

On Intuitionistic β -Continuous Functions

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Abstract: The concept of intuitionistic fuzzy set and intuitionistic fuzzy topological space were defined by Atanassov. Later Coker introduced the concept of intuitionistic set and intuitionistic points. He also introduced the concept of intuitionistic topological space and investigated basic properties of continuous function and compactness. In a recent paper, the concept of intuitionistic β -open, intuitionistic β -closure and intuitionistic β -interior in intuitionistic topological space were defined by Singaravelan. Also some basic properties of intuitionistic β -open set were discussed. The purpose of this paper is to introduce and study the concept of intuitionistic β -continuous functions in intuitionistic topological space and studied its relation with other existing intuitionistic continuous functions.

Keywords: $I\beta$ -open sets, $I\beta$ -closed sets, $I\beta$ -closure, $I\beta$ -interior, $I\beta$ -continuous.

I. Introduction

In 1986, Atanassov[4] introduced the concept of Intuitionistic fuzzy sets as a generalization of fuzzy sets. Later in 1996, Coker [9] introduced the concept of Intuitionistic set and Intuitionistic points. This is a discrete form of intuitionistic fuzzy sets where all the sets are crisp set. In 2000, Coker [11] also introduced the concept of "intuitionistic topological space" and investigated basic properties of continuous functions and compactness. In general topological space N. Levine [16] introduced semi open sets and semi continuity in topological space and M.E. Abd El. Monsef et.al [1] introduced " β -open sets and β -continuous mapping" and discussed some basic properties. D.Andrijevic[3] introduced and discussed some more properties of semi pre open set in topological space. A. Csaszar[5, 6] introduced and discussed generalized open set, γ -interior and γ -closure in topological space.

Gnanambal Ilango and Selvanayagi [14], introduced generalized pre regular closed sets in intuitionistic topological spaces. A. Singaravelan [21] introduced intuitionistic β -open set in intuitionistic topological space which called intuitionistic semi pre open in ITS(X).

In this paper, properties of intuitionistic β -continuous mappings are discussed.

II. Preliminaries

Let us recall some basic definitions and results which are useful for this sequel. Throughout the present study, a space X means an intuitionistic topological space.

Definition 2.01 [9]

Let X is a non empty set. An intuitionistic set (IS for short) A is an object having the form $A = \langle X, A_1, A_2 \rangle$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \emptyset$. The set A_1 is called the set of members of A, while A_2 is called the set of non-members of A.

Definition 2.02 [9]

Let X be a non empty set and let A, B are intuitionistic sets in the form $A = \langle X, A_1, A_2 \rangle$, $B = \langle X, B_1, B_2 \rangle$ respectively. Then

- $A \subseteq B$ iff $A_1 \subseteq B_1$ and $B_2 \subseteq A_2$
- $A = B$ iff $A \subseteq B$ and $B \subseteq A$
- $A^c = \langle X, A_2, A_1 \rangle$
- $[] A = \langle X, A_1, (A_1)^c \rangle$
- $A - B = A \cap B^c$
- $\phi_- = \langle X, \phi, X \rangle$, $X_- = \langle X, X, \phi \rangle$
- $A \cup B = \langle X, A_1 \cup B_1, A_2 \cap B_2 \rangle$.
- $A \cap B = \langle X, A_1 \cap B_1, A_2 \cup B_2 \rangle$.

Furthermore, let $\{A_\alpha: \alpha \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_\alpha = \langle X, A_\alpha^{(1)}, A_\alpha^{(2)} \rangle$. Then

- $\bigcap A_\alpha = \langle X, \bigcap A_\alpha^{(1)}, \bigcup A_\alpha^{(2)} \rangle$.
- $\bigcup A_\alpha = \langle X, \bigcup A_\alpha^{(1)}, \bigcap A_\alpha^{(2)} \rangle$.

Definition 2.03 [11]

An intuitionistic topology (for short IT) on a non empty set X is a family of IS's in X satisfying the following axioms.

- (i) $\phi_{\sim}, X_{\sim} \in \tau$
- (ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.
- (iii) $\cup G_{\alpha} \in \tau$ for any arbitrary family $\{G_i: G_{\alpha}/ \alpha \in J\} \subseteq \tau$ where (X, τ) is called an intuitionistic topological space (for short ITS(X)) and any intuitionistic set in τ is called an intuitionistic open set (for short IOS) in X. The complement A^c of an IOS A is called an intuitionistic closed set (for short ICS) in X.

Definition 2.04[11] Let (X, τ) be an intuitionistic topological space (for short ITS(X)) and $A = \langle X, A_1, A_2 \rangle$ be an IS in X. Then the interior and closure of A are defined by

$$\text{Icl}(A) = \cap \{K: K \text{ is an ICS in } X \text{ and } A \subseteq K\},$$

$$\text{Iint}(A) = \cup \{G : G \text{ is an IOS in } X \text{ and } G \subseteq A\}.$$

It can be shown that $\text{Icl}(A)$ is an ICS and $\text{Iint}(A)$ is an IOS in X and A is an ICS in X iff $\text{Icl}(A) = A$ and is an IOS in X iff $\text{Iint}(A) = A$.

Definition 2.05[9] Let X be a non empty set and $P \in X$. Then the IS P defined by $P = \langle X, \{p\}, \{p\}^c \rangle$ is called an intuitionistic point (IP for short) in X. The intuitionistic point P is said to be contained in $A = \langle X, A_1, A_2 \rangle$ (i.e $p \in A$) if and only if $p \in A_1$.

Definition 2.06[14]

Let (X, τ) be an ITS(X). An intuitionistic set A of X is said to be

- (i) Intuitionistic semiopen if $A \subseteq \text{Icl}(\text{Iint}(A))$.
- (ii) Intuitionistic preopen if $A \subseteq \text{Iint}(\text{Icl}(A))$.
- (iii) Intuitionistic regular open if $A = \text{Iint}(\text{Icl}(A))$.
- (iv) Intuitionistic α -open if $A \subseteq \text{Iint}(\text{Icl}(\text{Iint}(A)))$.

The family of all intuitionistic pre open, intuitionistic regular open and intuitionistic α -open sets of (X, τ) are denoted by IPOS, IROS and I α OS respectively.

Definition 2.07[21]

A subset A of an intuitionistic topological space X is intuitionistic β -open set, if there exists a intuitionistic preopen set U in X, such that $U \subseteq A \subseteq \text{Icl}(U)$. The family of all intuitionistic β -open sets in X will be denoted by I β OS(X). The complement of intuitionistic I β -open set is I β -closed set.

Definition 2.08 [9,11]

Let $A, A_i (i \in J)$ be IS's in X, $B, B_j (j \in K)$ IS's in Y and $f: X \rightarrow Y$ a function. Then

- (a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$
- (b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
- (c) $A \subseteq f^{-1}(f(A))$ and if f is 1-1, then $A = f^{-1}(f(A))$.
- (d) $f(f^{-1}(B)) \subseteq B$ and if f is onto, then $f(f^{-1}(B)) = B$.
- (e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$.
- (f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$.
- (g) $f(\cup A_i) = \cup f(A_i)$.
- (h) $f(\cap A_i) \subseteq \cap f(A_i)$ and if f is 1-1, then $f(\cap A_i) = \cap f(A_i)$.
- (i) $f^{-1}(Y_{\sim}) = X_{\sim}$. (j) $f^{-1}(\phi_{\sim}) = \phi_{\sim}$.
- (k) $f(X_{\sim}) = Y_{\sim}$. If f is onto. (l) $f(\phi_{\sim}) = \phi_{\sim}$.
- (m) If f is onto, then $\overline{f(A)} \subseteq f(\overline{A})$; and if furthermore, f is 1-1, we have $\overline{f(A)} \subseteq f(\overline{A})$.
- (n) $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$
- (o) $B_1 \supseteq B_2 \Rightarrow f^{-1}(B_1) \supseteq f^{-1}(B_2)$.

Definition 2.09[11] Let (X, τ) and (Y, Φ) be two ITS's and let $f: X \rightarrow Y$ be a function. Then f is said to be continuous iff the preimage of each IS in Φ is an IS in τ .

Definition 2.10[5] Let (X, τ) and (Y, Φ) be two ITS's and let $f: X \rightarrow Y$ be a function. Then f is said to be open iff the preimage of each IS in τ is an IS in Φ .

Definition 2.11 Let (X, τ) and (Y, Φ) be two ITS's and let $f: X \rightarrow Y$ is called intuitionistic semi continuous if for every intuitinistic set V of Y, $f^{-1}(V)$ is semi open in X.

Definition 2.12 Let (X, τ) and (Y, Φ) be two ITS's and let $f: X \rightarrow Y$ is called intuitionistic regular continuous if for every intuitinistic open set V of Y, $f^{-1}(V)$ is regular open in X.

Definition 2.13 Let (X, τ) and (Y, Φ) be two ITS's and let $f: X \rightarrow Y$ is called intuitionistic pre continuous if for every intuitinistic open set V of Y, $f^{-1}(V)$ is pre open in X.

Definition 2.14 Let (X, τ) and (Y, Φ) be two ITS's and let $f: X \rightarrow Y$ is called intuitionistic α - continuous if for every intuitinistic open set V of Y, $f^{-1}(V)$ is α - open in X.

Definition 2.15[11] Let (X, τ_1) and (Y, τ_2) be two ITS's on X . then τ_1 is said to be contained in τ_2 (in symbols, $\tau_1 \subseteq \tau_2$), if $G \in \tau_2$ for each $G \in \tau_1$. In this case, we also say that τ_1 is coarser than τ_2 .

Definition 2.16 [21]

Let (X, τ) be an intuitionistic topological space and let $A = \langle X, A_1, A_2 \rangle$ be the subset of X . then $I\beta\text{-cl}(A) = \bigcap \{F : F \text{ is intuitionistic } \beta\text{-closed in } X \text{ and } A \subseteq F\}$.

Definition 2.17 [21]

Let (X, τ) be an intuitionistic topological space and let $A = \langle X, A_1, A_2 \rangle$ be the subset of X . then $I\beta\text{-int}(A) = \bigcup \{F : F \text{ is intuitionistic } \beta\text{-open in } X \text{ and } F \subseteq A\}$.

Proposition 2.18[21] A subset $A = \langle X, A_1, A_2 \rangle$ of an ITS(X) is intuitionistic β -open set iff $A \subseteq \text{Icl}(\text{Int}(\text{Icl}(A)))$

Lemma 2.19[21] Let A and B be subsets of ITS(X), then the following results are obvious.

- (i) $I\beta\text{-cl}(X_\sim) = X_\sim$ and $I\beta\text{-cl}(\phi_\sim) = \phi_\sim$
- (ii) If $A \subseteq B$, then $I\beta\text{-cl}(A) \subseteq I\beta\text{-cl}(B)$
- (iii) $I\beta\text{-cl}(I\beta\text{-cl}(A)) = I\beta\text{-cl}(A)$

III. On Intuitionistic β -Continuous Functions

Here intuitionistic β -continuous functions is defined and its relation with existing intuitionistic continuous functions is shown and its basic properties are given.

Definition 3.1: A mapping $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ is said to be intuitionistic β -continuous (briefly $I\beta$ -continuous), if the inverse image of each intuitionistic open set in Y is $I\beta$ -open in X .

Theorem 3.2: Let $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ be intuitionistic continuous, then f is $I\beta$ -continuous.

Proof: Let $A = \langle X, A_1, A_2 \rangle$ be a intuitionistic open set of Y . Since f is intuitionistic continuous, then $f^{-1}(A)$ is intuitionistic open in X , we know that every intuitionistic open set is $I\beta$ -open set, then $f^{-1}(A)$ is $I\beta$ -open in X . Thus f is $I\beta$ -continuous.

The converse of the above theorem need not be true from the following example.

Example 3.3 Let $X = \{a, b, c\} = Y, \tau_1 = \{\phi_\sim, X_\sim, \langle X, \{b\}, \{a, c\} \rangle, \langle X, \{a\}, \{b\} \rangle, \langle X, \{a, b\}, \phi \rangle\}$ and $\tau_2 = \{\phi_\sim, X_\sim, \langle X, \{c\}, \{a, b\} \rangle, \langle X, \{a\}, \{b, c\} \rangle, \langle X, \{a, c\}, \{b\} \rangle, \phi \rangle\}$.

Define $f: (X, \tau_1) \rightarrow (X, \tau_2)$ by $f(a) = c, f(b) = b, f(c) = a$, let $A = \langle X, \{b, c\}, \{a\} \rangle$ is $I\beta$ -open set but $f^{-1}(A) = \langle X, \{b, a\}, \{c\} \rangle$ is not a open in (X, τ_1) . Therefore f is not continuous. However f is $I\beta$ -continuous.

Theorem 3.4: Let $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ be intuitionistic regular continuous, then f is $I\beta$ -continuous.

Proof: Let $A = \langle X, A_1, A_2 \rangle$ be a intuitionistic open set of Y . Since f is intuitionistic regular continuous, then $f^{-1}(A)$ is intuitionistic regular open in X , as every intuitionistic regular open set is $I\beta$ -open, $f^{-1}(A)$ is $I\beta$ -open in X . Thus f is $I\beta$ -continuous.

The converse of the above theorem need not be true from the following example.

Example 3.5: Let $X = \{a, b, c\}, Y = \{a, b\}, \tau_1 = \{\phi_\sim, X_\sim, \langle X, \{c\}, \{a, b\} \rangle, \langle X, \{a\}, \{b, c\} \rangle, \langle X, \{a, c\}, \{b\} \rangle\}$ and $\tau_2 = \{\phi_\sim, X_\sim, \langle X, \{b\}, \phi \rangle, \langle X, \phi, \{a\} \rangle\}$.

Define $f: (X, \tau_1) \rightarrow (X, \tau_2)$ by $f(a) = b, f(b) = b, f(c) = a$, let $D = \langle X, \{b\}, \phi \rangle$ is open set and $f^{-1}(D)$ is $I\beta$ -open but not Ir-open set in (X, τ_1) . Therefore f is not Ir-continuous. However f is $I\beta$ -continuous.

Theorem 3.6: Let $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ be intuitionistic semi continuous, then f is $I\beta$ -continuous.

Proof: Let $A = \langle X, A_1, A_2 \rangle$ be a intuitionistic open set of Y . Since f is intuitionistic semi continuous, $f^{-1}(A)$ is intuitionistic semi open in X , as every intuitionistic semi open set is $I\beta$ -open, $f^{-1}(A)$ is $I\beta$ -open in X . Thus f is $I\beta$ -continuous.

The converse of the above theorem need not be true from the following example.

Example 3.7: Let $X = \{a, b, c\}, Y = \{a, b, c\}, \tau_1 = \{\phi_\sim, X_\sim, \langle X, \{c\}, \{a, b\} \rangle, \langle X, \phi, \{a, b\} \rangle, \langle X, \phi, \{b, c\} \rangle, \langle X, \{c\}, \{b\} \rangle, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \phi, \{b\} \rangle\}$ and $\tau_2 = \{\phi_\sim, X_\sim, \langle X, \{c\}, \{a, b\} \rangle, \langle X, \{a\}, \{b, c\} \rangle, \langle X, \{a, c\}, \{b\} \rangle\}$.

Define $f: (X, \tau_1) \rightarrow (X, \tau_2)$ by $f(a) = c, f(b) = a, f(c) = a$, let $D = \langle X, \{a, c\}, \{b\} \rangle$ is open set and $f^{-1}(D)$ is $I\beta$ -open but not IS-open set in (X, τ_1) . Therefore f is not IS-continuous. However f is $I\beta$ -continuous.

Theorem 3.8: Let $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ be intuitionistic pre continuous, Then f is $I\beta$ -continuous.

Proof: Let $A = \langle X, A_1, A_2 \rangle$ be a intuitionistic open set of Y . Since f is intuitionistic pre continuous, then $f^{-1}(A)$ is intuitionistic pre open in X , as every intuitionistic pre open set is $I\beta$ -open, $f^{-1}(A)$ is $I\beta$ -open in X . Thus f is $I\beta$ -continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 3.9: Let $X = \{a, b, c\}, Y = \{a, b\}, \tau_1 = \{\phi_\sim, X_\sim, \langle X, \{c\}, \{a, b\} \rangle, \langle X, \{a\}, \{b, c\} \rangle, \langle X, \{a, c\}, \{b\} \rangle\}$ and $\tau_2 = \{\phi_\sim, X_\sim, \langle X, \{b\}, \phi \rangle, \langle X, \phi, \{a\} \rangle, \langle X, \{a\}, \{b\} \rangle, \langle X, \{a, b\}, \phi \rangle\}$.

Define $f: (X, \tau_1) \rightarrow (X, \tau_2)$ by $f(a) = a, f(b) = b, f(c) = b$. Let $D = \langle X, \{b\}, \{a, c\} \rangle$ is open set and $f^{-1}(D)$ is $I\beta$ -open but not IP-open set in (X, τ_1) . Therefore f is not IP-continuous. However f is $I\beta$ -continuous.

Theorem 3.10: Let $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ be intuitionistic α -continuous, and then f is $I\beta$ -continuous.

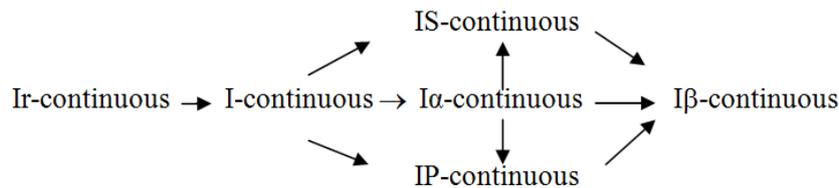
Proof: Let $A = \langle X, A_1, A_2 \rangle$ be an intuitionistic open set of Y . Since f is intuitionistic α -continuous, then $f^{-1}(A)$ is intuitionistic α -open in X , we know that every intuitionistic α -open set is $I\beta$ -open set, then $f^{-1}(A)$ is $I\beta$ -open in X . Thus f is $I\beta$ -continuous.

The converse of the above theorem need not be true from the following example.

Example 3.11: Let $X = \{a, b, c\} = Y$, $\tau_1 = \{\phi, X, \langle X, \{c\}, \{a, b\} \rangle, \langle X, \phi, \{a, b\} \rangle, \langle X, \phi, \{b, c\} \rangle, \langle X, \{c\}, \{b\} \rangle, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \phi, \{b\} \rangle\}$ and $\tau_2 = \{\phi, X, \langle X, \{c\}, \{a, b\} \rangle, \langle X, \{a\}, \{b, c\} \rangle, \langle X, \{a, c\}, \{b\} \rangle\}$.

Define $f: (X, \tau_1) \rightarrow (X, \tau_2)$ by $f(a) = a, f(b) = c, f(c) = c$. Let $D = \langle X, \{a\}, \{b, c\} \rangle$ is open set and $f^{-1}(D)$ is $I\beta$ -open but not $I\alpha$ -open set in (X, τ_1) . Therefore f is not $I\alpha$ -continuous. However f is $I\beta$ -continuous.

Remark 3.12: The following diagram shows the relationship between the some other existing intuitionistic continuous function.



(A \rightarrow B represents A implies B but not conversely)

Theorem 3.13: Let $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping, then the following condition are equivalent

- (i) f is $I\beta$ -continuous.
- (ii) The inverse image of intuitionistic closed set in Y is $I\beta$ -closed in X .

Proof: (i) \rightarrow (ii) Assume f is $I\beta$ -continuous. Let A be a intuitionistic closed subset of Y , then $Y-A$ is intuitionistic open in Y and $f^{-1}(Y-A) = X-f^{-1}(A)$, is $I\beta$ -open in X , which implies that $f^{-1}(A)$ is $I\beta$ -closed in X .

(ii) \rightarrow (i): Assume, the inverse of each intuitionistic closed set in Y is $I\beta$ -closed in X . Let B be an intuitionistic open set in Y , then $Y-B$ is a intuitionistic closed set in Y , which implies $f^{-1}(Y-B) = X-f^{-1}(B)$ is $I\beta$ -closed in X . Hence $f^{-1}(B)$ is $I\beta$ -open in X , which implies that f is $I\beta$ -continuous.

Theorem 3.14: Let $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping, where X and Y are intuitionistic topological spaces, then the following are equivalent.

- (i) The function f is $I\beta$ -continuous.
- (ii) The inverse image of intuitionistic closed set of Y is $I\beta$ -closed set in X .
- (iii) $f(I\beta-cl(A)) \subseteq Icl(f(A))$ for intuitionistic set A of X .
- (iv) $I\beta-cl(f^{-1}(B)) \subseteq f^{-1}(Icl(B))$ for each intuitionistic set of Y .

Proof: (i) \rightarrow (ii): Follows from theorem 3.13.

(ii) \rightarrow (iii): Let A be a intuitionistic set of X . then $Icl(f(A))$ is intuitionistic closed in Y . By (ii) $f^{-1}(Icl(f(A)))$ is $I\beta$ -closed in X and $f^{-1}(Icl(f(A))) = I\beta-cl(f^{-1}(Icl(f(A))))$. Since $A \subseteq f^{-1}(f(A))$ we have $I\beta-cl(A) \subseteq I\beta-cl(f^{-1}(f(A))) \subseteq I\beta-cl(f^{-1}(Icl(f(A)))) = f^{-1}(Icl(f(A)))$, $f(I\beta-cl(A)) \subseteq Icl(f(A))$.

(iii) \rightarrow (iv): Let B be a intuitionistic set of Y . Then by (iii) we have $f(I\beta-cl(f^{-1}(B))) \subseteq Icl(f(f^{-1}(B)))$. Hence $I\beta-cl(f^{-1}(B)) \subseteq f^{-1}(Icl(f(f^{-1}(B)))) \subseteq f^{-1}(Icl(B))$, $I\beta-cl(f^{-1}(B)) \subseteq f^{-1}(Icl(B))$.

(iv) \rightarrow (i): Let B be a intuitionistic set of Y . Then $B^c = C$ is intuitionistic closed subset in Y so that $Icl(C) = C$. Now by condition(iv) $I\beta-cl(f^{-1}(C)) \subseteq f^{-1}(Icl(C)) = f^{-1}(C)$. That is C is intuitionistic closed, we have $f^{-1}(C) \supseteq I\beta-cl(f^{-1}(C)) = (I\beta-int(f^{-1}(C^c)))^c$. Hence $f^{-1}(C)^c$ is $I\beta$ -open in X . That is $f^{-1}(c)$ is $I\beta$ -closed. Therefore f is $I\beta$ -continuous.

Theorem 3.15: Let $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping, where X and Y are intuitionistic topological spaces, and then the followings are equivalent.

- (a) The function f is $I\beta$ -continuous.
- (b) For each subset A of Y , $(f^{-1}(Iint(A))) \subseteq I\beta-int(f^{-1}(A))$

Proof: (a) \Rightarrow (b). Let $A = \langle X, A_1, A_2 \rangle$ be any intuitionistic set of Y , $Iint(A)$ is open set in Y and $f^{-1}(Iint(A))$ is a $I\beta$ -open set in X . since f is $I\beta$ -continuous. As $f^{-1}(Iint(A)) \subseteq f^{-1}(A)$, and $f^{-1}(Iint(A)) \subseteq I\beta-int(f^{-1}(A))$.

(b) \Rightarrow (c) Let A be any intuitionistic open subset of Y , so that $Iint(A) = A$. By condition $f^{-1}(Iint(A)) \subseteq I\beta-int(f^{-1}(A)) \Rightarrow f^{-1}(A) = f^{-1}(Iint(A)) \subseteq I\beta-int(f^{-1}(A)) \Rightarrow f^{-1}(A) \subseteq I\beta-int(f^{-1}(A))$

Hence $f^{-1}(A)$ is $I\beta$ -open, where A is intuitionistic open in Y . Therefore f is $I\beta$ -continuous.

Theorem 3.16: Let $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ be a single valued function, where X and Y are intuitionistic topological space s , then the followings are equivalent.

(i) The function f is β -continuous.

(ii) For each point $p \in X$ and each intuitionistic open set V in Y with $f(p) \in V$, there is a β -open set U in X such that $p \in U$, $f(U) \subseteq V$.

Proof: (i) \rightarrow (ii) Assume $f: X \rightarrow Y$ is a single valued function, where X and Y is $ITS(X)$. Let $f(p) \in V$ and $V \subseteq Y$ and intuitionistic open set, then $p \in f^{-1}(V) \in \beta$ -open set of X . Since f is β -continuous, let $U = f^{-1}(V)$, then $p \in U$ and $f(U) \subseteq V$.

(ii) \rightarrow (i): Let V be a intuitionistic open set in Y and $p \in f^{-1}(V)$, then $f(p) \in V$, there exists a $U_p \in \beta$ -open set of X , such that $p \in U_p$ and $f(U_p) \subseteq V$. then $p \in U_p \subseteq f^{-1}(V)$ and $f^{-1}(V) = \cup U_p$ by theorem every intuitionistic continuous function is β -continuous function. Therefore $f^{-1}(V)$ is intuitionistic β -open set in X . therefore f is β -continuous function.

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