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Fuzzy Processing Times using Credibility Theory and implementation in Job Sequencing

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Abstract
In this paper, a new algorithm to find an optimal sequence of ‘ ’ jobs that has to be processed through ‘ ’ machines is developed by treating the processing times of jobs on the machines as fuzzy (imprecise) variables. The algorithm is similar to that of Johnson (1954) which considers the processing times as precise in nature. The proposed algorithm makes use of the concepts of ‘Credibility Theory’ due to Liu (2008). An illustration of the proposed algorithm is given by representing the processing times as triangular fuzzy numbers.
Keywords: *Credibility measure, credibility distribution, membership function, fuzzy variable, expectation and variance of fuzzy variable, ranking of fuzzy variable, sequencing*

I. Introduction

Identifying an optimal sequence of jobs that will result in the minimum total processing time is a problem that has been addressed extensively in literature. This has potential applications in manufacturing industries. However,it is not only confined to manufacturing but also to other areas like human resource management that deals with allocation of different modules (jobs) of a project to various teams (machines)in a sequence such that the project is completed in the least possible time. Thus the main objective of job sequencing is to identify the sequence of jobs that will result in least total processing time when the jobs are processed through various machines. Johnson [1]has developed a basic algorithm that will give a sequence of ‘ ’ jobs when processed through two machines. An extension of Johnson’s algorithm for ‘ ’ machines can be found in [2–4]. One of the key assumptions in these algorithms is that the processing times of jobs under different machines can be determined precisely i.e., the processing times are assumed to be crisp in nature. However, most often, the processing times cannot be determined accurately but can only be expressed in terms of approximate values .This is because uncontrollable factors often play a role in altering the processing times. Therefore, it is reasonable to take into account the impreciseness of the processing times in order to find optimal sequence. The fuzzy set theory developed by Zadeh [5] gives a direction to handle impreciseness in data by treating them as fuzzy numbers [6–14]have developed new sequencing algorithms by treating processing time as fuzzy numbers. However, these algorithms make use of the concept of α –cut in dealing with fuzzy numbers. The choice of ‘α’ play a significant role in these algorithms. Recently, Liu [15] has developed a new mathematical theory called ‘Credibility Theory’ to deal with fuzzy phenomena. This theory gives an easy approach to perform various arithmetic and relational operations on fuzzy numbers without using the α-cut approach. In this paper, a new job sequencing algorithm is proposed using some of the concepts available in ‘Credibility Theory’ to obtain an optimal sequence of ‘ ’ jobs when processed through ‘ ’ machines by assuming the processing times as fuzzy variables. A numerical illustration of the proposed algorithm is provided by representing the processing times as triangular fuzzy numbers. The paper is organized as follows:

Section ‘Preliminaries’ contains the axioms, certain important definitions and results based on ‘Credibility Theory’. In section ‘New Sequencing Algorithm’, the proposed sequencing algorithm is described. Section ‘Numerical Illustration’ gives numerical illustration of the proposed algorithm. The ‘Conclusion’ of the paper is given in the last Section.

II. Preliminaries

The concepts and definitions in this section are based on [15]. Credibility Theory is a branch of mathematics for studying the behavior of fuzzy phenomena. A credibility measure like a probability measure is based on the following four axioms:

Let Θ be a nonempty set, and $\mathcal{P}$ the power set of Θ (i.e., the largest σ -algebra over Θ). Each element in $\mathcal{P}$ is called an event. In order to present an axiomatic definition of credibility, it is necessary to assign to each event A a number $Cr\{A\}$ which indicates the credibility that A will occur.

Axiom 1: $Cr\{\Theta\} = 1$.

Axiom 2: $Cr\{A\} \le Cr\{B\}$ whenever $A \subset B$.

Axiom 3: $Cr\{A\} + Cr\{A^C\} \equiv 1$ for any event A.

Axiom 4: $Cr\left\{\bigcup_i A_i\right\} = \sup_i Cr\{A_i\}$ for any events $\{A_i\}$ with $\sup_i Cr\{A_i\} < 0.5$.

The set function Cr is called a credibility measure if it satisfies the above four axioms. The triplet (Θ, P, Cr) is called a credibility space.

Fuzzy Variable

A fuzzy variable is a (measurable) function from a credibility space (Θ, P, Cr) to the set of real numbers. It is to be noted that any function of fuzzy variables defined on the same credibility space is again a fuzzy variable. That is the sum or product of two or more fuzzy variables is again a fuzzy variable.

Membership Function

The membership function μ of a fuzzy variable ξ defined on the credibility space (Θ, P, Cr) is derived from the credibility measure by $\mu(x) = \min\left(2Cr\{\xi = x\}, 1\right), x \in \Re$ Membership function represents the degree that the fuzzy variable ξ takes on some prescribed value.

Triangular Fuzzy Variable

A fuzzy variable fully determined by the triplet (a, b, c) of crisp numbers with $(a < b < c)$ and whose membership function is given by

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a}, & if \quad a \le x \le b \\ \dfrac{x-c}{b-c}, & if \quad b \le x \le c \\ 0, & otherwise \end{cases}$$

is called a triangular fuzzy variable. If $b - a = c - b$ then the triangular fuzzy variable is said to be symmetric.

Trapezoidal Fuzzy Variable

A fuzzy variable fully determined by the quadruplet (a, b, c, d) of crisp numbers with $(a < b < c < d)$ and whose membership function is given by

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a}, & if \quad a \le x \le b \\ 1, & if \quad b \le x \le c \\ \dfrac{x-d}{c-d}, & if \quad c \le x \le d \\ 0, & otherwise \end{cases}$$

is called a trapezoidal fuzzy variable. If $b - a = d - c$ then the trapezoidal fuzzy variable is said to be symmetric. Also if $b = c$ then the trapezoidal fuzzy variable reduces to a triangular fuzzy variable.

Credibility Distribution

The credibility distribution $\Phi : \Re \to [0,1]$ of a fuzzy variable ξ is defined by $\Phi(x) = Cr\{\theta \in \Theta \mid \xi(\theta) \le x\}$. That is, $\Phi(x)$ is the credibility that the fuzzy variable ξ takes a value less than or equal to x.

Remark 1: The credibility distribution of a triangular fuzzy variable $\xi = (a, b, c)$ is given by

$$\Phi(x) = Cr\{\xi \le x\} = \begin{cases} 0, & \text{if } x < a \\ \dfrac{x-a}{2(b-a)}, & \text{if } a \le x < b \\ \dfrac{x+c-2b}{2(c-b)}, & \text{if } b \le x < c \\ 1, & \text{if } x \ge c \end{cases}.$$

Remark 2: The credibility distribution of a trapezoidal fuzzy variable $\xi = (a, b, c, d)$ is given by

$$\Phi(x) = Cr\{\xi \le x\} = \begin{cases} 0, & \text{if } x < a \\ \dfrac{x-a}{2(b-a)}, & \text{if } a \le x < b \\ \dfrac{1}{2}, & \text{if } b \le x < c \\ \dfrac{x+d-2c}{2(d-c)}, & \text{if } c \le x \le d \\ 1, & \text{if } x \ge d \end{cases}$$

Credibility Density Function

The credibility density function $\phi : \Re \to [0, \infty)$ of a fuzzy variable ξ is a function such that

$$\Phi(x) = \int_{-\infty}^{x} \phi(y) dy, \forall x \in \Re .$$

Remark 3: The credibility density function of a triangular fuzzy variable $\xi = (a, b, c)$ is given by

$$\phi(x) = \begin{cases} \dfrac{1}{2(b-a)}, & \text{if } a \le x \le b \\ \dfrac{1}{2(c-b)}, & \text{if } b \le x \le c \\ 0, & \text{otherwise} \end{cases}.$$

Remark 4: The credibility density function of a trapezoidal fuzzy variable $\xi = (a, b, c, d)$ is given by

$$\phi(x) = \begin{cases} \dfrac{1}{2(b-a)}, & \text{if } a \le x \le b \\ \dfrac{1}{2(d-c)}, & \text{if } c \le x \le d \\ 0, & \text{otherwise} \end{cases}.$$

Expected Value and Variance of a Fuzzy Variable

The expected value and variance of a fuzzy variable ξ with Credibility distribution $Cr\{\xi \le r\}$ is defined (Liu and Liu, 2002)as $E[\xi] = \int_{0}^{+\infty} Cr\{\xi \ge r\} dr - \int_{-\infty}^{0} Cr\{\xi \le r\} dr$ and $V(\xi) = E(\xi - e)^2$ where e denotes the expected value of ξ .

Remark 5: Let $\xi = (a, b, c)$ be a symmetric triangular fuzzy variable. Then its expected value and variance are given by $E[\xi] = \dfrac{a + 2b + c}{4}$ and $V(\xi) = \dfrac{(c-a)^2}{24}$.

Remark 6: Let $\xi = (a, b, c, d)$ be a symmetric trapezoidal fuzzy variable. Then its expected value and variance are given by $E[\xi] = \frac{b+c}{2}$ and $V(\xi) = \frac{(d-a)^2 + (d-a)(c-b) + (c-b)^2}{24}$.

Ranking Fuzzy Variables

Liet al. [16] have developed a method for ranking fuzzy variables based on their expected value and variance. Let ξ and η be two fuzzy variables defined on the credibility space (Θ, P, Cr).

The ranking of ξ and η is defined as follows:

1. $\xi \succ \eta$ if and only if $E[\xi] > E[\eta]$ or $E[\xi] = E[\eta]$ and $V(\xi) < V(\eta)$.
2. $\xi \prec \eta$ if and only if $E[\xi] < E[\eta]$ or $E[\xi] = E[\eta]$ and $V(\xi) > V(\eta)$.
3. $\xi \sim \eta$ if and only if $E[\xi] = E[\eta]$ and $V(\xi) = V(\eta)$.

Thus ξ is greater than or equal to η if and only if $\xi \succ \eta$ or $\xi \sim \eta$ and ξ is lesser than or equal to η if and only if $\xi \prec \eta$ or $\xi \sim \eta$.

Using the above concepts, in the next section, a new algorithm is proposed to find an optimal sequence of 'n' jobs that is processed through 'm' machines by assuming the processing times to befuzzy variables.

NEW SEQUENCING ALGORITHM

The proposed algorithm is based on [1]which gives an optimal sequence of 'n' jobs when processed through two machines. In the algorithm, the following assumptions are made.

1. The'n' jobs are processed through 'm' machines(one at a time) in the order $1, 2, \ldots, m$,
2. Each job has only one operation and can be processed on any machine.
3. Each machine can process only one job at a time.
4. All jobs are available for processing simultaneously at the starting time.
5. The processing times of jobs under various machines are imprecise.

The objective is to determine an optimal sequence of these 'n' jobs that will result in the total minimum processing time. The proposed algorithm is described below.

Algorithm

Let $\xi_{ij}, i = 1, 2, \ldots, n; j = 1, 2, \ldots, m$ represents the fuzzy processing time of i^{th} job under the j^{th} machine.

Step 1: Find the expected value and variance of $\xi_{ij}, i = 1, 2, \ldots, n; j = 1, 2, \ldots, m$ by usingthe definition of expectation and variance of fuzzy variables given in section'Preliminaries'.

Step2: Find $\min_i \xi_{i1}$, $\min_i \xi_{im}$ and maximum of each of $\xi_{i2}, \xi_{i3}, \ldots, \xi_{im-1}$ for $i = 1, 2, \ldots, n$ byusing the method of ranking fuzzy variables explained in section 'Preliminaries'.

Step3: If either of the following conditions, namely, $\min_i \xi_{i1} \geq \max_i \xi_{ij}$ for $j = 2, 3, \ldots, m-1$ or

$\min_i \xi_{im} \geq \max_i \xi_{ij}$ for $j = 2, 3, \ldots, m-1$

issatisfied,thenconvertthe m machine problem into a two machine problem by introducingtwofictitiousmachines G and H with respective fuzzy processing times ξ_{iG} and ξ_{iH} that are obtained as follows:

$\xi_{iG} = \xi_{i1} + \xi_{i2} + \ldots + \xi_{im-1}, i = 1, 2, \ldots, n$

and

$\xi_{iH} = \xi_{i2} + \xi_{i3} + \ldots + \xi_{im}, i = 1, 2, \ldots, n$.

Step 4: Select the overall minimum processing time (by using the method of rankingfuzzy variables)under machines G and H. If the overallminimum processing time is for the k^{th} job, $k \in \{1, 2, \ldots, n\}$ under machine G, then it is processed first i.e., it is placed first in the order of sequence. If it is under machine H, then it is processed last i.e., it is placed last in the order of the sequence. Marktheposition of the k^{th} job and remove the k^{th} job from further calculations.

Step 5: Repeat Step 4until all the jobs are processed and their positionsmarked in the sequence.

Thus an implementation of the above algorithm will result in an optimal sequence of jobs that will give the minimum processing time. It is emphasized that for finding the minimum and maximum values of fuzzy processing times under different jobs, the algorithm uses the method of ranking fuzzy variables based on their expected value and variance without using the α-cut approach thereby making the implementation simple.

NUMERICAL ILLUSTRATION

In this section, a numerical illustration of the proposed algorithm is given by considering a five job three machine problem. Table 1 gives the processing times (in hours) ofeach job on each of the machines by treating them as symmetric triangular fuzzy variables.

Table 1:*Processing Times.*

Job	**Machine**			
	1	2	3	4
1	(1,3,5)	(4,6,8)	(3,6,9)	(7,10,13)
2	(2,5,8)	(1,4,7)	(5,7,9)	(8,10,12)
3	(4,7,10)	(2,3,4)	(1,5,9)	(7,9,11)
4	(3,6,9)	(5,8,11)	(2,6,10)	(7,12,17)
5	(4,6,8)	(2,5,8)	(5,8,11)	(10,12,14)

The expected value and variance of the above processing times are obtained as given in Tables 2 and 3, respectively.

Table 2:*Expected Value of Processing Times.*

Job	**Machine**			
	1	2	3	4
1	3	6	6	10
2	5	4	7	10
3	7	3	5	9
4	6	8	6	12
5	6	5	8	12

Table 3:*Variance of Processing Times.*

Job	**Machine**			
	1	2	3	4
1	0.666	0.666	1.500	1.500
2	1.500	1.500	0.666	0.666
3	1.500	0.166	2.666	0.666
4	1.500	1.500	2.666	4.166
5	0.666	1.500	1.500	0.666

By using the method of ranking fuzzy variables discussed in section Preliminaries, the minimum among the processing times under machine 1 and 4 are, respectively obtained as (1, 3, 5) and (7, 9, 11). The maximum among the processing times under machine 2 and machine 3 are (5, 8, 11) and (5, 8, 11) respectively. Since the conditions given in Step 3 of the algorithm are satisfied, the given problem is reduced to a sequencing problem involving only two machines G and H . The processing times (Table 4) under machines G and H are as follows:

Table 4:*Processing Times of Jobs under Machines G and H .*

Job	**Machine**	
	G	H
1	(8,15,22)	(14,22,30)
2	(8,16,24)	(14,21,28)
3	(7,15,23)	(10,17,24)
4	(10,20,30)	(14,26,38)
5	(11,19,27)	(17,25,33)

The expected value and variance of the above processing times based on machines G and H are given below:

Table5:*Expected Values of Processing Times of Jobs under Machines* G *and* H .

Job	Machine	
	G	H
1	5	22
2	16	21
3	15	17
4	20	26
5	19	25

Table 6:*Variance of processing times of jobs under machines* G *and* H .

Job	Machine	
	G	H
1	8.166	10.666
2	10.666	8.166
3	10.666	8.166
4	16.666	24.000
5	10.666	10.666

Based onTables5 and6,it can be seen that the minimum value of the processing times is obtained at job 1 under machine G . Hence job 1 is marked first in the sequence. In a similar manner, the positions of the other jobs are determined. Thus the final sequence of jobs that will minimize the total processing time based on all the machines is obtained as:

1	**3**	**2**	**5**	**4**

III. Conclusion

In this paper, a new sequencing algorithm is proposed that is used to obtain an optimal sequence of jobs processed under various machines by treating the processing times as fuzzy variables. Certain important concepts from 'Credibility Theory' due to [15] together with some results pertaining to fuzzy variables that are developed on the lines of 'Credibility Theory' are used in order to develop the proposed algorithm. The algorithm uses a novel approach to determine the sequence of jobs by ranking fuzzyvariables based on the concepts of 'Credibility Theory' without using the α-cut approach. A numerical illustration of the proposed algorithm is provided by representing the processing times as triangular fuzzy numbers and the optimal sequence of jobs is determined.

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