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On Weak m-power Commutative Near – rings and weak (m,n) power Commutative Near –rings

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Abstract: *A right near – ring N is called weak commutative if xyz = xzy for every x,y,z ϵ N (Definition 9.4 [10]). A right near – ring N is called pseudo commutative (Definition 2.1 [11]) if xyz = zyx for all x,y,z ϵ N.A right near – ring N is called quasi – weak commutative (Definition 2.1 [7]) if xyz = yxz for all x,y,z ϵ N. We call a right near – ring N to be weak m – power commutative if $xy^m z = xz^m y$ for all x,y,z ϵ N. N is said to be weak (m,n) power commutative near – ring if$xy^mz^n = xz^my^n$ for all x,y,z ϵ N.In this paper we study and establish various results of weak m – power commutative near – ring and weak (m,n) power commutative near -- ring.*

I. Introduction

S.Uma,R.Balakrishnan and T.Tamizhchelvam [11] called a near- ring N to be pseudo commutative if xyz = zyx for every x,y,z ϵ N.G.Gopalakrishnamoorthy and S.Geetha [4] called a ring R to be m power commutative if $x^m y = y^m x$ for all x,y ϵ R where $m \geq 1$ is a fixed integer.They also called a ring R to be (m,n) power commutative if $x^m y^n = y^m x^n$ for all x,y ϵ R where $m \geq 1$ and $n \geq 1$ are fixed integers. G.Gopalakrishnamoorthy and R.Veega [6] called a near – ring N to be pseudo m- power commutative if $x^m y z = z y^m x$ for all x,y,z ϵ N where $m \geq 1$ is a fixed integer. G.Gopalakrishnamoorthy, N.Kamaraj and S.Geetha [7] defined a near – ring N to be Quasi – weak commutative if xyz = yxz for all x,y,z ϵ N.In this paper we define weak m – power commutative near – ring and weak (m,n) power commutative near – ring and establish some results.

II. Preliminaries

Throughout this paper N denotes a right near – ring with atleast two elements. For any non-empty set A ⊆ N , we denote A- {0} by A* .In this section we present some known definitions and results which are useful in the development of this paper.

2.1 Definition [10]:
A near – ring N is called weak-commutative if xyz = xzy for every x,y,z ϵ N.

2.2 Definition:
A right near- ring N is called weak anti-commutative if xyz = -xzy for every x,y,z ϵ N.

III. Weak m- power commutative near - rings

3.1 Definition:
Let N be a near – ring. N is said to be weak m- power commutative if $xy^m z = xz^m y$ for all x,y,z ϵ N,where $m \geq 1$ is a fixed integer.

3.2 Definition:
Let N be a near – ring. N is said to be weak m- power anti-commutative if $xy^m z = - xz^m y$ for all x,y,z ϵ N,where $m \geq 1$ is a fixed integer.

3.3 Lemma :
Let N be a distributive near – ring.If xyz = ± xzy for all x,y,z ϵ N then N is either Weak Commutative or Weak anti – Commutative.

Proof:
For each a ϵ N , let
$C_a = \{ x \in N / xaz = xza \ \forall z \in N \}$
$A_a = \{ x \in N / xaz = - xza \ \forall z \in N \}$
By the hypothesis of the lemma,
$N = C_a \cup A_a$
We note that if x,y ϵ C_a ,then x – y ϵ C_a.
For x,y ϵ C_a implies xaz = + xza ∀ z ϵ N → (1)
and yaz = + yza ∀ z ϵ N → (2)
(1) – (2) gives

$(x - y) az = (x - y) za \ \forall z \in N$
which implies $(x - y) \in C_a$.
Similarly, if $x,y \in A_a$, then $x - y \in A_a$.
We claim that either $N = C_a$ or $N = A_a$.
Suppose $N \neq C_a$ and $N \neq A_a$, then there are elements $b \in C_a - A_a$ and $d \in A_a - C_a$.
Now $b + d \in N = C_a \cup A_a$.
If $b + d \in C_a$ then $d = (b + d) - b \in C_a$, a contradiction.
If $b + d \in A_a$ then $b = (b + d) - d \in A_a$, again a contradiction.
Hence either $N = C_a$ or $N = A_a$.
Let $A = \{ a \in N / C_a = N \}$
and $B = \{ a \in N / A_a = N \}$
Clearly $N = A \cup B$.
We note that that if $x,y \in A$, then $x - y \in A$.
For if $x,y \in A \Rightarrow C_x = N$ and $C_y = N$.
This implies $xza = xaz$ and $yza = yaz$ for all $a,z \in N$,
So $(x - y) za = (x - y) az$ for all $a,z \in N$, which proves that $x - y \in A$.
Similarly, if $x,y \in B$, then $x - y \in B$.
We claim that either $N = A$ or $N = B$.
Suppose $N \neq A$ and $N \neq B$, there are elements $u \in A - B$ and $v \in B - A$.
Now, $u + v \in N = A \cup B$.
If $u + v \in A$, then $v = (u + v) - u \in A$, a contradiction.
If $u + v \in B$, then $u = (u + v) - v \in B$, again a contradiction.
Hence either $N = A$ or $N = B$.
This proves that N is either weak commutative or weak anti – commutative.

3.4 Lemma:
Let N be a near – ring (not necessarily associative).If $x y^m z = \pm x z^m y$ for all $x, y,z \in N$, then N is either weak m – power commutative or weak m – power anti – commutative.

Proof:
For each $a \in N$, let
$C_a = \{ x \in N / xa^m z = xz^m a \ \forall z \in N \}$
$A_a = \{ x \in N / xa^m z = - xz^m a \ \forall z \in N \}$
By the hypothesis of the lemma,
$N = C_a \cup A_a$
We note that, if $x,y \in C_a$ then $x - y \in C_a$
For $x,y \in C_a$ implies $xa^m z = xz^m a \ \forall z \in N$ $\longrightarrow (1)$
and $ya^m z = yz^m a \ \forall z \in N$ $\longrightarrow (2)$
Equation (1) – (2) gives,
$(x - y) a^m z = (x - y) z^m a \ \forall z \in N$.
$\Rightarrow (x - y) \in C_a$.
Similarly $x, y \in A_a$ implies $x - y \in A_a$.
We claim that either $N = C_a$ or $N = A_a$.
Suppose $N \neq C_a$ and $N \neq A_a$, there are elements $b \in C_a - A_a$ and $d \in A_a - C_a$.
Now, $b + d \in N = C_a \cup A_a$.
If $b + d \in C_a$ then $d = (b + d) - b \in C_a$, a contradiction.
Similarly, if $b + d \in A_a$, then $b = (b + d) - d \in A_a$, again a contradiction.
Hence either $N = C_a$ or $N = A_a$.
Let $A = \{ a \in N / C_a = N \}$
and $B = \{ a \in N / A_a = N \}$
Clearly $N = A \cup B$.
We note that if $x,y \in A$ implies $x- y \in A$.
For if $x,y \in A$ implies $C_x = N$ and $C_y = N$.
This implies $xz^m a = xa^m z$ and $yz^m a = ya^m z$ for all $a,z \in N$.
So, $(x - y) z^m a =(x-y) a^m z$ for all $a,z \in N$, which proves that $x - y \in A$.
Similarly $x,y \in B$ implies $x - y \in B$.
We claim that either $N = A$ or $N = B$.
Suppose $N \neq A$ and $N \neq B$, there are elements $u \in A - B$ and $v \in B - A$.
Now, $u + v \in N = A \cup B$.
If $u + v \in A$, then $v = (u + v) - u \in A$, a contradiction.

If $u + v \in B$, then $u = (u + v) - v \in B$, again a contradiction.
Hence either $N = A$ or $N = B$.
This proves that N is either weak m – power commutative or weak m- power anti – commutative.

3.5 Note:
When m = 1, we get Lemma 3.3.

3.6 Definition:
Let N be a near-ring and $m \geq 1$ and $n \geq 1$ be fixed integers.N is said to be weak – (m,n) Power commutative,if $xy^m z^n = xz^m y^n$ for all $x,y,z \in N$.

3.7 Definition:
Let N be a near-ring and $m \geq 1$ and $n \geq 1$ be fixed integers.N is said to be weak – (m,n) Power anti - commutative,if $xy^m z^n = - xz^m y^n$ for all $x,y,z \in N$.

3.8 Lemma:
Let N be a near – ring (not necessarily associative) satisfying $(x-y)^k = x^k - y^k$ for $k = m,n$ where $m \geq 1$ and $n \geq 1$ are fixed integers.If $xy^m z^n = \pm xz^m y^n$ for all $x,y,z \in N$, then N is either weak (m.n) power Commutative or weak - (m,n) power anti-commutative.

Proof:
For each $a \in N$, let
$C_a = \{ x \in N / xa^m z^n = xz^m a^n \ \forall z \in N \}$
$A_a = \{ x \in N / xa^m z^n = - xz^m a^n \ \forall z \in N \}$
By the hypothesis of the lemma,
$N = C_a \cup A_a$
We note that, if $x,y \in C_a$ then $x - y \in C_a$
For $x,y \in C_a$ implies $xa^m z^n = xz^m a^n \quad \forall z \in N \quad \longrightarrow (1)$
and $ya^m z^n = yz^m a^n \quad \forall z \in N \quad \longrightarrow (2)$
Equation (1) – (2) gives,
$(x - y) a^m z^n = (x - y) z^m a^n \quad \forall z \in N$.
$\Longrightarrow (x - y) \in C_a$.
Similarly $x, y \in A_a$ implies $x - y \in A_a$.
We claim that either $N = C_a$ or $N = A_a$. .
Suppose $N \neq C_a$ and $N \neq A_a$,there are elements $b \in C_a - A_a$ and $d \in A_a - C_a$.
Now, $b + d \in N = C_a \cup A_a$.
If $b + d \in C_a$ then $d = (b + d) - b \in C_a$, a contradiction.
Similarly, if $b + d \in A_a$,then $b = (b + d) - d \in A_a$, again a contradiction.
Hence either $N = C_a$ or $N = A_a$.
Let $A = \{ a \in N / C_a = N \}$
and $B = \{ a \in N / A_a = N \}$
Clearly $N = A \cup B$.
We note that if $x,y \in A$ implies $x- y \in A$.
For if $x,y \in A$ implies $C_x = N$ and $C_y = N$.
This implies $xz^m a^n = xa^m z^n$ and $yz^m a^n = ya^m z^n$ for all $a,z \in N$.
So, $(x - y) z^m a^n = (x-y) a^m z^n$ for all $a,z \in N$, which proves that $x - y \in A$.
Similarly $x,y \in B$ implies $x - y \in B$.
We claim that either $N = A$ or $N = B$.
Suppose $N \neq A$ and $N \neq B$, there are elements $u \in A - B$ and $v \in B - A$.
Now, $u + v \in N = A \cup B$.
If $u + v \in A$, then $v = (u + v) - u \in A$, a contradiction.
If $u + v \in B$, then $u = (u + v) - v \in B$, again a contradiction.
Hence either $N = A$ or $N = B$.
This proves that N is either weak (m,n) – power commutative or weak(m,n) – power anti – commutative.

3.9 Note :
When m = n = 1, we get Lemma 3.3.
When n = 1, we get Lemma 3.4.

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