

## Effect of Magnetic Field on Peristaltic Flow of Williamson Fluid in a Symmetric Channel

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**Abstract:** This paper deals with the influence of magnetic field on peristaltic flow of an incompressible Williamson fluid in a symmetric channel with heat and mass transfer. Convective conditions of heat and mass transfer are employed. Viscous dissipation and Joule heating are taken into consideration. Channel walls have compliant properties. Analysis has been carried out through long wavelength and low Reynolds number approach. Resulting problems are solved for small Weissenberg number. Impacts of variables reflecting the salient features of wall properties, concentration and heat transfer coefficient are pointed out. Trapping phenomenon is also analyzed.

**Keywords:** Williamson fluid, compliant walls.

### I. Introduction

Peristalsis is the form of fluid transport due to wave travelling along the walls of an inextensible tube/channel. This type of rhythmic contraction provides the basis of peristaltic pumps to transport fluid through different tubular parts without any direct contact (see studies [1]). [2] also discussed the influence of rotation and initial stresses on peristaltic flow of fourth grade fluid in an asymmetric channel. The peristaltic flow of nano fluid through a porous medium with mixed convection is analyzed by Nowar [3]. Kothandapani and Prakash [4] studied the combined effects of radiation and magnetic field on peristaltic transport of nanofluid. The studies related to peristalsis with heat and mass transfer for Newtonian fluid Have been structured in refs. [5, 6]. [7] discussed the peristaltic flow in a vertical channel filled with nanofluid. Peristaltic transport of fourth grade fluid is numerically discussed by Mustafa et. al.[8]. Kothandapani and Prakash [9] analyzed the peristaltic transport of nanofluid with thermal radiation and magnetic field in a tapered channel. Mehmood et al. [10] considered the partial slip effects on peristaltic transport in a channel with heat and mass transfer effects. Mathematical analysis has been carried out in the presence of magnetic field. In all of the above mentioned studies lubrication approach has been utilized to simplify the problems. It is seen that heat transfer in peristalsis has pivoted role.

### II. Basic Equations

The continuity, momentum, energy and concentration equation in the presence of viscous dissipation and joule heating are given by

$$\text{div}V = 0 \quad (1)$$

$$\rho \frac{dV}{dt} = \text{div}\tau' + \rho f \quad (2)$$

$$\rho C_p \frac{dT}{dt} = k \nabla^2 T + \frac{DK_T}{C_s} + \frac{1}{\sigma} (J \cdot J) \quad (3)$$

Where  $V$  is velocity vector,  $\tau'$  the Cauchy stress tensor,  $\rho$  the body force,  $d/dt$  the material time derivative,  $C_p$  the special heat at constant volume,  $J$  the current density,  $T$  the temperature of fluid  $D$  the coefficient of mass diffusivity,  $k$  the thermal diffusion ratio,  $C_s$  the concentration susceptibility. The constitutive equations fluid is

$$\tau' = -pI + s \quad (4)$$

$$s = \mu_0 [(1 + \Gamma \dot{\gamma})] A_1 \quad (5)$$

Where  $p$  is the pressure,  $s$  the extra stress tensor,  $\mu_0$  the zero shear rate viscosity,  $\Gamma$  the time constant and  $\dot{\gamma}$  is given by

$$\dot{\gamma} = \sqrt{\frac{1}{2} \pi}, \quad (6)$$

In which  $\pi = \text{tr}(A_1^2)$  where  $A_1 = \nabla \vec{V} + (\nabla \vec{V})^T$ . Here Eq.(5) corresponds to the viscous fluid when  $\Gamma = 0$ .

### III. Mathematical Formulation

Consider an incompressible magnetohydrodynamic (MHD) flow of Williamson fluid in a symmetric channel of width  $2d_1$ . Here x-axis is taken along the length of channel and y-axis transverses to it (see Fig 1). The induced magnetic field is neglected by assuming a very small magnetic Reynolds number. Also the electric field is taken absent. The flow is generated by sinusoidal waves propagating along the compliant walls of channel:

$$\bar{y} = \pm \bar{\eta}(\bar{x}, \bar{t}) = \pm [d + m\bar{x} + a \sin \frac{2\pi}{\lambda} (\bar{x} - c\bar{t})] \quad (7)$$

Where  $a$  is the wave amplitude,  $\lambda$  the wavelength,  $c$  the wave speed

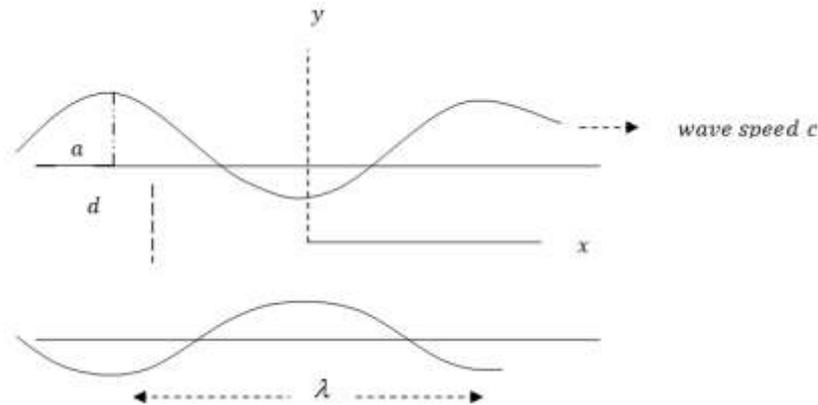


Figure 1 Diagrammatic of the problem

In wave frame, the equations which govern the flow are given by:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (8)$$

$$\rho \left( \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{u} \frac{\partial \bar{v}}{\partial \bar{y}} \right) = - \frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\partial}{\partial \bar{x}} \left[ 2\mu_0 (1 + \Gamma \bar{\gamma}) \frac{\partial \bar{u}}{\partial \bar{x}} \right] + \frac{\partial}{\partial \bar{y}} \left[ \mu_0 (1 + \Gamma \bar{\gamma}) \left( \frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}} \right) \right] - \sigma \beta_0^2 \bar{u} - \frac{\mu}{k} \bar{u} \quad (9)$$

$$\rho \left( \bar{v} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} \right) = - \frac{\partial \bar{p}}{\partial \bar{y}} + \frac{\partial}{\partial \bar{x}} \left[ \mu_0 (1 + \Gamma \bar{\gamma}) \left( \frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} \right) \right] + \frac{\partial}{\partial \bar{y}} \left[ 2\mu_0 (1 + \Gamma \bar{\gamma}) \frac{\partial \bar{v}}{\partial \bar{y}} \right] + \frac{\mu}{k} \bar{v} \quad (10)$$

$$\rho c_p \frac{d\bar{T}}{d\bar{t}} = k \left[ \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \right] +$$

$$\mu_0 (1 + \Gamma \bar{\gamma}) \left[ \left( \frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 + 2 \left( \frac{\partial \bar{u}}{\partial \bar{x}} \right)^2 + 2 \left( \frac{\partial \bar{v}}{\partial \bar{y}} \right)^2 \right] \quad (11)$$

where  $\rho$  is the density,  $B_0$  is the strength of the magnetic field,  $\bar{p}$  is the pressure,  $(\bar{u}, \bar{v})$  are the components of the velocity vector  $\bar{V}$  in the  $\bar{x}$  and  $\bar{y}$  directions respectively and  $\sigma$  the electrical conductivity. The corresponding boundary conditions are given by

$$\bar{u} = \pm \bar{\beta} \frac{\partial \bar{u}}{\partial \bar{y}} \text{ at } \bar{y} = \pm \bar{\eta} \quad (12)$$

$$\left[ -\tau \frac{\partial^3}{\partial \bar{x}^3} + m \frac{\partial^3}{\partial \bar{x} \partial \bar{t}^2} + \bar{d} \frac{\partial^2}{\partial \bar{x} \partial \bar{t}} B \frac{\partial^5}{\partial \bar{x}^5} + H \frac{\partial}{\partial \bar{x}} \right] (\bar{\eta}) = \frac{\partial}{\partial \bar{x}} 2\mu_0 (1 + \Gamma \bar{\gamma}) \frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial}{\partial \bar{y}} \left[ \mu_0 (1 + \Gamma \bar{\gamma}) \left( \frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} \right) \right] \text{ at } \bar{y} = \pm \bar{\eta} \quad (13)$$

$$T = T_0 \text{ at } \bar{y} = \pm \bar{\eta} \quad (14)$$

In above equations  $\tau$  is elastic tension in the membrane,  $m$  the mass per unit area,  $\bar{d}$  the coefficient of viscous damping and  $H$  the spring stiffness.

#### IV. Dimensionless Analysis

Defining velocity component  $u$  and  $v$  in terms of stream function, introduce the dimensionless variables

$$x = \frac{\bar{x}}{\lambda}, y = \frac{\bar{y}}{d}, \epsilon = \frac{a}{d}, u = \frac{\bar{u}}{c}, v = \frac{\bar{v}}{c\delta}, p = \frac{d^2 \bar{p}}{c\lambda\mu_0}, t = \frac{c\bar{t}}{\lambda}, We = \Gamma \frac{c}{a}, Br = Pr Ec, Pr = \mu_0 C P k, Ec = c 2 T_0 C P, E_1 = -\tau d 3 \lambda 3 \mu_0 C, E_2 = m c d 3 \lambda 3 \mu_0, E_3 = d 3 d \lambda 3 \mu_0, E_4 = \frac{B d^3}{c \lambda^3 \mu_0}, E_5 = \frac{H d^3}{c \lambda \mu_0}, Re = \frac{c d \rho}{\mu_0}, \bar{\gamma} = \dot{\gamma} \frac{d}{c}, M = \sqrt{\frac{\sigma}{\mu_0}} \beta_0 d, \bar{\gamma} = \dot{\gamma} \frac{d}{c}, \theta = \frac{T - T_0}{T_0} \quad (15)$$

And use long wave length assumption ( $\delta \ll 1$ ) the motion equation and temperature equation can be written as

$$\frac{\partial^2}{\partial y^2} \left[ (1 + We \dot{\gamma}) \left( \frac{\partial^2 \psi}{\partial y^2} \right) \right] - M^2 \frac{\partial \psi}{\partial y} - \frac{1}{k} \frac{\partial \psi}{\partial y} \quad (16)$$

$$\frac{\partial^2 \theta}{\partial y^2} + Br (1 + We \dot{\gamma}) \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 = 0 \quad (17)$$

$$\frac{\partial \psi}{\partial y} = \pm \beta \frac{\partial^2 \psi}{\partial y^2} aty = \pm \eta(18)$$

$$\left[ E_1 \frac{\partial^3}{\partial x^3} + E_2 \frac{\partial^3}{\partial x \partial t^2} + E_3 \frac{\partial^2}{\partial x \partial t} + E_4 \frac{\partial^5}{\partial x^5} + E_5 \frac{\partial}{\partial x} + \right] (\eta) =$$

$$\frac{\partial}{\partial y} \left[ (1 + We\dot{\gamma}) \left( \frac{\partial^2 \psi}{\partial y^2} \right) \right] aty = \pm \eta(19)$$

$$\theta = 0 \quad aty = \pm \eta(20)$$

$$\dot{\gamma} = \frac{\partial^2 \psi}{\partial y^2}(21)$$

$$\text{Here } \eta = 1 + \varepsilon \sin[2\pi(x - t)]$$

### V. Solution Procedure

It seems difficult to solve Eqs.(16) to (21) in closed form . Thus we aim to find series solutions for small Weissenberg number and write

$$\psi = \psi_0 + We\psi_1(22)$$

$$\theta = \theta_0 + We\theta_1(23)$$

Here we have considered the zeroth and first order systems only due to the fact that the perturbation is taken small ( $We \ll 1$ ) so that contribution of higher order terms are negligible due to order analysis.

#### Zeroth-order system with its boundary conditions

Substitute (22) and (23) in to equations (16) to (21) and equating the coefficients of equal power of Weissenberg number we get zero and first order system and they are as :

$$\frac{\partial^4 \psi_0}{\partial y^4} - A^2 \frac{\partial^2 \psi_0}{\partial y^2} = 0(24)$$

$$\left[ E_1 \frac{\partial^3}{\partial x^3} + E_2 \frac{\partial^3}{\partial x \partial t^2} + E_3 \frac{\partial^2}{\partial x \partial t} + E_4 \frac{\partial^5}{\partial x^5} + E_5 \frac{\partial}{\partial x} + \right] (\eta) =$$

$$\frac{\partial}{\partial y} \left[ \left( \frac{\partial^2 \psi_0}{\partial y^2} \right) \right] aty = \pm \eta(25)$$

$$\frac{\partial \psi_0}{\partial y} = \pm \beta \frac{\partial^2 \psi_0}{\partial y^2} aty = \pm \eta(26)$$

$$\frac{\partial^2 \theta_0}{\partial y^2} + Br \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2 = 0(27)$$

$$\theta_0 = 0 aty = \pm \eta(28)$$

$$\text{Here } A^2 = M^2 + \frac{1}{k} \text{ and } \eta = 1 + \varepsilon \sin[2\pi(x - t)].$$

#### And the first-order system with its boundary conditions

$$\frac{\partial^4 \psi_1}{\partial y^4} - A^2 \frac{\partial^2 \psi_1}{\partial y^2} + 2 \left( \frac{\partial^3 \psi_0}{\partial y^3} \right)^2 + 2 \frac{\partial^2 \psi_0}{\partial y^2} \frac{\partial^4 \psi_0}{\partial y^4} = 0(29)$$

$$\frac{\partial \psi_1}{\partial y} = \pm \beta \frac{\partial^2 \psi_1}{\partial y^2} aty = \pm \eta(30)$$

$$0 = \left[ \frac{\partial^3 \psi_1}{\partial y^3} + 2 \left( \frac{\partial^2 \psi_0}{\partial y^2} \right) \left( \frac{\partial^3 \psi_0}{\partial y^3} \right) \right] aty = \pm \eta(31)$$

$$\frac{\partial^2 \theta_1}{\partial y^2} + Br \frac{\partial^2 \psi_0}{\partial y^2} \left( 2 \frac{\partial^2 \psi_1}{\partial y^2} + \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2 \right) = 0(32)$$

$$\theta_1 = 0 \quad aty = \pm \eta(33)$$

$$\text{Here } A^2 = M^2 + \frac{1}{k} \text{ and } \eta = 1 + \varepsilon \sin[2\pi(x - t)].$$

Solve the zero and first order systems it is found them the stream function and temperature are given by

$$\psi = c3 + \frac{c2 e^{-Ay}}{A^2} + \frac{c1 e^{Ay}}{A^2} + c4y + We(d3 - \frac{e^{-2Ay}(c2^2 + c1^2 e^{4Ay} - 3e^{Ay}(d2 + d1 e^{2Ay}))}{3A^2} + d4y)$$

$$(34)$$

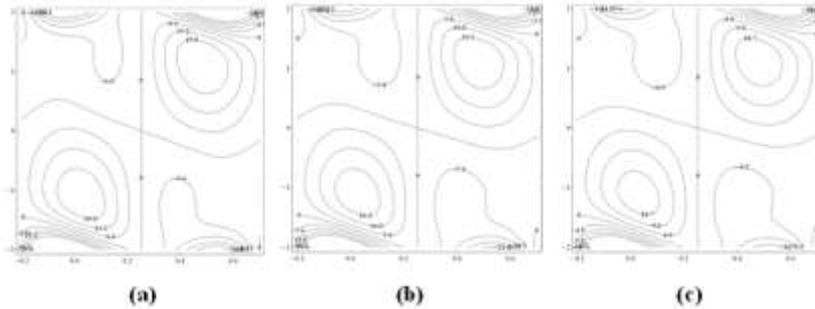
$$\theta =$$

$$b1 + b2y - \frac{Br(c2^2 e^{-2Ay} + c1^2 e^{2Ay} + 4c1c2A^2 y^2)}{4A^2} +$$

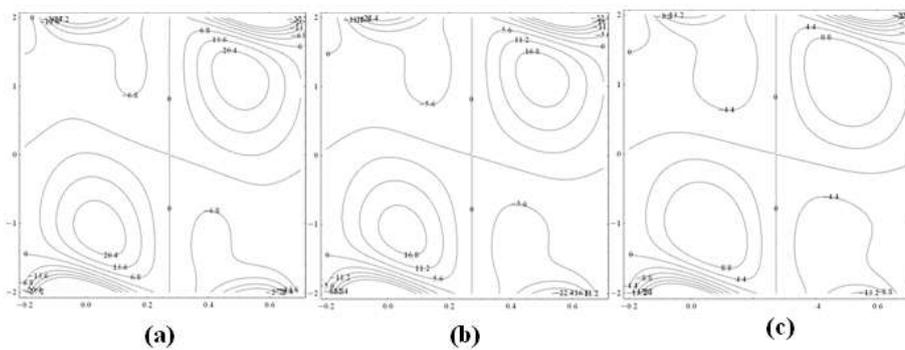
$$We \left( \frac{Br(10c2^3 e^{-3Ay} - 18c1c2^2 e^{-Ay} + 9c2(-3d2 e^{-2Ay} - 2c1^2 e^{Ay} - 6d1A^2 y^2) + c1(-27d1 e^{2Ay} + 10c1^2 e^{3Ay} - 54d2A^2 y^2))}{54A^2} \right) + z1 +$$

$$yz2 (35)$$

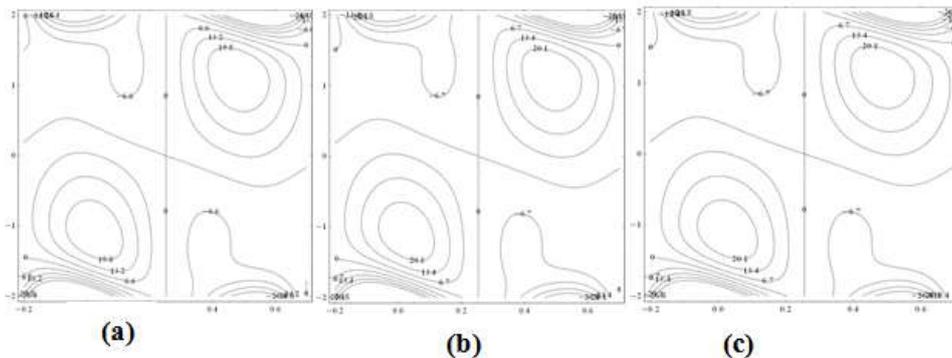
Here the constants  $c_1, c_2, c_3, c_4, d_1, d_2, d_3, d_4, b_1, b_2, z_1$  and  $z_2$  can be evaluated using MATHEMATICA



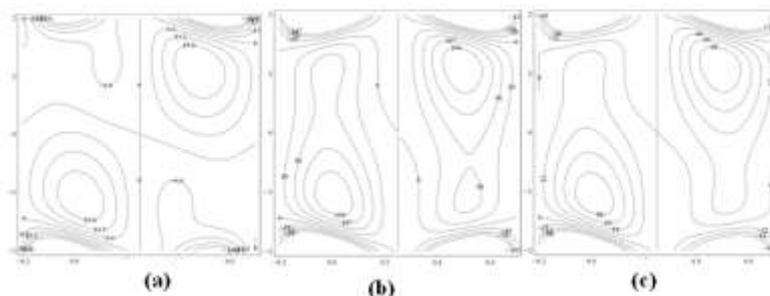
**Figure 2.** Stream line for different values of  $E_1$  (a)  $E_1 = 0.5$ , (b)  $E_1 = 0.7$ , (c)  $E_1 = 0.8$  and the other parameters  $E_2 = 0.2, E_3 = 0.1, E_4 = 0.05, E_5 = 0.3, We = 0.03, \varepsilon = 0.2, t = 0.02, M = 1, m = 0.01, \beta = 0.01$ .



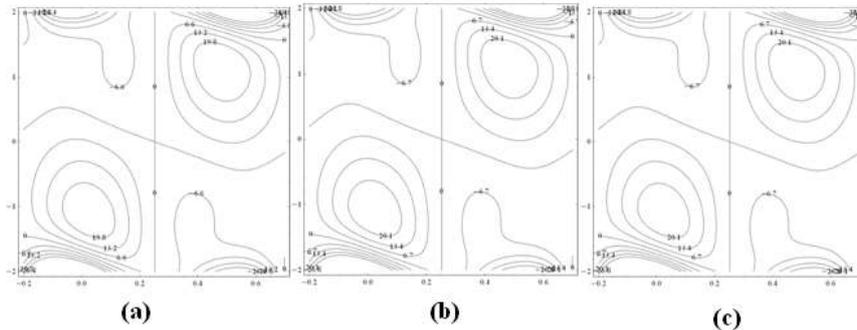
**Figure 3.** Stream line for different values of  $E_2$  (a)  $E_2 = 0.2$ , (b)  $E_2 = 0.4$ , (c)  $E_2 = 0.6$  and the other parameters  $E_1 = 0.5, E_3 = 0.1, E_4 = 0.05, E_5 = 0.3, We = 0.03, \varepsilon = 0.2, t = 0.02, M = 1, m = 0.01, \beta = 0.01$ .



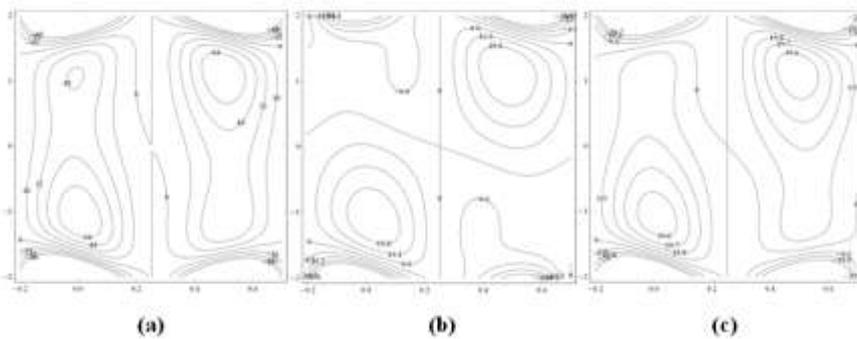
**Figure 4.** Stream line for different values of  $E_3$  (a)  $E_3 = 0.1$ , (b)  $E_3 = 0.2$ , (c)  $E_3 = 0.3$  and the other parameters  $E_1 = 0.5, E_2 = 0.2, E_4 = 0.05, E_5 = 0.3, We = 0.03, \varepsilon = 0.2, t = 0.02, M = 1, m = 0.01, \beta = 0.01$ .



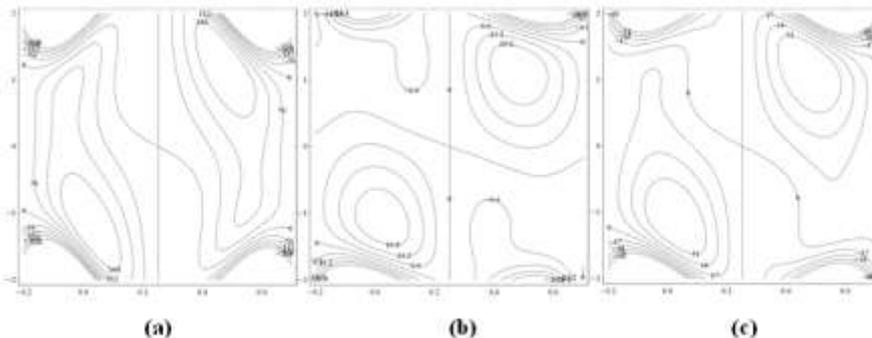
**Figure 5.** Stream line for different values of  $E_4$ (a) $E_4 = 0.05$ , (b) $E_4 = 0.10$ , (c) $E_4 = 0.15$  and the other parameters  $E_1 = 0.5$ ,  $E_2 = 0.2$ ,  $E_3 = 0.1$ ,  $E_5 = 0.3$ ,  $We = 0.03$ ,  $\varepsilon = 0.2$ ,  $t = 0.02$ ,  $M = 1$ ,  $m = 0.01$ ,  $\beta = 0.01$ .



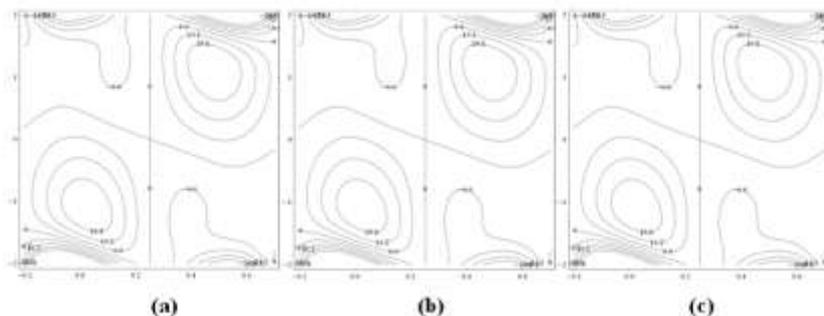
**Figure 6.** Stream line for different values of  $E_5$ (a) $E_5 = 0.3$ , (b) $E_5 = 0.6$ , (c) $E_5 = 0.9$  and the other parameters  $E_1 = 0.5$ ,  $E_2 = 0.2$ ,  $E_3 = 0.1$ ,  $E_4 = 0.05$ ,  $We = 0.03$ ,  $\varepsilon = 0.2$ ,  $t = 0.02$ ,  $M = 1$ ,  $m = 0.01$ ,  $\beta = 0.01$ .



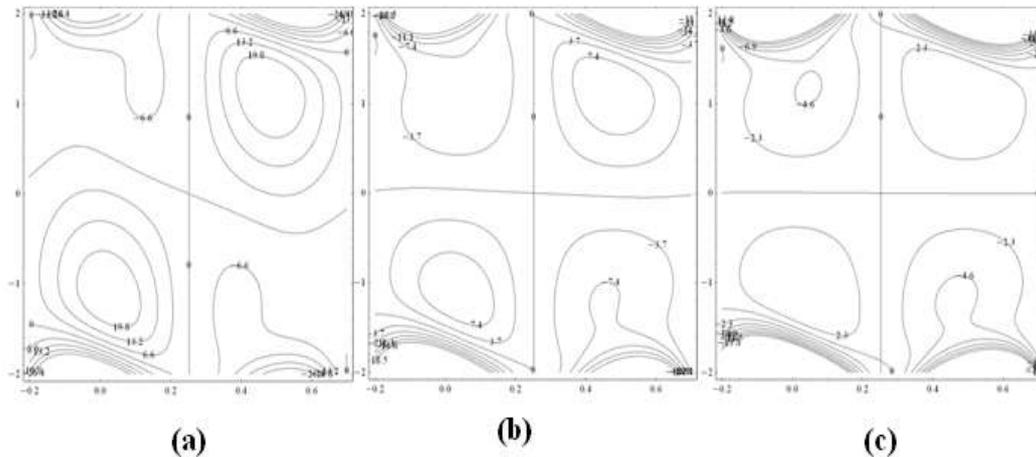
**Figure 7.** Stream line for different values of  $We$ (a) $We = 0.03$ , (b) $We = 0.09$ , (c) $We = 0.18$  and the other parameters  $E_1 = 0.5$ ,  $E_2 = 0.2$ ,  $E_3 = 0.1$ ,  $E_4 = 0.05$ ,  $E_5 = 0.3$ ,  $\varepsilon = 0.2$ ,  $t = 0.02$ ,  $M = 1$ ,  $m = 0.01$ ,  $\beta = 0.01$ .



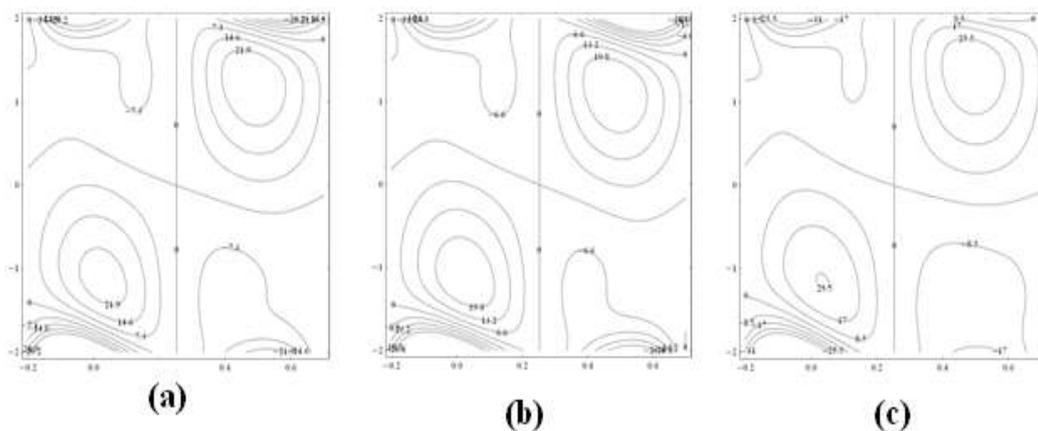
**Figure 8.** Stream line for different values of  $\varepsilon$ (a) $\varepsilon = 0.2$ , (b) $\varepsilon = 0.4$ , (c) $\varepsilon = 0.8$  and the other parameters  $E_1 = 0.5$ ,  $E_2 = 0.2$ ,  $E_3 = 0.1$ ,  $E_4 = 0.05$ ,  $E_5 = 0.3$ ,  $We = 0.03$ ,  $t = 0.02$ ,  $M = 1$ ,  $m = 0.01$ ,  $\beta = 0.01$ .



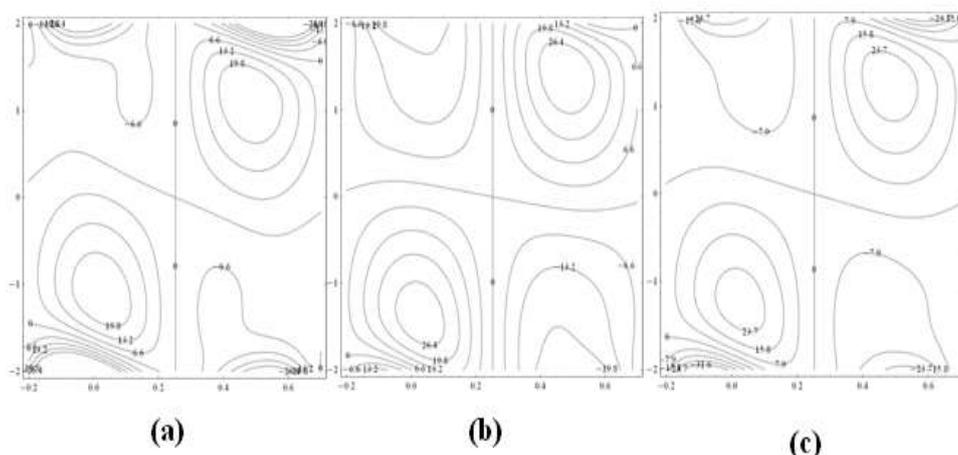
**Figure 9.** Stream line for different values of  $t$  (a)  $t = 0.02$ , (b)  $t = 0.04$ , (c)  $t = 0.06$  and the other parameters  $E_1 = 0.5, E_2 = 0.2, E_3 = 0.1, E_4 = 0.05, E_5 = 0.3, We = 0.03, \varepsilon = 0.02, M = 1, m = 0.01, \beta = 0.01$ .



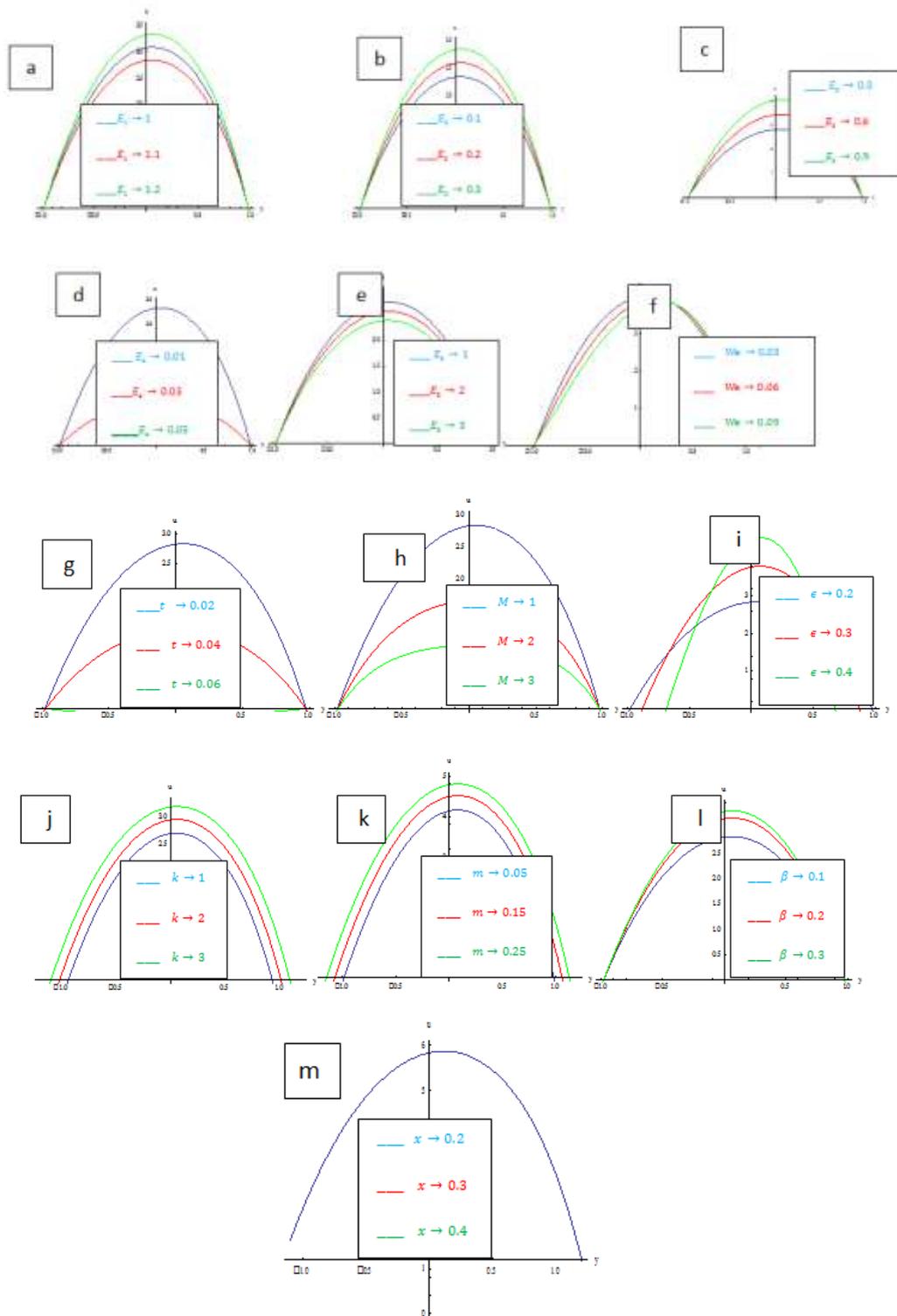
**Figure 10.** Stream line for different values of  $M$  (a)  $M = 1$ , (b)  $M = 2$ , (c)  $M = 3$  and the other parameters  $E_1 = 0.5, E_2 = 0.2, E_3 = 0.1, E_4 = 0.05, E_5 = 0.3, We = 0.03, \varepsilon = 0.2, t = 0.02, m = 0.01, \beta = 0.01$



**Figure 11.** Stream line for different values of  $m$  (a)  $m = 0.01$ , (b)  $m = 0.02$ , (c)  $m = 0.03$  and the other parameters  $E_1 = 0.5, E_2 = 0.2, E_3 = 0.1, E_4 = 0.05, E_5 = 0.3, We = 0.03, \varepsilon = 0.2, t = 0.02, M = 1, \beta = 0.01$ .

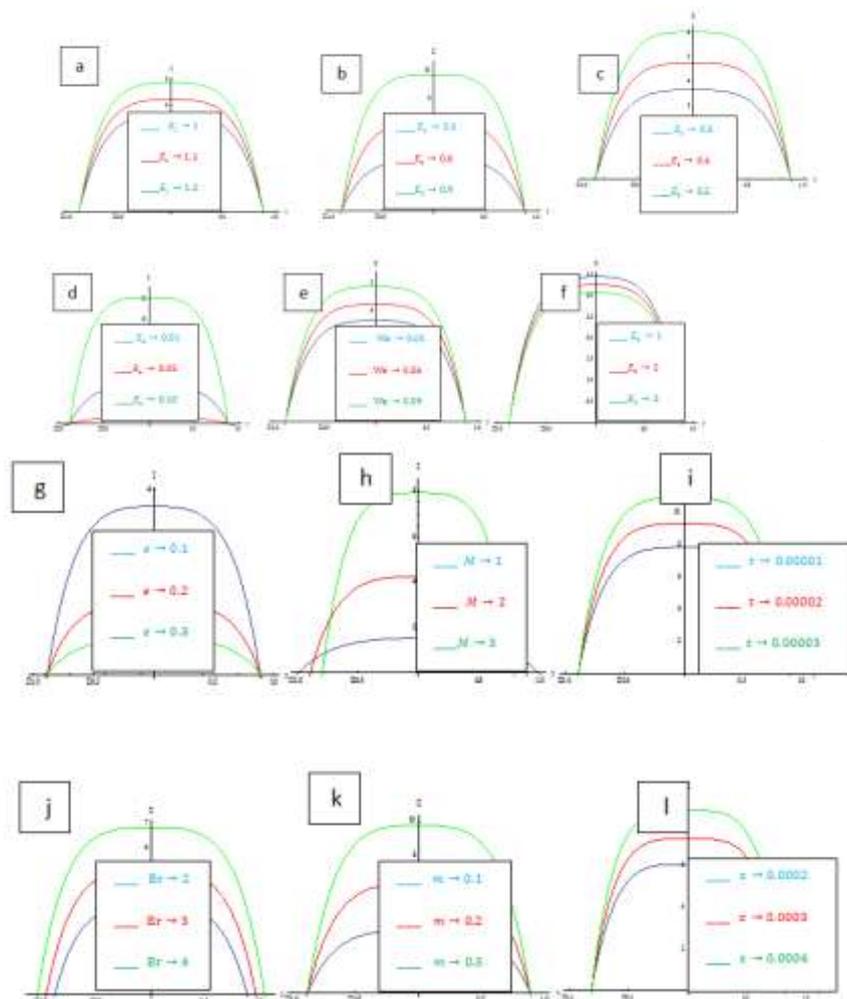


**Figure 12.** Stream line for different values of  $\beta$  (a)  $\beta = 0.01$ , (b)  $\beta = 0.02$ , (c)  $\beta = 0.03$  and the other parameters  $E_1 = 0.5, E_2 = 0.2, E_3 = 0.1, E_4 = 0.05, E_5 = 0.3, We = 0.03, \varepsilon = 0.2, t = 0.02, M = 1, m = 0.01$ .



**Figure 13.**(a) Variation of  $u$  with  $y$  for different values of  $E_1$  at  $E_2 = 0.3, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, k = 1, m = 0.1, \beta = 0.1, x = 0.8$ . (b)- Variation of  $u$  with  $y$  for different values of  $E_2$  at  $E_1 = 1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, k = 1, m = 0.1, \beta = 0.1, x = 0.8$ . (c)- Variation of  $u$  with  $y$  for different values of  $E_3$  at  $E_1 = 1, E_2 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, k = 1, m = 0.1, \beta = 0.1, x = 0.8$ . (d)- Variation of  $u$  with  $y$  for different values of  $E_4$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_5 = 0.01, We = 0.03,$

$\epsilon = 0.2, t = 0.02, M = 1, k = 1, m = 0.1, \beta = 0.1, x = 0.8$ . (e)- Variation of  $u$  with  $y$  for different values of  $E_5$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, k = 1, m = 0.1, \beta = 0.1, x = 0.8$ . (f)- Variation of  $u$  with  $y$  for different values of  $We$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, \epsilon = 0.2, t = 0.02, M = 1, k = 1, m = 0.1, \beta = 0.1, x = 0.8$ . (g)- Variation of  $u$  with  $y$  for different values of  $t$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, M = 1, k = 1, m = 0.1, \beta = 0.1, x = 0.8$ . (h)- Variation of  $u$  with  $y$  for different values of  $M$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, k = 1, m = 0.1, \beta = 0.1, x = 0.8$ . (i)- Variation of  $u$  with  $y$  for different values of  $\epsilon$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, M = 1, t = 0.02, k = 1, m = 0.1, \beta = 0.1, x = 0.8$ . (j)- Variation of  $u$  with  $y$  for different values of  $k$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, m = 0.1, \beta = 0.1, x = 0.8$ . (k)- Variation of  $u$  with  $y$  for different values of  $m$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, k = 1, \beta = 0.1, x = 0.8$ . (l)- Variation of  $u$  with  $y$  for different values of  $\beta$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, k = 1, m = 0.1, x = 0.8$ . (m)- Variation of  $u$  with  $y$  for different values of  $x$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, k = 1, m = 0.1, \beta = 0.1$ .



**Figure 14.**(a) Variation of  $\theta$  with  $y$  for different values of  $E_1$  at  $E_2 = 0.3, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, m = 0.1, x = 0.8$ . (b)- Variation of  $\theta$  with  $y$  for different values of  $E_2$  at  $E_1 = 1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, m = 0.1, x = 0.8$ . (c)- Variation of  $\theta$  with  $y$  for different values of  $E_2$  at  $E_1 = 1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, m = 0.1, x = 0.8$ . (d)- Variation of  $\theta$  with  $y$  for different values of  $E_4$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, m = 0.1, x = 0.8$ . (e)- Variation of  $\theta$  with  $y$  for different values of  $We$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, \epsilon = 0.2, t = 0.02, M = 1, m = 0.1, x = 0.8$ . (f)- Variation of  $\theta$  with  $y$  for different values of  $E_5$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, m = 0.1, x = 0.8$ . (g)- Variation

of  $\theta$  with  $y$  for different values of  $\epsilon$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, t = 0.02, M = 1, m = 0.1, x = 0.8$ . (h)- Variation of  $\theta$  with  $y$  for different values of  $Ma$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, m = 0.1, x = 0.8$ . (i)- Variation of  $\theta$  with  $y$  for different values of  $ta$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, M = 1, m = 0.1, x = 0.8$ . (j)- Variation of  $\theta$  with  $y$  for different values of  $Br$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, m = 0.1, x = 0.8$ . (k)- Variation of  $\theta$  with  $y$  for different values of  $ma$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, Br = 2, x = 0.8$ . (l)- Variation of  $\theta$  with  $y$  for different values of  $x$  at  $E_1 = 1, E_2 = 0.1, E_3 = 0.3, E_4 = 0.01, E_5 = 0.01, We = 0.03, \epsilon = 0.2, t = 0.02, M = 1, m = 0.1, .$

## VI. Graphical Results And Discussion

In this chapter, we have presented a set of figures to observe the behavior of sundry parameters involved in the expressions of longitudinal velocity ( $u = \psi_{0y} + We\psi_{1y}$ ), temperature  $\theta$  and stream function  $\psi$ . Figs 13.(a) to 13.(m) display the effects of various physical parameters on the velocity profile ( $y$ ). Figs 13.(a) and 13.(b) depicts that the velocity increases when  $E_1$  and  $E_2$  enhanced. It due to the fact that less resistance is offered to the flow because of the wall elastance and thus velocity increases. However reverse effect is observed for  $E_3, E_4$  and  $E_5$ . As  $E_3$  represent the damping which is resistive force so velocity decreases when  $E_3$  is increased. Similar behavior is observed for the velocity in case of rigidity and stiffness due to presence of damping force. Fig 13.(h) shows that the velocity decreases by increasing Hartman number. It is due to the fact that magnetic field applied in the trasverse direction shows damping effect on the flow. Fig 13.(f) illustrates that velocity profile decreases for  $We$  in the region  $[-1,0]$  whereas it has opposite behavior in the region  $[0,1]$ . Figs 13.(j), 13.(k) and 13.(l) depicts that the velocity increases when  $m, ka$  and  $\beta$  enhanced. Effects of pertinent parameters on temperature profile can be visualized through Figs 14.(a) to 14.(l). The variation of compliant wall parameters is studied in Figs 13.(a) to 17.(e). As temperature is the average kinetic energy of the particles and kinetic energy depends upon the velocity. Therefore increase in velocity by  $E_1$  and  $E_2$  leads to temperature enhancement. Similarly decrease in velocity by  $E_3, E_4$  and  $E_5$  shows decay in temperature. Fig 14.(h) reveals that the temperature profile  $\theta$  decreases when Hartman number  $M$  is increased. Effect of  $Br$  on temperature can be observed through Fig 14.(j). The Brinkman number  $Br$  is the product of the Prandtl number  $Pr$  and the Eckert number  $Ec$ . Here Eckert number occurs due to the viscous dissipation effects and the temperature enhances. Fig 14.(k) depict that the Temperature increases when  $m$  enhanced. The function of an internally circulating bolus of fluid by closed stream lines is shown in Figs 12. Fig 10 depict that the size of trapped bolus increases when there is an increase in the Hartman number. We have analyzed from Figs 2 and 3 that the size of trapped bolus increases when  $E_1$  and  $E_2$  are enhanced. However it decreases when  $E_3$  is increased. Also when we decrease the values of  $E_4$  and  $E_5$  the trapped bolus size increases. Fig 7 depict the stream lines pattern for different values of Weissenberg number.

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