{c},{d},{a, b},{b, c},{a, c},{a, d},{b, c}, {b, d},{c, d},{a, b, c},{b, c, d}, X}. Hence {a, d} are $I_{rwg}lc^*$ sets but not Ilc set.

I_{rwg} cl(A) is the smallest I_{rwg}-closed set containing A.

Theorem 3.7: Let A be any subset of X, then

(i) A is I_{rwg}-closed in X if and only if A = I_{rwg} cl(A)

(ii) I_{rwg}-cl(A) is I_{rwg}-closed in X

(iii) x ∈ I_{rwg} cl(A) if and only if A ∩ U ≠ ∅ for every I_{rwg}-open set U containing x.

Proof: (i) and (ii) are trivially true.

(iii) Suppose that there exists an I_{rwg}-open set U containing x such that A ∩ U = ∅. Since X − U is I_{rwg}-closed and A ⊆ X − U, I_{rwg}cl(A) ⊆ X − U. Therefore x ∉ I_{rwg}cl (A). Conversely suppose that x ∉ I_{rwg} cl(A). Then U = X − I_{rwg} cl(A) is I_{rwg} -open set containing x and A ∩ U = ∅.

Theorem 3.8: For a subset A of (X, τ, I), the following statements are equivalent.

(i) A ∈ I_{rwg}LC(X ,τ)

(ii) A = U ∩ I_{rwg} cl(A) for some I_{rwg}-open set U.

(iii) I_{rwg}cl(A) − A is I_{rwg}-closed.

(iv)A ∪ (X − I_{rwg} cl(A)) is I_{rwg}-open.

Proof: (i) ⇒ (ii) Suppose A ∈ I_{rwg}LC(X, τ). Then there exists an I_{rwg}-open subset U and I_{rwg}- closed subset F such that A = U ∩ F. Since A ⊆ U and A ⊆ I_{rwg} cl(A), A ⊆ U ∩ I_{rwg} cl(A). Also by Theorem 3.7, I_{rwg}cl(A) is I_{rwg}-closed in X. Hence I_{rwg} cl(A) ⊆ F and U ∩ I_{rwg}cl(A) ⊆ U ∩ F = A. Therefore A = U ∩ I_{rwg}cl(A).

(ii)⇒ (i) By Theorem 3.7, I_{rwg}cl(A) is I_{rwg}-closed and hence A = U ∩ I_{rwg} cl(A) ∈ I_{rwg}LC(X, τ).

(iii) ⇒ (iv) Let S = I_{rwg}cl(A) −A. Then, by assumption S is I_{rwg}-closed which implies X − S is I_{rwg}-open and X− S =X ∩ (X – S) = X ∩ ((X – (I_{rwg}cl(A) – A)) = A ∪ (X− I_{rwg}cl(A)). Thus A ∪ (X − I_{rwg}cl(A)) is I_{rwg}-open.

(iv) ⇒ (iii) Let W = A ∪ (X − I_{rwg}cl(A)). Then W is I_{rwg}-open. This implies that X −W is I_{rwg}-closed and X −W = X – (A ∪ (X− I_{rwg}cl(A)) = I_{rwg}cl(A) ∩ X − A = I_{rwg}cl(A) −A. Thus I_{rwg}cl(A) − A is closed.

(iv) ⇒ (ii) Let U = A ∪ (X − I_{rwg}cl(A)). Then U is I_{rwg}–open. U ∩ I_{rwg}cl(A) = (A ∪ (X − I_{rwg}cl(A))) ∩ I_{rwg}cl(A) = (I_{rwg}cl(A) ∩ A) ∪ (I_{rwg}cl(A) ∩ X− I_{rwg}cl(A)) = A ∪ ∅ = A. Therefore, A = U ∩ I_{rwg}cl(A) for some I_{rwg} -open set U.

(ii) ⇒ (iv) Let A = U ∩ I_{rwg} cl(A), for some I_{rwg}-open set U. A ∪ (X − I_{rwg}cl(A)) = (U ∩ I_{rwg}cl(A) ∪ (X − I_{rwg}cl(A)) = U ∩ (I_{rwg} cl(A) ∪ (X − I_{rwg}cl(A))) = U ∩ X = U, is I_{rwg}-open.

Theorem 3.9: For a subset A of (X ,τ , I), the following statements are equivalent.

(i) A ∈ $I_{rwg}LC^*$(X, τ)

(ii) A = U ∩ cl(A) for some I_{rwg}-open set U.

(iii) cl^*(int(A)) − A is I_{rwg}-closed.

(iv) A ∪ (X − cl^*(int(A))) is I_{rwg}-open.

Proof: The proof is similar to that of above theorem

Theorem 3.10: Let A be a subset of (X, τ, I). If A ∈ $I_{rwg}LC^{**}$(X, τ) then I_{rwg}cl(A) −A is I_{rwg}-closed and A ∪ (X − I_{rwg} cl(A)) is I_{rwg}-open.

Proof: Let A ∈ $I_{rwg}LC^{**}$(X, τ). Then there exists an open set U such that A = U ∩ I_{rwg}cl(A). A ∪ (X − I_{rwg} cl(A)) = (U ∩ I_{rwg} cl(A)) ∪ (X − I_{rwg} cl(A)) = U ∩ (I_{rwg} cl(A) ∪ (X − I_{rwg}cl(A))) = U ∩ X = U, is open. Since every open set is I_{rwg}-open, A ∪ (X − I_{rwg} cl(A)) is I_{rwg}-open. Let W = A ∪ (X − I_{rwg}cl(A)). Then W is I_{rwg}-open implies X −W is I_{rwg}-closed and X −W = X − (A ∪ (X − I_{rwg}cl(A))) = I_{rwg}cl(A) ∩ X − A = I_{rwg}cl(A) − A. Thus I_{rwg}cl(A) − A is I_{rwg}-closed.

Theorem 3.11: Let A and B be subsets of (X, τ, I). If A ∈ I_{rwg}LC (X, τ) and B is I_{rwg}-open, then A ∩ B ∈ I_{rwg}LC(X ,τ).

Proof: Let A ∈ I_{rwg}LC(X, τ). Then A = U ∩ F where U is I_{rwg}-open and F is I_{rwg}-closed. So A ∩ B = U ∩ F ∩ B = U ∩ B ∩ F. This implies that A ∩ B ∈ I_{rwg}LC(X, τ).

Theorem 3.12: Let A and B be subsets of (X, τ, I). If A ∈ $I_{rwg}LC^*$ (X ,τ) and B ∈ $I_{rwg}LC^*$ (X, τ) then A ∩ B ∈ $I_{rwg}LC^*$ (X, τ).

Proof: Let A and B ∈$I_{rwg}LC^*$(X, τ). Then there exists I_{rwg}-open sets P and Q such that A = P ∩ cl(A) and B = Q ∩ cl(B). Therefore A ∩ B = P ∩ cl(A) ∩ Q ∩ cl(B) = P ∩ Q ∩ cl(A) ∩ cl(B) where P ∩ Q is I_{rwg}-open and cl(A) and cl(B) is closed. This shows that A ∩ B ∈ $I_{rwg}LC^*$ (X, τ).

Theorem 3.13: If A ∈ $I_{rwg}LC^{**}$ (X, τ) and B is I_{rwg}-open, then A ∩ B ∈ $I_{rwg}LC^{**}$ (X, τ).

Proof: Let A ∈ $I_{rwg}LC^{**}$ (X, τ). Then there exists an open set U and an I_{rwg}-closed set F such that A = U ∩ F. So A ∩ B = U ∩ F ∩ B = U ∩ B ∩ F. This proves that A ∩ B ∈ $I_{rwg}LC^{**}$ (X, τ).

Theorem 3.14: Let A and Z be subsets of (X, τ, I) and let A ⊆ Z. If Z is I_{rwg}-open in (X, τ, I) and A ∈ $I_{rwg}LC^*$ (Z, τ/Z), then A ∈ $I_{rwg}LC^*$(X, τ).

Proof: Suppose that A is $I_{rwg}lc^*$, then there exist an I_{rwg}-open set U of (Z, τ/Z) such that A = U ∩ cl_z(A). But cl_Z(A) = Z ∩ cl(A). Therefore, A = U ∩ Z ∩ cl(A) where U ∩ Z is I_{rwg}-open. Thus A ∈ $I_{rwg}LC^*$(X, τ).

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