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Numerical Study of Instabilities in Porous Media Using Finite Difference Method

M.S.Prajapati[1], Priti Tandel[2], P.H.Bhathawala[3]
[1]*Associate Professor, Muni. Art & U.B.Science College, Mehsana-384002, Gujarat, India*
[2]*Department of Mathematics, V.N.S.G.U., Surat-394510, Gujarat, India*
[3]*UGC Visiting Professor, Dept. Of Mathematics, HNGU Patan, Gujarat, India*

Abstract: *The present paper deals with the instabilities takes place in two immiscible phase flow through homogeneous porous media. If the flow of two immiscible fluids is considered unidirectional in a large medium, it can be investigated easily. Corresponding to the movement direction, if parameters like pressures, saturations, fluid speeds, etc. changes only in a single space direction then it can be easily investigated. Solution is obtained by finite difference method.*

Keywords: *Instabilities, Porous media, Immiscible, Finite difference method.*

I. Introduction

The vertical component of the velocities is taken in an account as the porous medium is considered thicker. The region of the medium where saturation of injecting part rises sharply, the interfaces known as front. On the large scale, the front encroachment are called the tongue phenomenon, but on a smaller scale, called fingering. These encroachments satisfied the conditions of stability of instability. Instabilities occur due to the different viscosities of flowing fluids.

Many authors like Scheidegrer , A.E[1,2,3], Verma [4], Coss [5], G. Hu, P.A.
Zegeling [6], Singh T.[7], R.N.Borana, V.H.Pradhan, and M. N. Mehta[8], D.E.HillandJ.Y. Parlange[9] discussed this phenomena in different ways.

Capillary pressure and common mean pressure is taken into consideration to obtain the solution. Fingers are determined by a statistical treatment. The governing equation is solved using finite difference method known as Successive over Relaxation method.

II. Statement of the problem

Instead of regular displacement of the whole front, when another fluid of lesser viscosity is injected into a porous medium containing a fluid, protuberance may occur which shoot through the porous medium at relatively great speed. Those protuberances are called fingers and the phenomenon is called fingering. A finite cylindrical piece of homogeneous porous medium of length L is considered. It is fully saturated with a fluid known as native fluid, which is displaced by injecting the another fluid. This gives rise to protuberance known as fingers. Complete saturation is assumed at the initial boundary.

The cross- sectional area occupied covered by fingers has been is considered. In the statistical treatment of fingers only the average behavior of the two fluids involved is taken into consideration.

III. Mathematical Formulation of the problem

According to Darcy's law, the seepage velocity of injected fluid (V_i) and native fluid (V_n) are :

$$V_i = -\frac{k_i}{\mu_i} k \frac{\partial P_i}{\partial x} \tag{1}$$

$$V_n = -\frac{k_n}{\mu_n} k \frac{\partial P_n}{\partial x} \tag{2}$$

Here k-permeability of the homogeneous medium,k_i,k_n- relative permabilities of injected fluid and native fluid respectively , P_i, P_n- pressures of injected fluid and native fluid respectively and μ_i,μ_n - viscosities of injected fluid and native fluid respectively. Again k_i and k_n are assumed to be functions of saturations S_i and S_n respectively.

The continuity equations are:

$$\varphi \frac{\partial S_i}{\partial t} + \frac{\partial V_i}{\partial x} = 0 \tag{3}$$

$$\varphi \frac{\partial S_n}{\partial t} + \frac{\partial V_n}{\partial x} = 0 \tag{4}$$

Where φ is the porosity of the medium.

The porous medium is considered to be fully saturated.

Therefore from the definition of phase saturation,

$$S_i + S_n = 1 \quad (5)$$

The capillary pressure(P_c), across their common interfaceis:

$$P_c = P_n - P_i \quad (6)$$

Assuming the relationship between the relative permabilities, phase saturation and capillary pressure as follow:

$$k_i = S_i,\ k_n = S_n = 1 - S_i \quad (7)$$

$$P_c = -\beta S_i \quad (8)$$

Here negative sign shows the direction of saturation of water is opposite to capillary pressure, where $\beta$is however small.

The pressure of native fluid (P_n)can be written as

$$P_n = \frac{1}{2}(P_n + P_i) + \frac{1}{2}(P_n - P_i)$$

$$P_n = \overline{P} + \frac{P_c}{2}\ ,\ \overline{P} = \frac{P_n + P_i}{2} \quad (9)$$

Where $\overline{P}$ is the mean pressure which is assumed as a constant.

Substituting(1) and (2) into (3) and (4)

$$\varphi\,\frac{\partial S_i}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_i}{\mu_i}k\frac{\partial P_i}{\partial x}\right] \quad (10)$$

$$\varphi\frac{\partial S_n}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_n}{\mu_n}k\frac{\partial P_n}{\partial x}\right] \quad (11)$$

Eliminating $\frac{\partial P_i}{\partial x}$ from equations (11) and (6) ,we get

$$\varphi\,\frac{\partial S_i}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_i}{\mu_i}k\left\{\frac{\partial P_n}{\partial x} - \frac{\partial P_c}{\partial x}\right\}\right] \quad (12)$$

From(5),(11) and (12), we obtain

$$\frac{\partial}{\partial x}\left[\left\{\frac{k_i}{\mu_i} + \frac{k_n}{\mu_n}\right\}k\frac{\partial P_n}{\partial x} - \frac{k_i}{\mu_i}k\frac{\partial P_c}{\partial x}\right] = 0 \quad (13)$$

Integrating equation (13) with respect to x , we get

$$\left[\left\{\frac{k_i}{\mu_i} + \frac{k_n}{\mu_n}\right\}k\frac{\partial P_n}{\partial x} - \frac{k_i}{\mu_i}k\frac{\partial P_c}{\partial x}\right] = -c \quad (14)$$

Where c is the constant of integration (negative sign on right hand side is considered for our convenience).

Simplifying equation (14), we get

$$\frac{\partial P_n}{\partial x} = -\frac{A}{k\left\{\frac{k_i}{\mu_i}+\frac{k_n}{\mu_n}\right\}} + \frac{k_i}{\mu_w\left\{\frac{k_i}{\mu_i}+\frac{k_n}{\mu_n}\right\}}\frac{\partial P_c}{\partial x} \quad (15)$$

Now substituting equation (15) into equation (12), we have

$$\varphi\,\frac{\partial S_i}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_i}{\mu_i}k\left\{-\frac{A}{k\left\{\frac{k_i}{\mu_i}+\frac{k_n}{\mu_n}\right\}} + \frac{k_w}{\mu_w\left\{\frac{k_i}{\mu_i}+\frac{k_n}{\mu_n}\right\}}\frac{\partial P_c}{\partial x} - \frac{\partial P_c}{\partial x}\right\}\right]$$

$$\varphi\frac{\partial S_i}{\partial t} + \frac{\partial}{\partial x}\left[\frac{k_n}{\mu_n}k\frac{\partial P_c}{\partial x}\frac{1}{\left\{1+\frac{k_i k_n}{\mu_i \mu_n}\right\}} + \frac{A}{\left\{1+\frac{k_i k_n}{\mu_i \mu_n}\right\}}\right] = 0 \quad (16)$$

Now from (9), $\frac{\partial P_n}{\partial x} = \frac{1}{2}\frac{\partial P_c}{\partial x}$

Substituting this value in (14), we have

$$A = \left\{\frac{k_i}{\mu_i} - \frac{k_n}{\mu_n}\right\}\frac{k}{2}\frac{\partial P_c}{\partial x} \quad (17)$$

Substituting the value of A from (17) into (16) , we get

$$\varphi\frac{\partial S_i}{\partial t} + \frac{\partial}{\partial x}\left[k\frac{k_i}{2\mu_i}\frac{\partial P_c}{\partial x}\right] = 0$$

$$\varphi\frac{\partial S_i}{\partial t} + \frac{\partial}{\partial x}\left[k\frac{k_i}{2\mu_i}\frac{dP_c}{dS_i}\frac{\partial S_i}{\partial x}\right] = 0 \quad (18)$$

From equations (7) ,(8) and (18) , we have

$$\varphi\frac{\partial S_i}{\partial t} - \frac{\beta k}{2}\frac{k_i}{\mu_i}\frac{\partial^2 S_i}{\partial x^2} = 0 \quad (19)$$

which is the governing equation of our problem.

A set of boundary conditions are written as

$$S_i(0,t) = s_1,$$

$$S_i(L,t) = s_2 \quad (20)$$

$$S_i(x,0) = 0,\ \ 0 \le x \le L \quad (21)$$

Also we assume that there is no flow across the face $x = L$ as the face at $x = L$ is assumed to be imperable), that is, Let $X = \frac{x}{L}$, $T = \frac{k_i}{2\mu_i}\frac{k}{\varphi L^2}t$

From (19) , we have

$$\frac{\partial S_i}{\partial T} - \beta\frac{\partial^2 S_i}{\partial X^2} = 0 \quad (22)$$

with $S_i(0,T) = s_1$, $S_i(1,T) = s_2$ (23)

$S_i(X,0) = 0, 0 \le X \le 1$ (24)

IV. Mathematical Solution

Using S.O.R. method [11, 12], we have

$$s_{i_{m,n+1}} = s_{i_{m,n}} + \frac{\beta k}{2h^2}\left(s_{i_{m+1,n}} - 2s_{i_{m,n}} + s_{i_{m-1,n}} + s_{i_{m+1,n+1}} - 2s_{i_{m,n+1}} + s_{i_{m-1,n+1}}\right)$$

Let $r = \frac{k}{h^2}$,

$$\lambda_m = s_{i_{m,n}} + \frac{\beta r}{2}\left(s_{i_{m+1,n}} - 2s_{i_{m,n}} + s_{i_{m-1,n}}\right)s_{i_{m,n+1}}$$

$$= (1-\omega)s_{i_{m,n}} + \omega\left[\frac{\beta r}{2(1+\beta r)}\left(s_{i_{m+1,n}} + s_{i_{m-1,n+1}}\right) + \frac{\lambda_m}{(1+\beta r)}\right]$$

Choose k = 0.1, h = 0.1, β = 0.1, $\omega = 1.47$, $s_1 = 0.75$, $s_2 = 0$

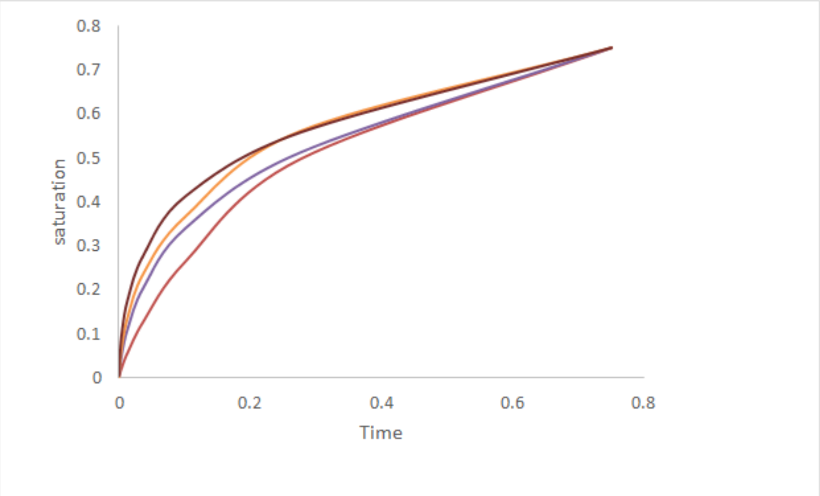
$$s_{i_{m,n+1}} = -0.47 s_{i_{m,n}} + 1.47\left[0.25\left(s_{i_{m+1,n}} + s_{i_{m-1,n+1}}\right) + \frac{\lambda_m}{(2)}\right]$$

Where $\lambda_m = 0.5\left(s_{i_{m+1,n}} + s_{i_{m-1,n}}\right)$

V. Table

T→	T=0.1	T=0.2	T=0.3	T=0.4	T=0.5
X↓			S_i		
0	0.75	0.75	0.75	0.75	0.75
0.1	0.275625	0.496156008	0.511642798	0.560391605	0.55734135
0.2	0.101292188	0.263382478	0.339610503	0.366633202	0.412415537
0.3	0.037224879	0.126577152	0.204343785	0.236688818	0.278114779
0.4	0.013680143	0.057462757	0.113083915	0.140215119	0.179800003
0.5	0.005027453	0.025140091	0.058737247	0.076885424	0.110651122
0.6	0.001847589	0.010717262	0.029081976	0.039703013	0.064650801
0.7	0.000678989	0.004481861	0.013877096	0.019580321	0.035991621
0.8	0.000249528	0.001846735	0.006431706	0.009312034	0.019168384
0.9	9.17017E-05	0.000752047	0.00290513	0.004248097	0.009622856
1.0	3.37004E-05	0.000294238	0.00120572	0.001568582	0.004360342

VI. Graphical Representation



VII. Conclusion

At the length X=0, 75% saturation is considered. It is clear that as time increases the saturation (S_i) increases. Also it is clear from the table that as length increases, saturation decreases.

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