

Study of Mhd Boudary Layer Flow of Nano Fluid Past a Stretching Sheet With Slip Effect

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Abstract: The present paper deals with the study of magneto hydrodynamics boundary layer flow of nanofluid past a stretching sheet. Here second order velocity slip boundary condition is considered. The governing equations are transformed into nonlinear coupled ordinary differential equations with the help of suitable similarity transformations. These equations were then solved numerically by using an implicit finite difference well known method as Kellerbox scheme. The effects of various parameters such as magnetic parameter (M), Prandtl number (Pr), Brownian motion parameter (Nb), thermoporesis parameter (Nt), slip parameters A and B effect on velocity, temperature and concentration profiles were discussed, also the numerical values for the skin friction, local Nusselt number and local Sherwood number for several values of governing parameters are calculated and they are represented through tables and graphically.

Keywords: Nanofluid, magnetohydrodynamic, boundary layer flow, stretching sheet, slip condition.

I. Introduction

The nanofluids have many applications in the industries since materials of nanometer size have unique physical and chemical properties. Nanofluids are solid-liquid composite materials consisting of solid nanoparticles or nanofibers with sizes typically of 1-100 nm suspended in liquid. Convective heat transfer fluids, including oil, water ethylene glycol mixture are weak transfer fluids since the thermal conductivity of these fluids plays an important role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface. Therefore numerical methods to improve the thermal conductivity of these fluids by suspending nano/micro or larger-sized particle materials in liquids [1].

The study of magneto hydrodynamics, boundary layer flow with heat and mass transfer from a stretching surface has many applications in industrial and engineering fields. Magneto hydro dynamics (MHD) is the study of the interaction of conducting fluids with electromagnetic phenomena. The flow of an electrically conducting fluid in the presence of a magnetic field is important in various areas of technology and engineering such as MHD power generation, MHD flow, meters, MHD pumps, etc. The flow over a stretching surface is an important problem in many engineering processes with applications such as extrusion, melt spinning, the hot rolling, wire drawing, manufacturing of plastic, glass fibre production, cooling of large metallic plate in a bath, rubber sheets and filaments are manufactured by continuous extrusion of the polymer from a die to a wind up roller which is located at a finite distance way. The thin polymer sheet constitutes a continuously moving surface with non-uniform velocity through an ambient fluid [2]. Recent experiments show that the velocity of the stretching surface is approximately proportional to the distance from the orifice [3]. This problem is particularly interesting since an exact solution of the two-dimensional Navier–Stokes equations has been obtained by Crane [4]. When this pioneering work the flow field over a stretching surface has drawn considerable attention and good amount of literature has been generated [5-9].

The no-slip boundary condition is known as the main manifestation of the Navier–Stokes theory of fluid dynamics. But there are situations wherein such condition is not appropriate. Especially no-slip condition is inadequate for most non-Newtonian liquids and nanofluids, as some polymer melt often shows microscopic wall slip and that in general is governed by a non-linear and monotone relation between the slip velocity and the traction. The liquids exhibiting boundary slip find applications in technological problems such as polishing of artificial heart valves and internal cavities. The earlier studies that took into account the slip boundary condition over a stretching sheet were conducted by Andersson [10]. He gave a closed form solution of a full Navier–Stokes equations for a magnetohydrodynamics flow over a stretching sheet. Following Andersson, Wang [11] found the closed form similarity solution of a full Navier–Stokes equations for the flow due to a stretching sheet with partial slip. Furthermore, Wang [12] investigated stagnation slip flow and heat transfer on a moving plate. Similarly, Fang et al. [13] studied slip magnetohydrodynamics viscous flow over a stretching sheet analytically. Still further, Hayat et al. [14] expanded the problem of the previous researchers by incorporating thermal slip condition and discussed unsteady magneto hydrodynamic flow and heat transfer over a permeable stretching sheet with slip condition. In a similar way, Aziz [15] studied hydrodynamic and thermal slip boundary layer flow over a flat plate with constant heat flux boundary condition. The above mentioned literature

discussed the slip boundary conditions when the first order velocity slip boundary conditions were used. However, Fang et al. [16] found a closed form solution for viscous flow over a shrinking sheet using the second order velocity slip flow model. Similarly, Mahantesh et al. [17] studied flow and heat transfer over a stretching sheet by considering second order velocity slip boundary condition. Recently many authors obtained analytical and numerical solutions for boundary layer flow and heat transfer due to a stretching sheet with slip boundary conditions also many others worked on nanofluids with different conditions [18-24].

II. Formulation of the Problem

Consider the steady two-dimensional flow of a viscous incompressible nanofluid over a stretching sheet coinciding with the plane $y = 0$, with surface temperature T_w and concentration C_w . The fluid occupies the upper half plane ($y > 0$). The sheet is stretched horizontally by applying two equal and opposite forces along x -axis keeping the origin fixed. The stretching velocity of the sheet is $U_w(x) = ax$, where the x -component of the velocity varies non-linearly along it, $a > 0$ is proportionality constant which gives stretching rate, long with these the fluid is permitted by magnetic field. The induced magnetic field is assumed to be small compared to the applied magnetic field, so it is neglected. The ambient temperature and concentration are respectively T_∞ and C_∞ . T is the temperature and C is the concentration of the nanofluid in the boundary layer.

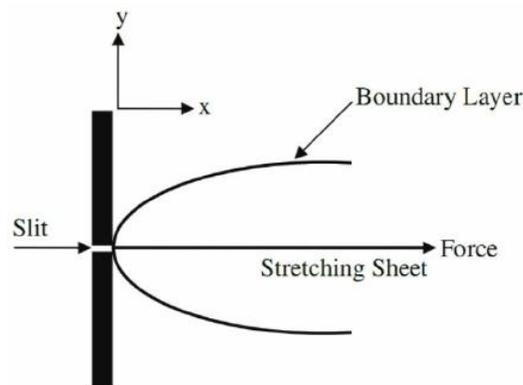


Figure-1. Physical model and co-ordinate system.

Under the above assumptions, the governing equations of the conservation of mass, momentum, energy and concentration in the presence slip conditions past a stretching sheet can be expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho} u \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau D_B \left\{ \left(\frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right\} \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left\{ \left(\frac{\partial^2 C}{\partial y^2} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial y^2} \right) \right\} \tag{4}$$

Where u and v denotes the velocities in the direction of x -and y -respectively, ν is the kinematic viscosity, ρ_f is the density of the base fluid, σ is the electrical conductivity, α is the thermal diffusivity, $(\rho c)_p$ is the effective heat capacity of the nano particles, $(\rho c)_f$ is the heat capacity of the base fluid, τ is the ratio of the nano particle heat capacity and base fluid heat capacity, D_B is the Brownian motion diffusion coefficient and D_T is the thermophoretic diffusion coefficient, the magnetic field $B(x) = B_0(x)$ where B_0 is the constant magnetic field.

The boundary conditions corresponding to the problem are as follows.

$$U = U_w + U_{slip} = U_w + A_1 \frac{\partial u}{\partial y} + B_1 \frac{\partial^2 u}{\partial y^2}, \quad v = 0, \quad T = T_w, \quad C = C_w \text{ at } y = 0 \tag{5}$$

$$u = v = 0, \quad T = T_\infty, \quad C = C_\infty \text{ as } y \rightarrow \infty \tag{6}$$

Introducing the following similarity transformations as

$$\psi = (av)^{1/2} xf(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \eta = \left(\frac{a}{v}\right)^{1/2} y$$

ψ represent the stream function and u, v are defined as

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = - \frac{\partial \psi}{\partial x} \tag{7}$$

so that Eq.(1) is satisfied identically. By employing the similarity transformations (7), the governing equations (2)-(4) reduced to the following ordinary differential equations:

$$f''' + ff'' - f'^2 - Mf' = 0 \tag{8}$$

$$\frac{1}{Pr} \theta'' + f\theta' + Nb\theta'\phi' + Nt\theta'^2 + f\theta' = 0 \tag{9}$$

$$\phi'' + Le\phi' + \frac{Nt}{Nb}\theta'' = 0 \tag{10}$$

By using (7) the transformed boundary conditions are:

$$f(0) = 0, \quad f'(0) = 1 + Af''(0) + Bf'''(0), \quad \theta(0) = 0, \quad \phi(0) = 0 \tag{11}$$

$$f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \tag{12}$$

The involved physical parameters are defined as follows:

$$M = \frac{\sigma B_0^2}{a\rho_f} \text{ is magnetic parameter, } Pr = \frac{\nu}{\alpha} \text{ is Prandtl number, } Nb = \frac{\tau D_B (C_w - C_\infty)}{\nu} \text{ is the brownian}$$

$$\text{motion parameter, } Nt = \frac{\tau D_T (T_w - T_\infty)}{T_\infty \nu} \text{ is the themoporesis parameter } A = A_1 \sqrt{\frac{a}{v}} \text{ is the first order velocity}$$

$$\text{slip parameter, } B = B_1 \frac{a}{\nu} \text{ is the second order velocity slip parameter,}$$

$$Le = \frac{\nu}{D_B} \text{ is the Lewis number.}$$

The Skin friction coefficient, Nusselt number and Sherwood numbers are given by

$$C_f = \frac{\tau_w}{\rho u_w^2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \quad Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)} \tag{13}$$

$$\text{Here } \tau_w = \mu_B \left(\frac{\partial u}{\partial y}\right)_{y=0} \text{ is the local shering stress of the surface, } q_w = -k \left(\frac{\partial T}{\partial y}\right)_{y=0} \text{ is the heat flux of the}$$

$$\text{surface and the mass flux of the surface is } q_m = -D_B \left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{14}$$

Where k is thermal conductivity of the nanofluid.

By substituting equation (7) into equations (13)-(14), we will get

$$C_f Re^{\frac{1}{2}} = f''(0), \quad Nu_x Re^{-\frac{1}{2}} = -\theta'(0), \quad Sh_x Re^{-\frac{1}{2}} = -\phi'(0) \tag{15}$$

where 'Re' local Reynolds number.

NUMERICAL METHOD:

The higher order ordinary differential equations with the boundary conditions are solved numerically by using implicit finite difference scheme known as Keller-Box method, the following steps are involved to achieve the Numerical solution.

- Reduce the non-linear higher order ordinary differential equations into a system of first order ordinary differential equations.
- Write the finite differences for the first order equations.
- Linearize the algebraic equations by Newton's method, and write them in matrix-vector form. Solving the linear system by the block tri-diagonal elimination technique.

- In order to solve the above differential equations numerically, we adopt Matlab software which is very efficient in using the well known Keller box method.
- For getting accuracy of this method to choose appropriate initial guesses.

$$f(\eta) = \frac{1}{1+A-B} [1 - e^{-\eta}], \quad \theta(\eta) = e^{-\eta}, \quad \phi(\eta) = e^{-\eta}$$

The step size delta is used to obtain numerical solution with four decimal place accuracy as criterion of convergence. Accuracy of this numerical method presented in Table-1 is being validated by direct comparison with the numerical results reported by Khan and Pop, Wang. The numerical calculations for the governing parameters of Pr, Le, Nt, Nb, M, A, B and comparison for the reduced nusselt number of different Pr values shown in Table1. Neglecting the effect of Nb and Nt numbers the result for the reduced nusselt number are compared with those obtained by khan and pop, Wang. We notice that the comparison shows very closely agree for each value of Pr. keeping Pr =10, Le = 10 for different values of Nb=0.1, 0.2, 0.3, 0.4, 0.5 and Nt=0.1, 0.2, 0.3, 0.4, 0.5 values are presented in Table 2(a), 2(b) results for Nur and Shr respectively.

III. Results And Discussion

Table-1: Comparison of results for the reduced nusselt number $-\theta'(0)$, Nb=Nt=0

Pr	Khan & Pop	Wang	Present value
0.07	0.0656	0.0663	0.0656
0.20	0.1688	0.1691	0.1691
0.70	0.4544	0.4539	0.4543
2.00	0.9114	0.9113	0.9114
7.00	1.8954	1.8954	1.8954
20.0	3.3542	3.3539	3.3542
70.0	6.4645	6.4641	6.4642

Table-2(a): Variation of Nur&Shr with Nb and Nt for Pr=10 and Le=10

	Nb =0.1 &Nur	Nb=0.2 &Nur	Nb=0.3 &Nur	Nb=0.4 &Nur	Nb=0.5 &Nur
Nt = 0.1	0.9524	0.5055	0.2521	0.1193	0.0541
Nt = 0.2	0.6932	0.3653	0.1815	0.0858	0.0389
Nt = 0.3	0.5210	0.2730	0.1354	0.0640	0.0389
Nt = 0.4	0.4026	0.2109	0.1045	0.0494	0.0224
Nt = 0.5	0.3210	0.1680	0.0832	0.0393	0.0179

Table -2(b)

	Nb =0.1 &Shr	Nb=0.2 &Shr	Nb=0.3 &Shr	Nb=0.4 &Shr	Nb=0.5 &Shr
Nt = 0.1	2.1294	2.3820	2.4101	2.3998	2.3837
Nt = 0.2	2.2741	2.5154	2.4809	2.5152	2.4469
Nt = 0.3	2.5288	2.6557	2.6090	2.5488	2.4985
Nt = 0.4	2.7954	2.7820	2.6878	2.6040	2.5400
Nt = 0.5	3.0354	2.8886	2.7521	2.6484	2.5734

The Figure 2 represent the effect of magnetic field parameter on velocity of the nanofluid. As the magnetic field parameter M increases, it improves the opposite force to the flow of nanofluid direction called 'Lorentz force'. This Lorentz force opposes the motion of the nanofluid, as a result the velocity of the fluid decreases. From Figure 3 temperature profile is increases for increasing M. The effects of first order velocity slip parameter on the dimensionless velocity $f'(\eta)$ and temperature $\theta(\eta)$ is displayed in Figure 4 and 5 respectively. From the Figures velocity decreases with increase in the values of slip parameter, but it is reverse in the case of temperature, i.e. temperature increases with the increase of first order slip parameter. Physically, in the presence of slip the slipping fluid shows a decrease in the surface skin-friction between the stretching sheet and the fluid. So the flow velocity decreases when the value of A increases. Friction force is generate as slip parameter increases, as a result, the speed of the fluid flow decreases at the surroundings of the sheet. Hence, the temperature increases neighbourhood of the sheet. Figure 6 and 7 represents the effect of second order slip parameter B on temperature and velocity. For increasing the values of B the temperature decreases, but the reverse result appears in the case of velocity.

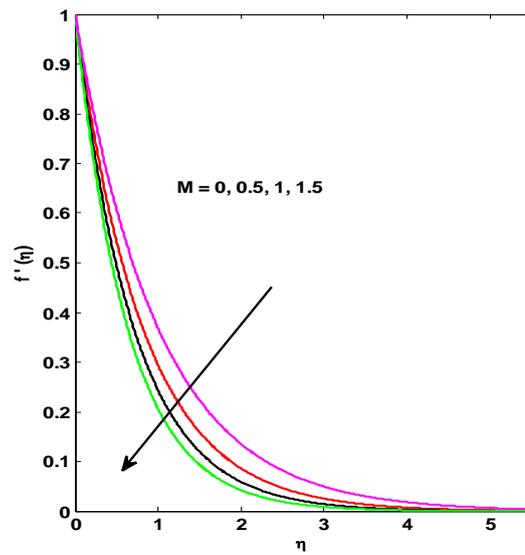


Figure.2.Effect of several values of magnetic parameter M on velocity profile.

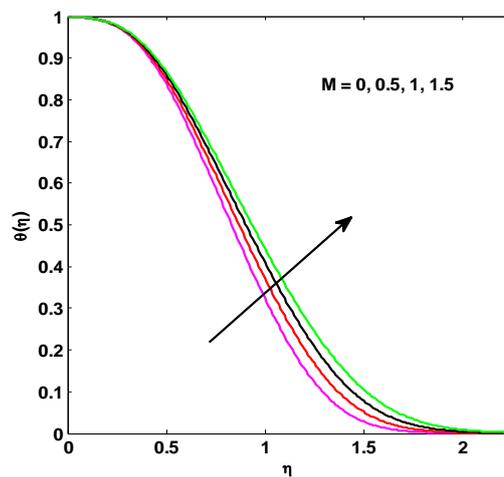


Figure.3.Effect of several values of magnetic parameter M on temperature profile.

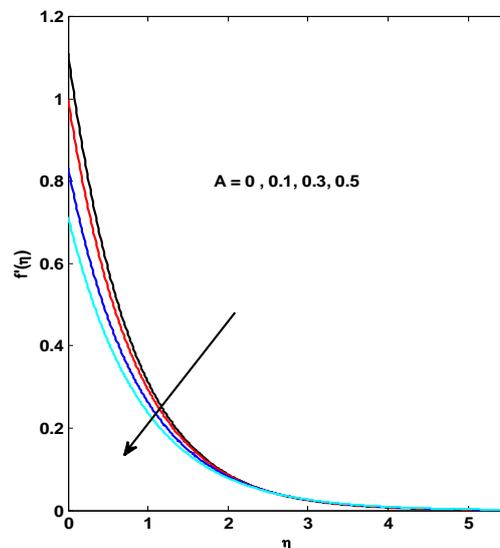


Figure.4.Effect of several values of slip parameter A on velocity profile.

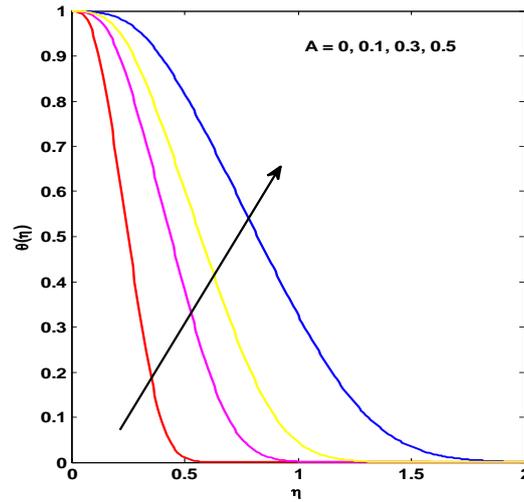


Figure.5. Effect of several values of slip parameter A on temperature profile.

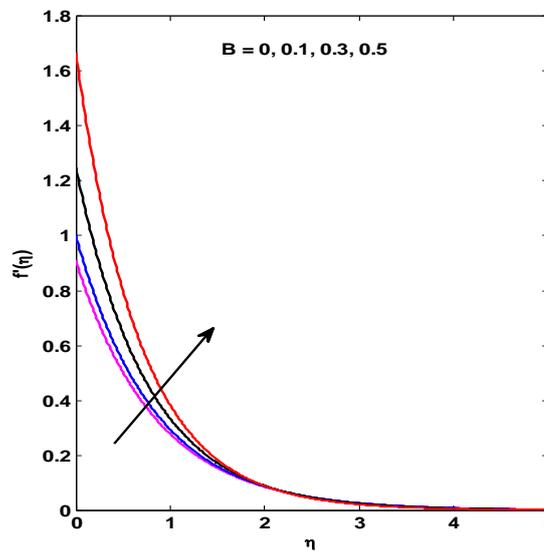


Figure.6. Effect of several values of slip parameter B on velocity profile.

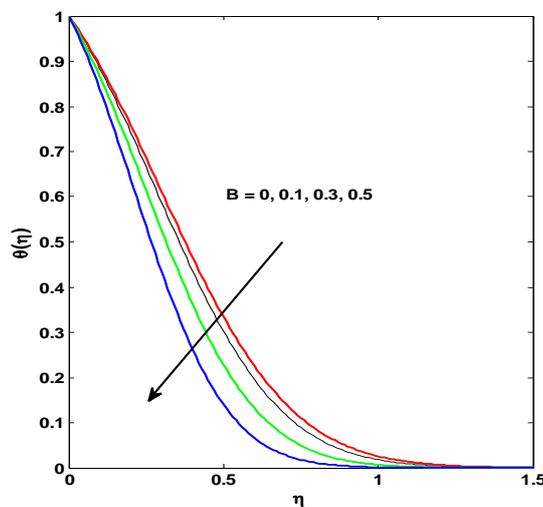


Figure.7. Effect of several values of slip parameter B on temperature profile.

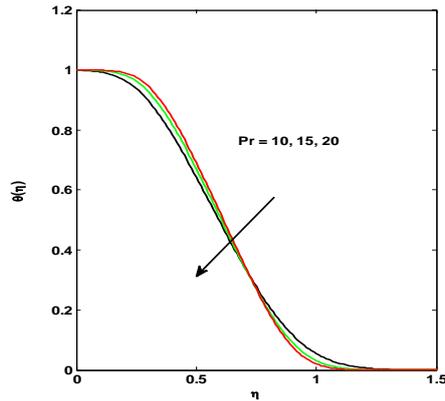


Figure.8.Effect of several values of Prandtl number Pr on concentration profile.

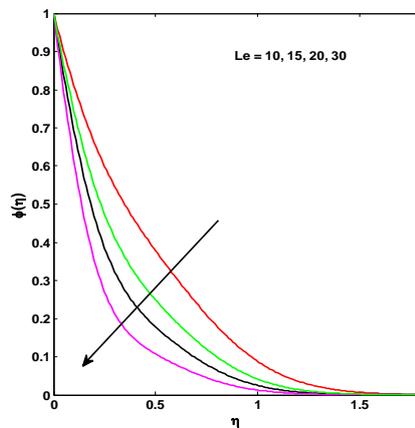


Figure.9.Effect of several values of Lewis number Le on concentration profile.

The effect of Prandtl number Pr on the heat transfer process is shown by the Figure 8. This figure reveals that an increase in Prandtl number Pr, the temperature field decreases. An increase in the values of Pr reduces the thermal diffusivity, because Prandtl number is a dimensionless number which is defined as the ratio of momentum diffusivity to thermal diffusivity, that is $Pr = \nu/\alpha$. Increasing the values of Pr implies that momentum diffusivity is higher than thermal diffusivity. Therefore thermal boundary layer thickness is a decreasing function of Pr. Figure 9 shows the impact of Lewis number Le on concentration profile. Actually, a higher value of Lewis number $Le = \alpha/D_B$ represents a lower nanoparticle diffusivity (Brownian motion) and a higher thermal diffusivity. If $Le > 1$, the thermal diffusion rate exceeds the Brownian diffusion rate. Lower Brownian diffusion leads to less mass transfer rate, as a result, the nanoparticle volume fraction (concentration) graph and the concentration boundary layer thickness both are decreases.

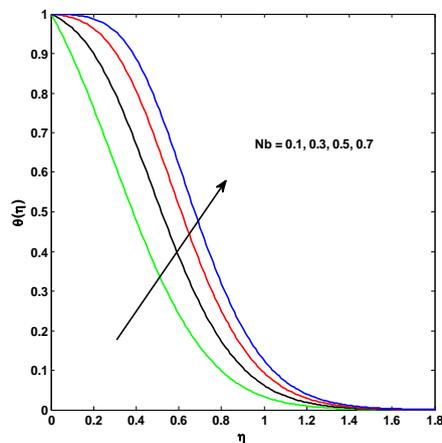


Figure.10. Effect of several values of Brownian motion parameter Nb on temperature profile.

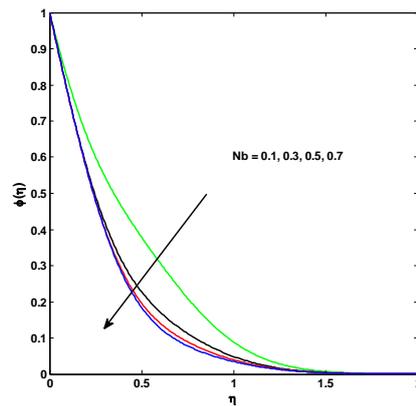


Figure.11. Effect of several values of Brownian motion parameter Nb on concentration profile.

The effect of Brownian motion parameter Nb on temperature and concentration are presented in Figure 10 and 11. The figure 10 represents the variation of temperature with the Brownian motion parameter Nb. Increase in Nb values gives both the temperature graph and thermal boundary layer thickness increases. As increase in Nb, due to movement of nanoparticles, the result increase in the kinetic energy of the nanoparticle, thus rises the temperature. But it is reverse in the case of nano particle volume fraction profile, i.e. as the values of Nb increase, the concentration boundary layer thickness decrease. Figure 12 and 13 shows the impact of thermophoresis parameter Nt on temperature and nanoparticle concentration profile respectively. It is found that an increase in the thermophoresis parameter Nt leads to increase in both temperature and nanoparticle concentration. As the thermophoretic effect increases, nanoparticles are migrated from the hot surface to cold ambient fluid, as a result the temperature enhanced in the boundary layer. This will helps in the thickening of the thermal boundary layer. Figure 13 reveals the variation of concentration profile and it can be seen from the graph concentration profile is increases, as the Nt increases and concentration boundary layer thickness also increases.

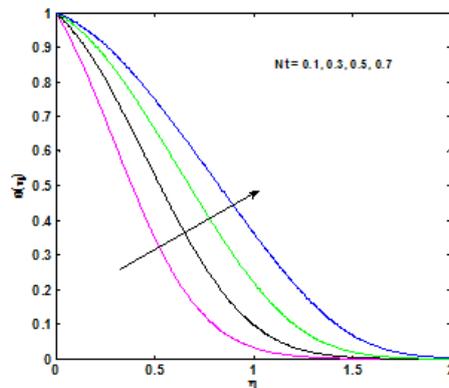


Figure.12.Effect of several values of themoporesis parameter Nt on temperature profile.

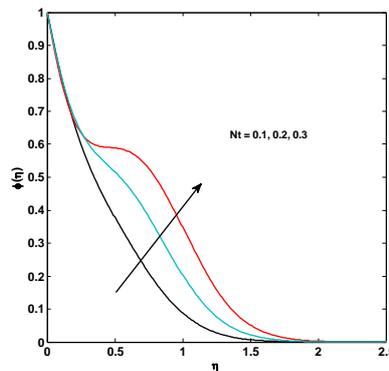


Figure.13.Effect of several values of themoporesis parameter Nt on concentration profile.

Figure 14 indicates the variation of skin friction coefficient for magnetic parameter M versus velocity 1st and 2nd order skin parameters (A, B). Figure. 15 gives the variation of Nusselt with the Prandtl number and thermoporesis parameter M .

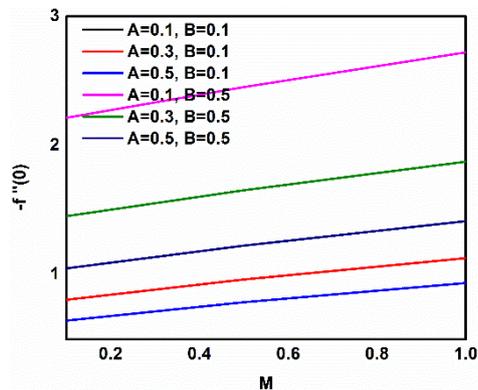


Figure.14. Effect of M and A,B on skin friction coefficient.

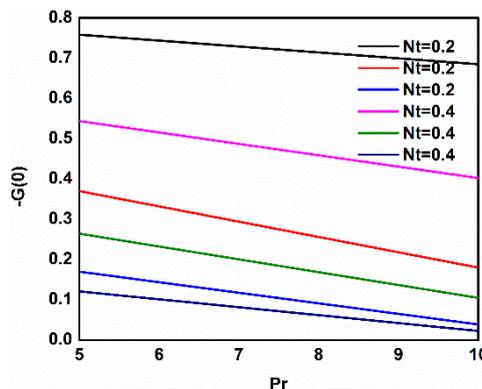


Figure.15. Effect of Pr, Nt on Nusselt number.

IV. Conclusion

In this work, the MHD boundary layer flow of a nanofluid past a stretching sheet with slip conditions was analysed. The resulting partial differential equations, describing the problem, are transformed into ordinary differential equations by using similarity transformations. These equations were solved numerically by well-known Keller-Box method. The effects of magnetic parameter M , slip parameters A and B , Prandtl number Pr , Lewis number Le , Brownian motion parameter Nb , thermophoresis parameter Nt on the fluid flow and heat transfer characteristics of the MHD boundary layer flow of a nanofluid past a stretching sheet with slip conditions were investigated. The numerical results obtained are agreed very well with the previously published data in limiting condition and for some particular cases of the present study: An increase in slip parameter A is to decrease in the velocity profile but is to increase the temperature profiles. An increase in slip parameter B is to increase in the velocity profile but is to decrease the temperature profiles. As Lewis number Le increases, concentration profile decreases. As magnetic parameter M increases, velocity profile decreases but temperature profile increases. As Nb increases, the temperature of nanofluid increases, and the concentration decreases. As Nt increases, the temperature of nanofluid increases, and the concentration increases.

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