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The Multisection Of Any Arbitrary Angle With Straightedge And Compass Only

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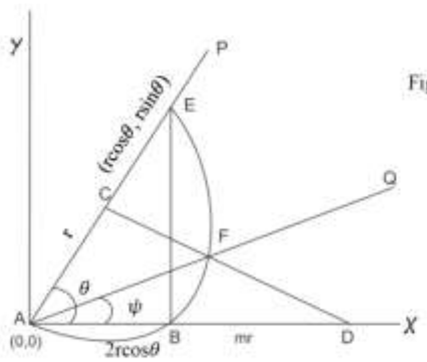
Preface
At school level students learn about bisection of any arbitrary angle or repeated bisections if required with the help of straightedge and compass only. They do not come across about angle trisection or other higher multisections with the help of only these tools.In 1837 P. Wantzel a French mathematician proved the impossibility of trisection of any arbitrary angle with only stranghtedge and compass except some specified angles.In my present work I have endeavoured to demostrate methods for drawing very approximate multisections of any arbitrary angles lying between 0 to 90^0 with the help of only stranghtedge and compass.
I came to know about impossibility of angle trisection at my B.Sc. standard in 1970 through a mathematical journal and since then I was working on it. Thus my present work is the result of many years of dedicated efforts.

I. Multisection of an arbitrary angle with the help of straightedge and compass only

1. Before proceeding further we shall first derive the formula or results (4) & (5) worked out in the following pages.



Given AX & AY are X & Y axis.
AP a line making an angle θ with X axis such that ($0^0 < \theta \leq 90^0$)
C any point on the line AP,a semi-circle ABFE is drawn taking C as centre and AC as radius*r*.
The semi circle cuts X axis on pt. B. AE is diameter.
BD a length is cut equal to mr (m times of radius r). points C & D are joined.
To find the locus of point F where the line CD cuts the semicircle at F.
Point C is (r cosθ, r sinθ)
Point E is (2r cosθ, 2r sinθ)
So length A B is = 2r cosθ& BD = mr
Hence coordinates of pt. D is (2r cosθ+mr, o)
So equation of line CD is
$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$
Or $y - r\sin\theta = \frac{0-r\ sin\theta}{2rcos\theta + mr - rcos\theta}(x - r\cos\theta)$
Or $y - r\sin\theta = \frac{-r\ sin\theta}{r\ cos\theta + mr}\ (x - rcos\theta)$
Or $(y - r\sin\theta)(r\cos\theta + mr) = r^2 \sin\cos\theta - xr\mathrm{Sin}\theta$
Or $(y - r\sin\theta) \times r\ (cos\theta + m) = r\ (rsin\theta cos\theta - xsin\theta)$
Or $(y - rsin\theta)(cos\theta + m) = rsin\theta cos\theta - x\ sin\theta$
Or $y\ cos\theta + my - r\ sin\theta cos\theta = r\ sin\theta cos\theta - x\ sin\theta + mrsin\theta$

Or $y\ cos\theta + my + xsin\theta = 2r\ sin\theta cos\theta + mr\ sin\theta$------------------- (1)
Now equation of the circle is
$(x-x_1)^2 + (y-y_1)^2 = r^2$
$= (x - rcos\theta)^2 + (y - rsin\theta)^2 = r^2$-------------------------- (2)
$= x^2+r^2cos^2\theta-2xrcos\theta+y^2+r^2sin^2\theta-2yr\ sin\theta= r^2$
Or $x^2+y^2+r^2\ (cos^2\theta+sin^2\theta)- 2r\ (x\ cos\theta+ y\ sin\theta) = r^2$
Or $x^2+y^2+r^2 – 2r\ (x\ cos\theta+y\ sin\theta)= r^2$
Or $x^2+y^2 = 2r\ (x\ cos\theta+y\ sin\theta)$------------------------------ (3)
Now from (1) $r = \frac{y\ cos\theta + x\ sin\theta + my}{2sin\theta cos\theta\ + msin\theta}$
Putting this value of r in eq. (3) we have
$x^2+y^2 = \frac{2(y\ cos\theta + x\ sin\theta + my)(xcos\theta + ysin\theta)}{2sin\theta cos\theta\ + msin\theta}$
or $(x^2+y^2)\ (2sin\theta cos\theta + msin\theta)$
$= 2(ycos\theta + xsin\theta + my)\ (x\ cos\theta + y\ sin\theta)$
L.H.S. = $2x^2sin\theta cos\theta + x^2msin\theta + 2y^2sin\theta cos\theta + my^2sin\theta$
R.H.S. = $2yxcos^2\theta + 2y^2sin\theta cos\theta + 2x^2sin\theta cos\theta + 2xysin^2\theta + 2xymcos\theta + 2my^2sin\theta$
= $2xy(cos^2\theta + sin^2\theta) + 2y^2\ sin\theta cos\theta + 2x^2sin\theta cos\theta + 2xymcos\theta + 2my^2sin\theta$
Simplifying LHS & RHS we get:-
$mx^2sin\theta + my^2sin\theta = 2xy + 2xymcos\theta + 2my^2sin\theta$
OR $mx^2sin\theta = 2my^2sin\theta - my^2sin\theta + 2xymcos\theta + 2xy$
OR $my^2sin\theta + 2xy(1 + mcos\theta) - mx^2sin\theta = 0$
OR $y = \frac{-2x(1+mcos\theta) \pm \sqrt{4x^2(1+mcos)^2 + 4m^2x^2sin^2\theta}}{2msin\theta}$
By solving for y-
OR $y = \frac{-2x(1+mcos\theta) \pm \sqrt{4x^2(1+m^2cos^2\theta + 2mcos\theta + 4m^2x^2sin^2\theta}}{2msin\theta}$

$$= \frac{-2x(1 + mcos\theta) \pm \sqrt{4x^2\{1 + (m^2cos^2\theta + m^2sin^2\theta) + 2mcos\theta\}}}{2msin\theta}$$

OR $y = \frac{-2x(1+mcos\theta) \pm \sqrt{4x^2(1+m^2+2mcos\theta)}}{2msin\theta}$

$$= \frac{-2x(1 + mcos\theta) \pm 2x\sqrt{1 + m^2 + 2mcos\theta}}{2msin\theta}$$

OR $y = \frac{x\sqrt{1+m^2+2mcos\theta} - x(1+mcos\theta)}{msin\theta}$
Only taking (+) sign. (since$0^0<\theta \leq 90^0$)

$$= x\left\{\frac{\sqrt{1 + m^2 + 2mcos\theta} - (1 + mcos\theta)}{msin\theta}\right\}$$

Hence Locus of the point F, the point of intersection of line CD & circle ABFE is a straight line passing through origin A(0,0) and making an angle ψ with X axis.
such that,
$\tan\psi = \frac{\sqrt{1+m^2+2mcos\theta} - (1+mcos\theta)}{msin\theta}$ **……………………………………….(4)**
which is independent of r and a function of θ& m only.
This is an important result and the basis of angle multisections.
Note that the length BD is = m × r
that is m times or radius r
if BD is $\frac{m}{n}$ times of r then (4) becomes
$\tan\psi = \frac{\sqrt{m^2+n^2+2mncos\theta} - (n+mcos\theta)}{msin\theta}$**…………………………..…………….(5)**

1.1 Corollary-1
if BD = mr and m=1
then BD = r so from (4)
$\tan\psi = \frac{\sqrt{1^2+1^2+2.1cos\theta} - (1+cos\theta)}{sin\theta}$
or = $\frac{\sqrt{2+2cos\theta} - (1+cos\theta)}{sin\theta}$

$= \frac{\sqrt{2(1+cos\theta)}-(1+cos\theta)}{sin\theta}$

$= \frac{\sqrt{2.2cos^2\frac{\theta}{2}-2cos^2\frac{\theta}{2}}}{2sin\frac{\theta}{2}cos\frac{\theta}{2}}$

$= \frac{2cos\frac{\theta}{2}-2cos^2\frac{\theta}{2}}{2sin\frac{\theta}{2}cos\frac{\theta}{2}}$

$= \frac{2cos\frac{\theta}{2}\left(1-cos\frac{\theta}{2}\right)}{2sin\frac{\theta}{2}cos\frac{\theta}{2}}$

$= \frac{1-cos\frac{\theta}{2}}{sin\frac{\theta}{2}} = \frac{2sin^2\frac{\theta}{4}}{2sin\frac{\theta}{4}cos\frac{\theta}{4}}$

$= \frac{sin\frac{\theta}{4}}{cos\frac{\theta}{4}} = tan\frac{\theta}{4}$

So $tan\psi = tan\frac{\theta}{4}$

Or $\psi = \frac{\theta}{4}$

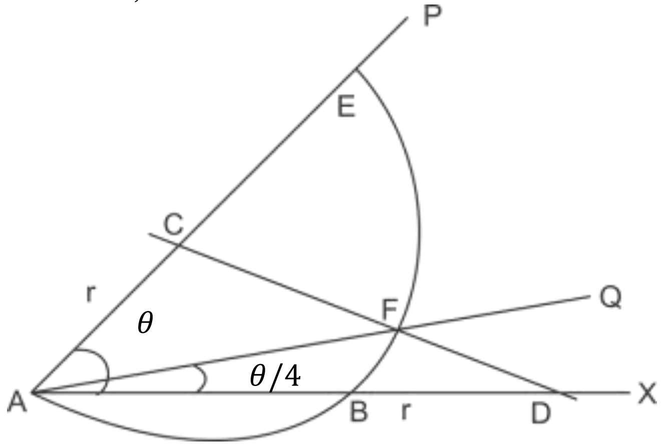
Hence if length BD = r (radius) then

$\angle QAD = \frac{1}{4}\angle PAD = \frac{\theta}{4}$ ……………………………………………………(6)

This is another important result as it gives us a method for constructing $\frac{1}{4}^{th}$ of any original angle without bisecting it with the help of straightedge and compass only.

In Fig.2

$\angle EAD = \theta, m = 1$



$Fig. 2$

$m = 1$

$BD = AC = r$

By corollary 1 $\angle FAD = \frac{1}{4}\theta$

1.2 Corollary-2

If D proceeds further & BD becomes larger $\angle FAD$ increases gradually and finally when BD is infinite line CD becomes parallel to AD or X axis then equation of line CD becomes.

$y = rsin\theta$

and eq. of circle is $(x - rcos\theta)^2 + (y - rsin\theta)^2 = r^2$

or $(x - rcos\theta)^2 - 0^2 = r^2$

or $\left(x - \frac{ycos\theta}{sin\theta}\right)^2 = \left(\frac{y}{sin\theta}\right)^2$

or $x - \frac{ycos\theta}{sin\theta} = \frac{y}{sin\theta}$

or $x\,sin\theta - ycos\theta = y$

or $x\,sin\theta = y(1 + cos\theta)$

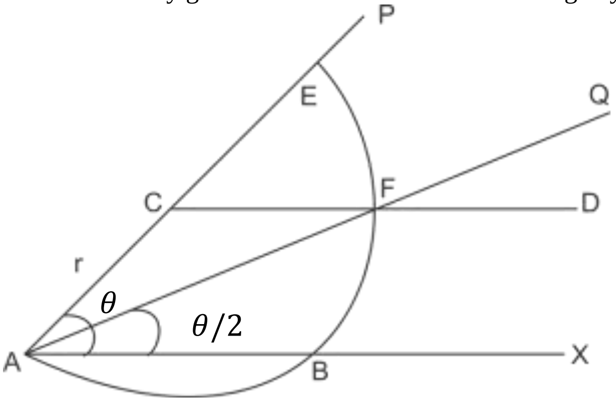
or $y \times 2cos^2\frac{\theta}{2} = x \times 2sin\frac{\theta}{2}cos\frac{\theta}{2}$

or $y \times cos\frac{\theta}{2} = x \times sin\frac{\theta}{2}$

or $y = x.tan\frac{\theta}{2}$

or $tan\psi = tan\frac{\theta}{2}$

or $\psi = \frac{\theta}{2}$ ……………………………………………………………(7)
So in this case the line AF bisects the angle E A B or θ.
So this corollary gives us another method of bisecting any given angle.



$Fig.3$

$m = \infty$

Line CD is parallel to AX
$\angle FAB = \frac{1}{2}\angle EAX = \frac{\theta}{2}$
When the length BD = 0, pt. F concides with pt. B and $\angle FAB = 0$ and as per corollary 2 when BD is infinite $\angle FAB = \frac{1}{2}\angle EAB$
So by taking different values of m we can multisect the given angle θ with the help of straight edge and compass only.

II. Some results of taking different values of *m* and θ and ψ

A for trisection of an angle

θ	m or $\frac{m}{n}$	ψ
90^0	$\sqrt{3}$	30^0exactly
60^0	$\frac{15}{8} or \left(2-\frac{1}{8}\right)$	19.983^0approximately
30	2	10.05^0approx.
20	2	6.682^0approx.
10	2	3.335^0approx.
6	2	2.00043^0approx.

From above if will be seen, when$\theta \to 0^0$, "2"is the limiting value of *'m'* for trisection of any angle.
So the values from $\theta = 0^0$ to 90^0, m ranges between 2 to $\sqrt{3}$
For *'m'* formula (4) and for $\frac{m}{n}$ formula (5) can be taken to obtain the value of tan ψ or ψ

B. other results-

	θ	m or $\frac{m}{n}$	ψ
Voluntary	90^0	$\sqrt{3}$	30^0 exactaly
	90	1.2 or $\frac{6}{5}$	25^0approximately
	90	.84 or $\frac{21}{25}$	20^0approx.
	60	4.5or $\frac{9}{2}$	25^0approx.
Pentasection	90	$\sqrt{3}-1$	18.1^0approx.
	60	$\frac{9}{13}$	12.003 approx.
	30	$\frac{2}{3}$	5.97^0

3. Special Case:when $\theta = 90^0, cos\theta = 0, sin\theta = 1$ $then\ formula\ (4) becomes$

$$\boldsymbol{tan\psi = \frac{\sqrt{1+m^1}-1}{m}}$$ **……………………………………………………………..(8)**

put$m = \sqrt{3}$ then

$$tan\psi = \frac{\sqrt{1+3}-1}{\sqrt{3}}$$

$= \frac{2-1}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^0$

or$\psi = 30^0 = \frac{1}{3}\ of\ 90^0\ which\ is\ true\ as\ per\ results\ of\ A\ \&\ B.$

Let$tan\psi = \frac{\sqrt{1+m^2}-1}{m}$ = k

So $\sqrt{1+m^2} - 1 = mk$

or$\sqrt{1+m^2} - 1 = mk$

or$1 + m^2 = 1 + m^2k^2 + 2mk$

or$m^2 - m^2k^2 = 2mk$

or$m^2(1-k^2) = 2mk$

or$\boldsymbol{m = \frac{2k}{1-k^2} = tan2\psi}$ **…………………………………………………………(9)**

So for given k, m can be evaluated exact or approx.for $\theta = 90^0$and also for any angle θ applying the results (4) or (5). Nearest fractional values can be obtained by applying continued fractions methods.

For Example if θ = 90^0 then for trisection

ψ = 30^0 So by result (9)

m = tan2ψ = tan 2.30^0

= tan60^0 = $\sqrt{3}$ as in result A

For ψ = 15^0 m = tan2.15^0

= tan30 = $\frac{1}{\sqrt{3}}$

For ψ = 20^0, m = tan40^0

= .84 = $\frac{21}{25}$ approximately as in result B

For ψ = 25^0 m = tan50^0

= 1.19 = $\frac{6}{5}$ approximately As in result B

4. How to draw lines equal to

BD = mr with the help of unmarked straight edge and compass only.

We know that by simple school level geometry methods any line can be divided into any number of equal parts. This method can be used here.

i) When m is an integer like 2, 3, 4, 5……… simply BD = m times of radius and can be drawn by compass only.

ii) When m = $\frac{p}{q}$ a fractional form then BD = $\frac{pr}{q}$ = p × $\left(\frac{r}{q}\right)$ in this cuse r can be divided into q equal parts and p part of it will be take to draw BD. All this exercise can be done with the help of straight edge and compass only.

e.g. $\frac{15r}{8}$ = $\left(1+\frac{7}{8}\right)r = \left(2-\frac{1}{8}\right)r$

BD = $\left(r+\frac{7}{8}r\right)\ or\ \left(2r-\frac{1}{8}r\right)$ can be draw in parts.

iii) When m = $\sqrt{p}$ likes $\sqrt{2}, \sqrt{3}, \sqrt{5}$ etc.

Then BD = $\sqrt{p}$ r like $\sqrt{2}$ r, $\sqrt{3}$ r……….

in right angled triangle Fig.4

ABC if AB = BC = r

then AC = $\sqrt{2}$ r

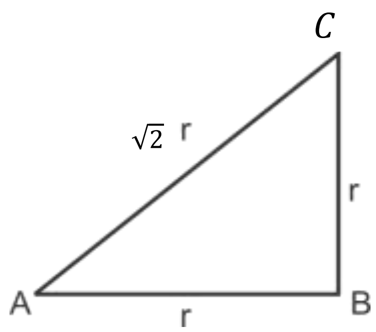


Fig.4

in fig. 5 rt angled triangle if AC = 2 r

CB = r then

AB = $\sqrt{3}$ r

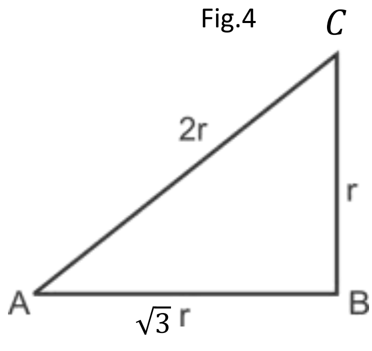


Fig.5

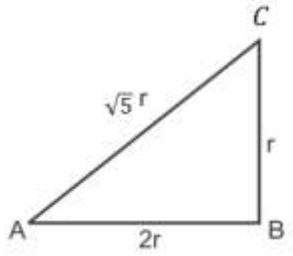


Fig.6

In fig.6
BC = r AB = 2r
Then AC = $\sqrt{5}$ r

In fig.7
If AB = $\sqrt{5}$ r
BC = r
Then AC = $\sqrt{6}$ r

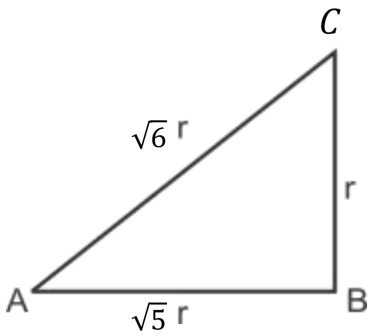


Fig.7

Similarly any line equal to $\sqrt{p}$ r can be drawn.

iv) If m = $\frac{\sqrt{b}}{\sqrt{q}}$

The BD = $\frac{\sqrt{b}}{\sqrt{q}}$ r = $\frac{\sqrt{pq}}{q}$ r

Then $\sqrt{pq}$ r can be drawn as in (iii) and then divided into q equal parts.

e.q. $\frac{\sqrt{3}}{\sqrt{2}}$ r = $\frac{\sqrt{6}}{2}$ r

in fig. (7) AC = $\sqrt{6}$ r

so BD = $\frac{\sqrt{6}}{2}$ r = $\frac{AC}{2}$

v) If m is like $\sqrt{3}$ – 1………

BD = $(\sqrt{3} - 1)$ r

= $\sqrt{3}$ r – r which can be draw n easily so BD can be drawn with the help of straights edge and compass only for all above type of values of m.

5. To find the limiting values of m when $\boldsymbol{\theta}$ is small and tends to zero

As in result (4) we have

$$\tan\psi = \frac{\sqrt{1+m^2+2m\cos\theta} - (1+m\cos\theta)}{m\sin\theta}$$

$$= \frac{\left\{1+m^2+2m\left(1-\frac{\theta^2}{!2}+\frac{\theta^4}{!4}\ldots\ldots\right)\right\}^{1/2} - \left\{1+m\left(1-\frac{\theta^2}{!2}+\frac{\theta^4}{!4}\ldots\ldots\right)\right\}}{m\left(\theta-\frac{\theta 3}{!3}+\frac{\theta 5}{!5}\ldots\ldots\right)}$$

Since $\theta$is small we can discard θ^3 and other higher powers of θ

Then,

$$\mathrm{Tan}\psi = \frac{\{1+m^2+2m-m\theta^2\}^{1/2} - \left\{1+m-\frac{m\theta^2}{2}\right\}}{m\theta}$$

$$= \frac{\{(1+m)^2-m\theta^2\}^{1/2} - \left\{1+m-\frac{m\theta^2}{2}\right\}}{m\theta}$$

$$= \frac{\left\{(1+m)^2\left(1-\frac{m\theta^2}{(1+m)^2}\right)\right\}^{1/2} - \left\{1+m-\frac{m\theta^2}{2}\right\}}{m\theta}$$

$$= \frac{\left\{(1+m)\left(1-\frac{1}{2}\frac{m\theta^2}{(1+m)^2}\right)\right\} - \left\{1+m-\frac{m\theta^2}{2}\right\}}{m\theta}$$

$= \frac{\left\{1+m-\frac{m\theta^2}{2(1+m)}\right\}-\left\{1+m-\frac{m\theta^2}{2}\right\}}{m\theta}$

$= \frac{1+m-1-m+\frac{m\theta^2}{2}-\frac{m\theta^2}{2(1+m)}}{m\theta}$

$= \frac{\frac{m\theta^2}{2}-\frac{m\theta^2}{2(1+m)}}{m\theta}$

$= \frac{\frac{m\theta^2}{2}\left(1-\frac{1}{1+m}\right)}{m\theta}$

$= \frac{\theta}{2} \times \frac{m}{m+1}$

Now since θ is small

Tanψ = ψ

So,

$\psi = \frac{\theta}{2} \times \frac{m}{1+m}$……………………………………….. (10)

This equation gives limiting values of m for different multisections of θ when θ is small

i) For trisection $\psi = \frac{\theta}{3}$

So, $\frac{\theta}{3} = \frac{\theta}{2} \times \frac{m}{1+m}$

or$\frac{1}{3} = \frac{m}{2(Hm)}$

or 3m = 2(1+m)

or 3m – 2m = 2

or m= 2

So as θ becomes smaller and smaller 2 is the limiting value of m as shown in result A.

ii) For penta section

$\psi = \frac{\theta}{5}$

So $\frac{\theta}{5} = \frac{\theta}{2} \cdot \frac{m}{(1+m)}$

or 5m = 2m + 2

or 3m = 2

or m = $\frac{2}{3}$ is the limiting value of m

Similarity for other multisections limiting values can be derived when θ goes smaller in general for $\frac{\theta}{n}$th section n≥2

m= $\frac{2}{n-2}$ is limiting value ……………………… (11)

It can be verirfied that these limiting values provide very good approximations for multisections of angles up to 30^0.

6. Chart showing limiting values of 'm' for angle θ= 90^0 and whenθ is small.

Multisection	**θ= 90^0**	**θ is small**	**Difference/trend**
$\frac{\theta}{2}$	∞	∞	Nil
$\frac{\theta}{3}$	$\sqrt{3}$	2	2-$\sqrt{3}$/increasing
$\frac{\theta}{4}$	1	1	0/static
$\frac{\theta}{5}$	tan36^0	$\frac{2}{3}$	.05984/decreasing
$\frac{\theta}{6}$	tan30^0	$\frac{1}{2}$	.07735/decreasing
$\frac{\theta}{10}$	tan18^0	$\frac{1}{4}$	.07492/do
$\frac{\theta}{n}$	tan$\frac{2\theta^0}{n}$	$\frac{2}{n-2}$	do
$\frac{\theta}{n}$ (when n is very large)	$\frac{\pi^{rad}}{n}$ (as per formula (9))	$\frac{2}{n}$ as per formula (11)	$\frac{\pi-2}{n}$/do

SOME EXAMPLES

Ex-1 Given $\angle PAQ = \theta$

Construct an angle equal to $\frac{\theta}{4}$ at point A

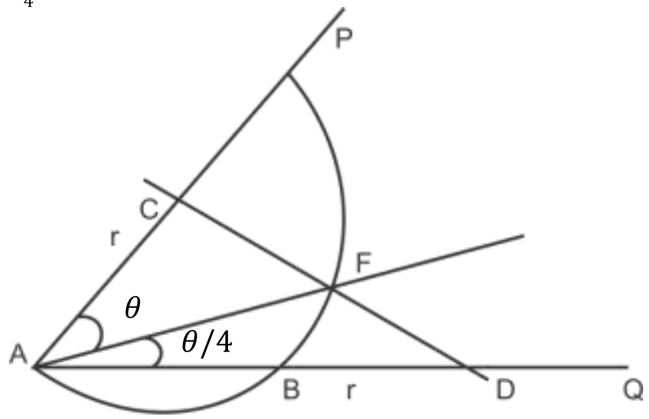


Figure-8

Construction- given $\angle PAQ = \theta$

Take any point C on AP and draw a semi circle taking c as centre and AC as radius r which cuts the line AQ at B.

Cut BD = radius r and join ptsC an D by a line .Let Line CD cuts the semi circle at F. Then $\angle FAD = \frac{\theta}{4}$

Since here as per corollary-1

m=1 and BD = mr = r

nence$\angle FAD = \frac{\theta}{4}$

Ex-2 Given $\angle PAQ = 90^0$, trisect the $\angle PAQ$

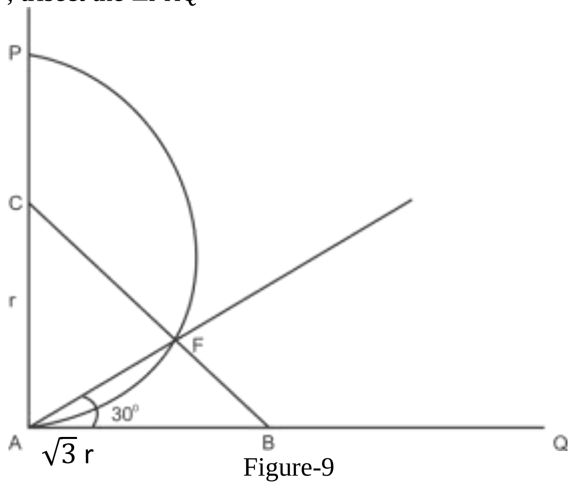


Figure-9

Construction : Given $\angle PAQ = 90^0$

Take C any point on AP

Draw a semi circle taking C as centre and CA as radius (r)

Draw CB = 2r by compass (2r = diameter of the circle)

Point B being on the line AQ

Let the line CB cuts the semi circle on F join AF. Then $\angle FAB = 90^0/2 = 30^0$

By Pythagoras theonem if AC = r; CB = 2r

AB = $\sqrt{3}$ r so here m = $\sqrt{3}$

By corollary 3 and formula (9) we have

$if\theta = 90^0$ then for $\frac{\theta}{3}\psi = 30^0$

m = tan2ψ

= tan2.30

= tan60 = $\sqrt{3}$

Hence $\angle FAB = 30^0$

Ex-3 Given $\angle PAQ = 20^0$
taking m = 2 trisect it and find the value of tan ψ&ψ approximately

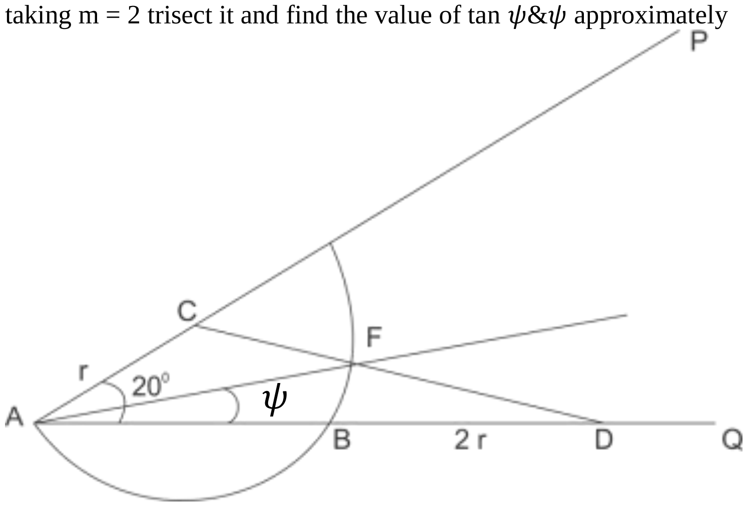


Figure-10

Given $\angle PAQ = 20^0$, m = 2 so BD = 2r
$\angle FAB = \psi$
Now from formula (4) we have

Tanψ = $\frac{\sqrt{1+m^2+2mcos\theta} - (1+mcos\theta)}{msin\theta}$

= $\frac{\sqrt{1+2^2+2.2cos\,20^0} - (1+2cos\,20^0)}{2sin\,20^0}$

= $\frac{\sqrt{5+4\times.93969262} - (1+2\times.93969262)}{2\times.34202014}$

= $\frac{2.95952200 - 2.87938524}{.68404028}$

= .117152107 = tan6.68186^0

So. ψ = 6.68186^0 = $\frac{20^0}{3}$ approximately

With a difference of $(\frac{20}{3} - 6.68186)$

= - .015193^0; slightly higher than $\frac{20^0}{3}$

So m = 2 approximately trisects 20^0 angle
Also for m = 2; θ is 30^0, ψ = 10.05^0 with a slight difference than 10^0 *i.e.* approximately trisects 30^0.
Note: when angle θ is > 30^0 and < 90^0
Than for trisection of angle θ following steps can be adopted.

i) Draw $\frac{1}{4}$ of θ using corollary 1
ii) Trisect $\frac{\theta}{4}$ to find $\frac{\theta}{12}$ taking m=2
iii) Make an angle $4\times\frac{\theta}{12}$ i,e 4 limes of the angle obtained in step (ii) this will give very near approximation of $\frac{\theta}{3}$.

Ex-4 Trisect 80^0 angle
Step:
i) Draw $\frac{80}{4}$ = 20^0
ii) Trisect 20^0 as in Ex-3
iii) Make angle 4 times of (ii)

Thus ψ = $\left\{\left(\frac{1}{4}\times 80\right)\times\frac{1}{3}\right\}\times 4$
= 6.68186×4
= 26.727^0 = $\frac{80}{3}$approximately

Defference = 26.727- 26.667
= .06^0

Ex-5 Given angle θ = 10^0
draw $\frac{\theta}{5}$
solution: since θ= 10^0 is a small angle using the formula (11)

m = $\frac{2}{5-2} = \frac{2}{3}$

so by formula (5)
we have

$\tan\psi = \frac{\sqrt{m^2+n^2+2mncos\theta} - (n+mcos\theta)}{msin\theta}$

putting m = 2, n = 3

$$\tan\psi = \frac{\sqrt{2^2+3^2+2.23cos10} - (3+2cos10)}{2sin10}$$

$$= \frac{\sqrt{13+12\times cos10} - (3+2cos10)}{2sin10}$$

$$= \frac{.01212044}{.34729636}$$

$$= .034899416$$

$$= \tan 1.998778^0$$

$= \tan 2^0$ approximately

So m = $\frac{2}{3}$ gives very near approximation of pentasection of 10^0 angle $\left(\frac{10^0}{5}\right)$ and can be drawn as in prev. examples.

Note: all the construction of angle multisection given in the previous examples can be done with the help of straight edge and compass only.

Reference:

(i) Ordinary geometry, aljebra, coordinate geometry and continued fractions.
(ii) Internet searches.