

Micropolar Fluid flow with Nano Particle Through Porous Plate with MHD Effects

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Abstract : *Unsteady naturally convective and thermally radiative micropolar fluid in presence of nano particle flow through a vertical porous plate with MHD has been studied. A flow model is established by employing the well-known boundary layer approximations. In order to obtain non-dimensional system of equations, different types of transformation is applied on the flow model. The coupled non-linear partial differential equations are solved by explicit finite difference method and the numerical results have been calculated by Compaq Visual 6.6a. The effects of various parameters entering into the problem on velocity, temperature and concentration are shown graphically.*

Keywords: *Micropolar Fluid, Nano Particle, Porous Plate, MHD, Thermal Radiation, Heat and Mass Transfer.*

I. Introduction

Theory of micropolar fluid can be used to describe many types of fluid such quid crystal, polymeric fluids, suspension solution, lubricants, animal blood etc. Magneto-micropolar fluid with nano particles plays a vital rule in the view of its wide applications in many engineering problems such as electric transformers, heating elements and nano technology. Kazimierz et al. [1] and [4] analyzed the unsteady natural convection in micropolar nano fluids, where the increasing heat exchange due to natural convection in water solutions of Al₂O₃, TiO₂ and Cu with properties of micropolar nano-fluids in the vicinity of vertical plate heated by heat flux of q_0 that rises suddenly. A vast investigation of convective heat transfer in nano-fluids by Buongiorno [2] predicted that the nano-particles' absolute velocity is roughly a sum of the base fluid velocity and the slip velocity. In Buongiorno [2] the author focused on inertia, Brownian diffusion, thermophoresis, diffusio-phoresis, Magnus effect, fluid drainage, and the gravity setting as the effective quantities and concluded that for the laminar type flow, only the Brownian diffusion and the thermophoresis have noticeable effects. Rehman et al. [3] investigated the incompressible flow of a micropolar nano fluid along a vertical permeable slender cylinder. The problem of boundary layer steady flow and heat transfer of a micropolar fluid solved using the homotopy analysis method. Nering et al. [5] analyzed the effect of nano particles added to heated micropolar fluid. The aim of this study is to evaluate cooling intensity of the heated vertical plate. The plate is heated by sudden rise of heat flux q_0 in time $\tau = 0$. Vertical plate is cooled by micropolar fluid containing nano-particles. The considerations take into account unsteady heat exchange process in natural convection in micropolar fluid. The properties of nano fluid change after addition of Al₂O₃ or Cu nano-particles with different size. Akbar et al. [6] analyzed the peristaltic flow of a nano fluid in a uniform tube for micropolar fluid. They studied about the regarding the micropolar fluid and nano fluid, presented the peristaltic flow of a micropolar fluid in a uniform tube with nano particles. Equations of momentum, energy, and concentrations are coupled so Homotopy perturbation method is used to get the solutions.

The behaviors of fluid that contain suspended, metal or dust particles in many practical situations are first observed by the micropolar fluid theory of Eringen (1966) [7] with internal structures in which coupling between the spin of each particle and the macroscopic velocity field is taken into account. Physically, the micropolar fluids contain dilute suspension of small, rigid, cylindrical macromolecules with individual motion and are influenced by spin inertia. Ferraro et al. [8], Hossain [12] and Cramer et al. [9] are notable authors for major contribution about MHD free convection flows and their significant application in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering, electronics, and so on. In addition, many transport processes exist in industries and technology where the transfer of heat and mass occurs simultaneously as a result of thermal diffusion and diffusion of chemical species. The growing needs for chemical reactions in chemical and hydrometallurgical industries require the study of heat and mass transfer with chemical reaction. The effect of the first order homogeneous chemical reaction of an unsteady flow past a vertical plate with the constant heat and mass transfer has been investigated by Das et al [10]. The chemical reaction effects on an

unsteady MHD free convection fluid flow past a semi-infinite vertical plate embedded in a porous plate with heat absorption have been studied by Anand Rao et al. [11].

Present studies concern with the theoretical and numerical investigation of micropolar fluid in presence of nano particles with thermal radiation, MHD, porous plate. This work presents unsteady MHD free convective flow past a vertical porous plate. The governing coupled non-linear partial differential equations are first transformed into a dimensionless momentum, angular momentum, energy and concentration equations and then the resultant non-linear set of equations has been solved numerically employing explicit finite difference technique. From the physical point of view, the numerical results for various parametric values have been presented graphically.

II. Mathematical Formulation

The fluid with the both micro-rotation and micro-inertia properties is known as micropolar fluid. Unsteady heat and mass transfer flow of viscoelastic fluid along a semi-infinite vertical porous plate $y = 0$ is considered in the presence of a uniform thermal radiation and magnetic field. The flow is considered to be in the x -direction which is taken along the plate in the upward x -direction and y -axis is normal to it. When, the plate velocity $U(t)$ is given as $u = U_0$. In initial step, it is considered that the plate as well as the fluid particle is at rest at the same temperature $T(= T_\infty)$ and the same concentration level $C(= C_\infty)$ at all points.

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum Equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + \left(\nu + \frac{\chi}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\chi}{\rho} \frac{\partial N}{\partial y} - \frac{\sigma B_0^2 u}{\rho} - \frac{\nu}{K'} u \tag{2}$$

Angular Momentum Equation:

$$\frac{\partial N}{\partial t} + u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{\gamma}{\rho j} \left(\frac{\partial^2 N}{\partial y^2} \right) - \frac{\chi}{\rho j} \left(2N + \frac{\partial u}{\partial y} \right) \tag{3}$$

Energy Equation:

$$\begin{aligned} \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = & \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{1}{c_p} \left(\nu + \frac{\chi}{\rho} \right) \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial N}{\partial y} \right)^2 \right] \\ & - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \tau \left\{ D_B \left(\frac{\partial T}{\partial y} \frac{\partial C}{\partial y} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right\} \end{aligned} \tag{4}$$

Concentration Equation:

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left(\frac{\partial^2 C}{\partial y^2} \right) + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \tag{5}$$

With boundary condition,

$$u = U_0 = bx, v = 0, \bar{N} = -s \frac{\partial u}{\partial y}, T = T_w, C = C_w \quad \text{at } y = 0$$

$$u = 0, \bar{N} \rightarrow \bar{N}_\infty, T \rightarrow T_\infty, C \rightarrow C_\infty \quad \text{at } y \rightarrow \infty$$

Where, u and v are the velocity component, B_0 is the magnetic field component, β is thermal expansion coefficient, β^* is concentration expansion coefficient, T_w denotes the wall temperature, C_w is the species concentration at the wall, b is the stretching constant, ν is the kinematic viscosity, ρ is density, κ is thermal conductivity, c_p is specific heat at constant pressure, q_r unidirectional radiative heat flux, D_B is Brownian diffusion coefficient, D_T thermophoresis diffusion coefficient. The value $s = 0$ corresponds to the case of the high density of liquid micro particles that prevents them from performing rotational movements. The value $s =$

0.5 is indicative of weak concentrations and the value $s = 1$ is representative for turbulent boundary layer. The radiative heat flux term by using the Rosseland approximation is given by $q_r = -\frac{4\sigma_s}{3k_e} \frac{\partial T^4}{\partial y}$.

Where, σ_s is the Stefan-Boltzmann constant and k_e is the mean absorption coefficient, respectively. If temperature differences within the flow are sufficiently small, then the q_r can be linearized by expanding T^4 into the Taylor series about T_∞ , which after neglecting higher order terms takes the form by $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$. Then the equation (4) becomes,

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa \partial^2 T}{\rho c_p \partial y^2} + \frac{1}{c_p} \left(\nu + \frac{\chi}{\rho} \right) \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial N}{\partial y} \right)^2 \right] + \frac{16\sigma_s T_\infty^3}{3k_e \rho c_p} \frac{\partial^2 T}{\partial y^2} + \tau \left\{ D_B \left(\frac{\partial T}{\partial y} \frac{\partial C}{\partial y} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right\} \quad (6)$$

From the governing equations (1) - (5) under the initial conditions and the boundary conditions will be based on the finite difference method it is required to make the equations dimensionless. For the purpose introducing the following dimensionless quantities:

$$X = \frac{xU_0}{\nu}, Y = \frac{yU_0}{\nu}, U = \frac{u}{U_0}, V = \frac{v}{U_0}, \tau = \frac{tU_0^2}{\nu}, N = \frac{\nu \bar{N}}{U_0^2}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty},$$

$$\text{So, } x = \frac{X\nu}{U_0}, y = \frac{Y\nu}{U_0}, u = UU_0, t = \frac{\tau\nu}{U_0^2}, T = T_\infty + \theta(T_w - T_\infty) \text{ and } C = C_\infty + \phi(C_w - C_\infty),$$

the dimensionless equations are obtained as follows:

Dimensionless Continuity Equation

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = 0 \quad (7)$$

Dimensionless Momentum Equation

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = G_r \theta + G_c \phi + (1 + \alpha) \frac{\partial^2 U}{\partial Y^2} + \alpha \frac{\partial N}{\partial Y} - MU - \frac{1}{D_a U} \quad (8)$$

Dimensionless Angular Momentum Equation

$$\frac{\partial N}{\partial \tau} U \frac{\partial N}{\partial X} + V \frac{\partial N}{\partial Y} = \beta \frac{\partial^2 N}{\partial Y^2} - \lambda \left(2N + \frac{\partial U}{\partial Y} \right) \quad (9)$$

Dimensionless Energy Equation

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial Y} U + V \frac{\partial \theta}{\partial Y} = \frac{1}{P_r} \left(1 + \frac{16R}{3} \right) \frac{\partial^2 \theta}{\partial Y^2} + (1 + \alpha) E_c \left[\left(\frac{\partial U}{\partial Y} \right)^2 + \left(\frac{\partial N}{\partial Y} \right)^2 \right] + Nb \left(\frac{\partial \theta}{\partial Y} \frac{\partial C}{\partial Y} \right) + Nt \left(\frac{\partial \theta}{\partial Y} \right)^2 \quad (10)$$

Dimensionless Concentration Equation

$$\frac{\partial \phi}{\partial \tau} + U \frac{\partial \phi}{\partial X} + V \frac{\partial \phi}{\partial Y} = \frac{1}{Le} \left[\frac{\partial^2 \phi}{\partial Y^2} + \left(\frac{Nt}{Nb} \right) \frac{\partial^2 \theta}{\partial Y^2} \right] \quad (11)$$

$\tau \leq 0, U = 0, V = 0, \theta = 0, \phi = 0$ every where

$\tau > 0, U = 0, V = 0, \theta = 0, \phi = 0$ at $X = 0$

$$U = 1, N = -\frac{1}{2} \frac{\partial u}{\partial y}, T = 1, C = 1 \text{ at } y = 0$$

$$U = 0, N = 0, T = 0, C = 0 \text{ at } y \rightarrow \infty$$

Where the obtained physical parameters are given below:

Grashof number, $G_r = \frac{g\beta(T_w - T_\infty)\nu}{U_0^3}$, mass Grashof number, $G_r = \frac{g\beta^*(C_w - C_\infty)\nu}{U_0^3}$, micro-rotational number, $\alpha = \frac{\chi}{\rho\nu}$, magnetic parameter, $M = \frac{\sigma B_0^2\nu}{\rho U_0^2}$, Darcy number, $Da = \frac{K'U_0^2}{\nu^2}$, Spin Gradient viscosity, $\beta = \frac{\gamma}{\rho j\nu}$ and Vortex viscosity, $\lambda = \frac{\chi\nu}{\rho j\nu_0^2}$, Prandtl number, $P_r = \frac{\rho c_p\nu}{\kappa}$, radiation parameter, $R = \frac{\sigma T_\infty^3}{k_1 k}$, Eckert number, $E_c = \frac{U_0^2}{c_p(T_w - T_\infty)}$, Lewis number, $Le = \frac{\nu}{D_m}$, Brownian parameter, $Nb = \frac{\tau D_B(C_w - C_\infty)}{\nu}$ and thermophoresis parameter $Nt = \frac{\tau D_T}{T_\infty\nu}(T_w - T_\infty)$.

III. Numerical Solution

Coupled non-dimensional partial differential equations have been solved by the associated initial and boundary conditions. The method of explicit finite difference has been used to solve (7) - (11) subject to the initial and boundary conditions. For this reason the area within the boundary layer is divided by some perpendicular lines of Y -axis, where the normal of the medium is Y -axis as shown in Fig-2. It is assumed that the maximum length of boundary layer $Y_{\max} = 20$ as corresponds to $Y \rightarrow \infty$. i.e. Y vary from 0 to 20 and the number of grid spacing in Y directions are $m(=100)$ and $n(=300)$, with the smaller time step $\Delta\tau = 0.005$. Using the explicit finite difference approximation we have,

Continuity Equation

$$\frac{U_{i,j} - U_{i,j-1}}{\Delta X} + \frac{V_{i,j} - V_{i,j-1}}{\Delta Y} = 0 \tag{12}$$

Angular Momentum Equation

$$\frac{N'_{i,j} - N_{i,j}}{\Delta\tau} + U_{i,j} \frac{N_{i,j} - N_{i-1,j}}{\Delta X} + V_{i,j} \frac{N_{i,j+1} - N_{i-1,j}}{\Delta Y} = \beta \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} - \lambda(2N_{i,j} + \frac{U_{i,j+1} - U_{i,j}}{\Delta Y}) \tag{13}$$

Momentum Equation

$$\frac{U'_{i,j} - U_{i,j}}{\Delta\tau} + U_{i,j} \frac{U_{i,j} - U_{i-1,j}}{\Delta X} + V_{i,j} \frac{U_{i,j+1} - U_{i-1,j}}{\Delta Y} = \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + G_r\theta_{i,j} + G_c\phi_{i,j} + (1+\alpha) \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + \alpha \frac{\bar{N}_{i,j+1} - \bar{N}_{i,j}}{\Delta Y} - (M + \frac{1}{D_a})U_{i,j} \tag{14}$$

Energy Equation

$$\frac{\theta'_{i,j} - \theta_{i,j}}{\Delta\tau} + U_{i,j} \frac{\theta_{i,j} - \theta_{i-1,j}}{\Delta X} + V_{i,j} \frac{\theta_{i,j+1} - \theta_{i-1,j}}{\Delta Y} = \frac{1}{P_r} (1 + \frac{16}{3}R) \frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta Y)^2} + (1+\alpha)E_c \left[\left(\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} \right)^2 + \left(\frac{N_{i,j+1} - N_{i,j}}{\Delta Y} \right)^2 \right] + N_b \left(\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta Y} \cdot \frac{\phi_{i,j+1} - \phi_{i,j}}{\Delta Y} \right) + N_t \left(\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta Y} \right)^2 \tag{15}$$

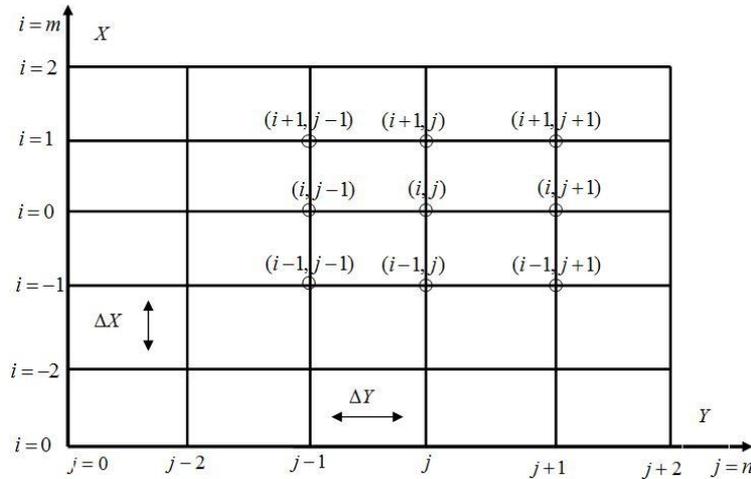


Fig-2: The finite difference space grid

Concentration Equation

$$\frac{\phi'_{i,j} - \phi_{i,j}}{\Delta\tau} + U_{i,j} \frac{\phi_{i,j} - \phi_{i-1,j}}{\Delta X} + V_{i,j} \frac{\phi_{i,j+1} - \phi_{i-1,j}}{\Delta Y} = \frac{1}{L_e} \left[\frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{(\Delta Y)^2} + \left(\frac{N_t}{N_b} \right) \frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta Y)^2} \right] \quad (16)$$

The initial and boundary condition with finite difference scheme as

$$U = 1, N = -\frac{1}{2} \frac{U - U}{(\Delta Y)^2}, \theta = 1, \phi = 1$$

$U = 0, N = 0, \theta = 0, \phi = 0$ Where, the subscripts i and j designate the grid points with X and Y coordinates respectively and a value of time, $\tau = n\Delta\tau$, where $n = 1, 2, 3, 4, \dots$

IV. Results And Discussion

This theoretical phenomenon of the flow model solved by finite difference solution and obtained by explicit procedure. The numerical values of coupled non-dimensional velocity, angular velocity, and temperature and concentration equation with the condition for different values of parameter have been computed by a FORTRAN program. The meaning of porosity is different from permeability.

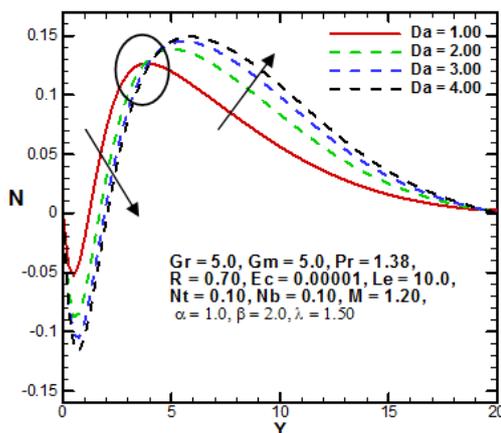


Fig. 1. Angular velocity profiles for Da.

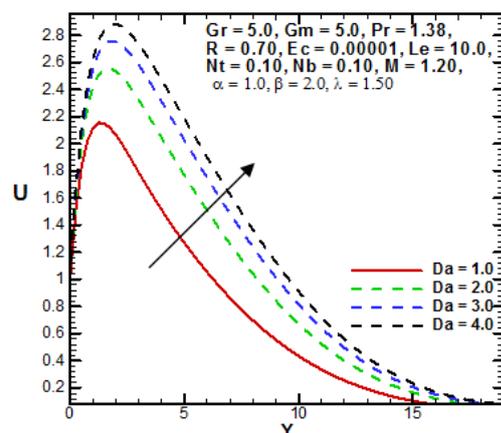


Fig. 2. Velocity profiles for Da.

It is a ratio of free volume to comprising the whole amount volume of the material. To add an additional term in Darcy's Law for inertial effects, permeability associated with that term. In Fig.1, the angular velocity increase with the increase of Da near $X = 0.1$ and then the profiles decrease. From Fig.2, the velocity profiles increase with the increase of Da.

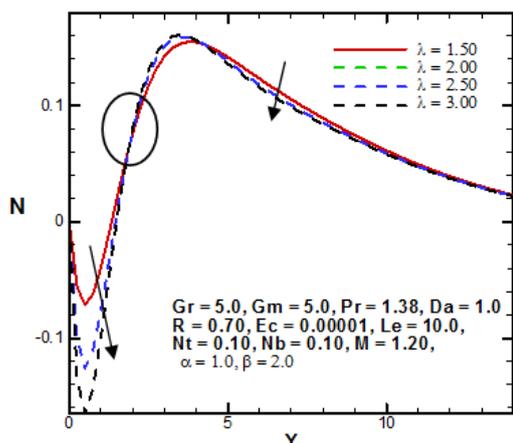


Fig. 3. Angular velocity profiles for λ

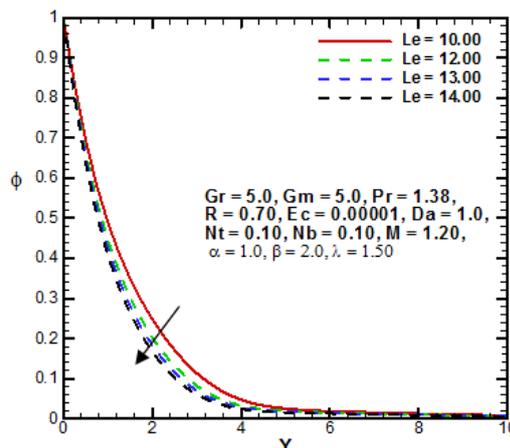


Fig. 4. Concentration profiles for Le

Fig. 3 shows that the angular velocity decrease below the $X = 0.05$ point near the plate and the increase for the increase of λ . The Fig. 4 presents that concentration profiles decrease with the increase of Le . When the magnetic force enhance the Lawrence force. For these reasons the velocity profiles in Fig. 6 decrease and for angular velocity Fig.5, near the plate increase and faraway from the plate decrease. For the large infect of M , temperature profiles decrease in Fig. 7. From Fig. 8, we observe that angular velocity decrease with the increase of α . In Fig. 9 and Fig. 10, the temperature profiles increase and concentration profile decrease with the increase of Nb . With the increase of Nt , temperature profiles decrease and concentration profiles increase for Nt in Fig. 11 and Fig. 12.

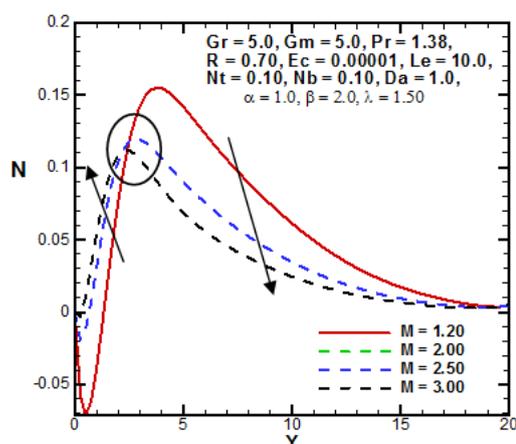


Fig. 5. Angular velocity profiles for M .

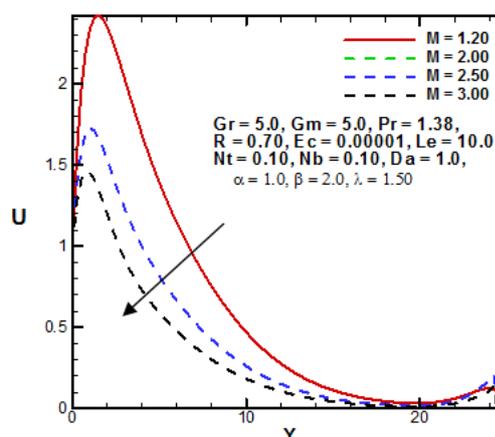


Fig. 6. Velocity profiles for M .

About the Prandtl number, $Pr \ll 1$ delineates the thermal diffusivity dominates. For the large values, $Pr \gg 1$, the momentum diffusivity dominates the behavior. For example, the liquid mercury point out that the heat conduction is more consequential compared to convection, so thermal diffusivity influences over others. For the increment of Pr , the velocity and temperature profiles decrease in Fig. 13 and Fig. 14. Thermal radiation for a medium which contains it inevitably has pressure and density gradients and the treatment requires the use of hydrodynamics. For the electromagnetic radiation by the thermal excitation increase the fluid velocity for the effect of thermal radiation particles movement increase and for these reasons the temperature profiles with the increase of R in Fig. 15. Angular velocity near the plate decrease for the increase of β in Fig. 16 and after few times the profiles increase with the increase of β .

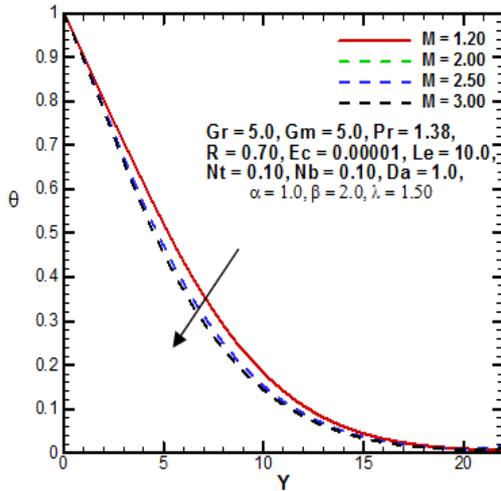


Fig. 7. Temperature profiles for M

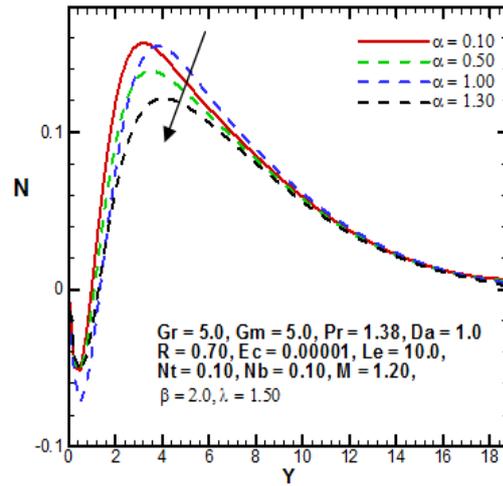


Fig. 8. Angular velocity profiles for α

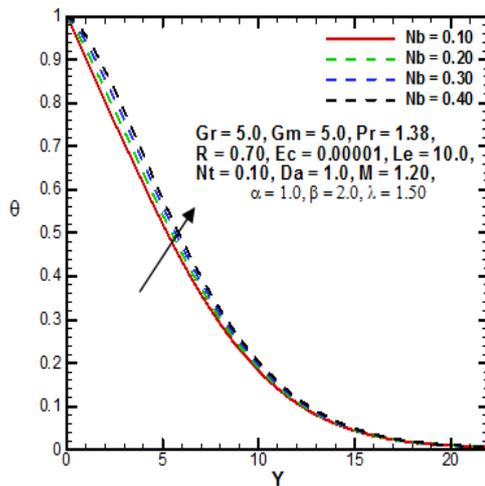


Fig. 9. Temperature profiles for Nb.

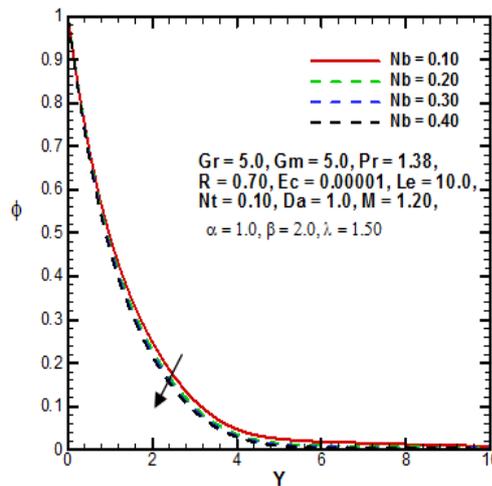


Fig. 10. Concentration profiles for Nb.

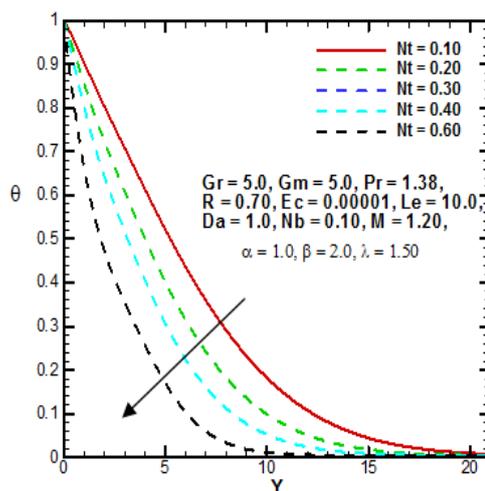


Fig. 11. Temperature profiles for Nt.

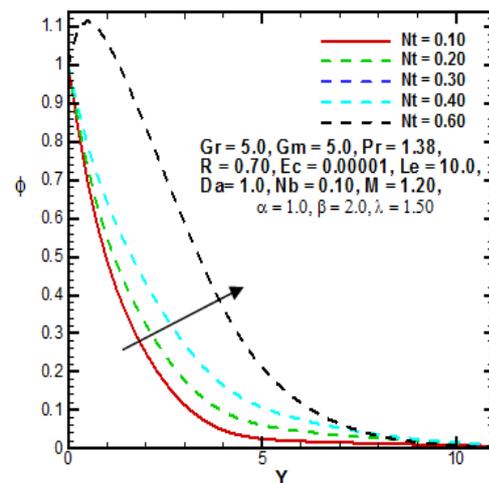


Fig. 12. Concentration profiles for Nt.

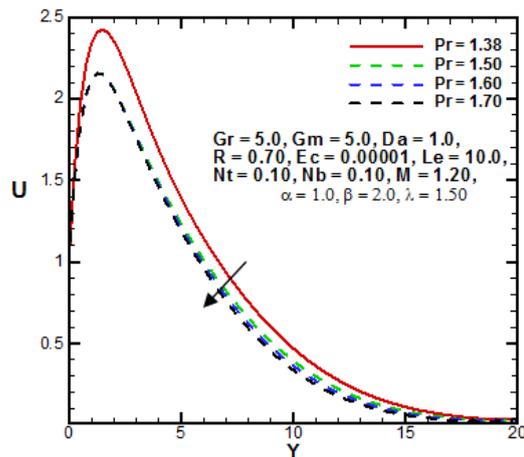


Fig. 13. Velocity profiles for Pr.

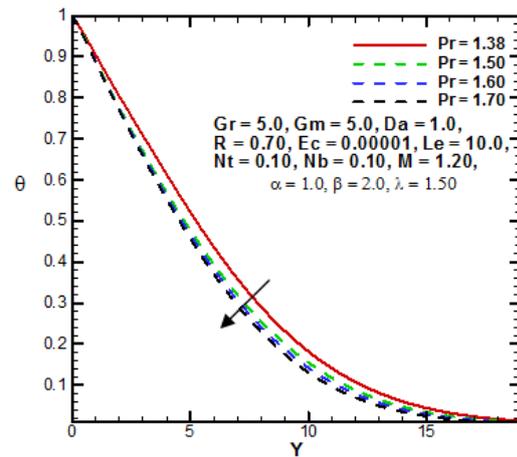


Fig. 14. Temperature profiles for Pr.

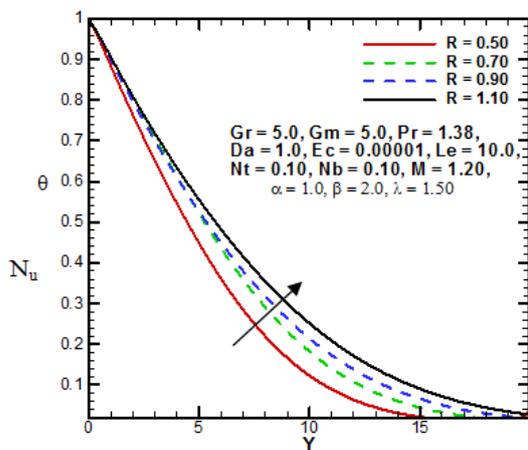


Fig. 15. Temperature profiles for R.

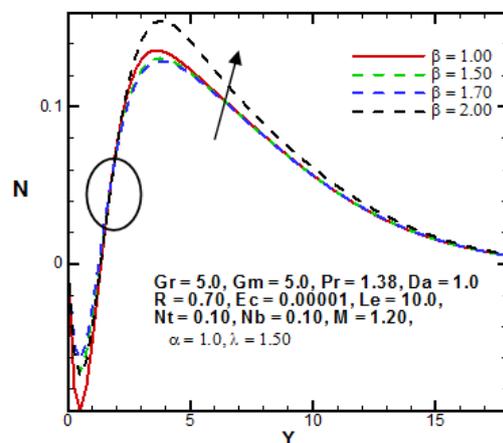


Fig. 16. Angular velocity profiles for β .

V. Conclusion

- Angular velocity and velocity profiles increase with the increase of Darcy number.
- Angular velocity decrease with the increment of Vortex viscosity.
- Concentration profiles decrease with the increase of Lewis number.
- For the large effects of magnetic parameter, angular velocity, velocity and temperature profiles decrease.
- With the increase of micro-rotational number, angular velocity decreases.
- Temperature profiles increase and concentration profiles decreases with the increase of Brownian parameter.
- Temperature profiles decrease and concentration profiles increases with the increase of Brownian parameter thermophoresis parameter.
- By the increase of Prandtl number, velocity and temperature profiles decrease.
- Angular velocity increase with the increment of Spin Gradient viscosity.

References

- [1]. Kazimierz Rup and Konrad Nering (2014), Unsteady natural convection in micropolar nanofluids, archives of thermodynamics, 35:3, 155–170.
- [2]. J. Buongiorno (2006). Convective Transport in Nanofluids, ASME J. Heat Transfer, 128, 240-250.
- [3]. Abdul Rehman and S. Nadeem (2012), Mixed Convection Heat Transfer in Micropolar Nanofluid over a Vertical Slender Cylinder, CHIN. PHYS. LETT., 29:12, 1-5.
- [4]. Kazimierz Rup and Konrad Nering (2016), Heat transfer enhancement in natural convection in micropolar nanofluids, Arch. Mech., 68:4,327–344.
- [5]. Konrad Nering and Kazimierz Rup (2016), The effect of nanoparticles added to heated micropolar fluid, Superlattices and Microstructures, 98, 283-294.
- [6]. Noreen Sher Akbar and S. Nadeem (2013), Peristaltic flow of a micropolar fluid with nano particles in small intestine, Appl. Nanosci, 3:461–468.
- [7]. A.C. Eringen (1966), Theory of micropolar fluids, Journal of Mathematics and Mechanics, 16, 1-18.

- [8]. Ferraro, V. C. A. and Plumpton, C., (1966), *An Introduction to Magneto Fluid Mechanics*, Oxford: Clarendon Press.
- [9]. Cramer, K. P. and Pai, S. I., (1973), *Magneto Fluid Dynamics for Engineers and Applied Physics*, New York: McGraw-HillBook Company.
- [10]. Das, U. N., Dekha, R. and Soungalgekar, V. M., (1994), Effects on mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. *Forschungim Ingenieurwesen*, 60, 284-287.
- [11]. Annad Rao, J., Sivaiah, S. and Srinivasa Raju, R., (1912), Chemical Reaction Effects on an Unsteady MHD Free Convection Fluid Flow past a Semi-infinite Vertical plate Embedded in a Porous Medium with Heat Absorption. *Journal of Applied Fluid Mechanics*, 3, 63-70.
- [12]. Hossain, M. A. and Ahmed, M., (1990), MHD Free Convection and Mass Transfer Flow Through a Porous Medium with Heat Source, *International Journal of Engineering and Science*, 33, 571.