

Transport of O₂ in A Red Blood Cell Involving A-Function

Kamal Kishore, Dr. S. S. Srivastava

Department of Mathematics SCD Govt. College, Ludhiana (Punjab)

Institute for Excellence in Higher Education Bhopal (M.P.)

Abstract: The aim of this paper is to discuss on Transport of O₂ in a Red Blood Cell involving A-function.

I. Introduction

The oxygen transport is essential to all of the cells of the body for sustaining metabolic process. The total surface area of the respiratory membrane is an astonishing 70m² for an average male adult, yet the volume of the blood in the lung capillaries at any instant is no more than that about 70 – 100 ml. Composition of the expired air will depend on the balance between the rates at which O₂ are being carried in and out by breathing and the rates at which O₂ is consumed and generated by metabolism. The transport of the gases through the membrane depends upon its thickness and surface area. The membrane offers a certain amount of resistance to the transport of respiratory gases. Vorp et al 1998 has performed computational analysis of O₂ diffusion from the lumen to the wall of the abdominal aortic aneurysm containing intraluminal thrombus. Their result demonstrates that the diameter of the abdominal aortic aneurysm bulge has little effect on the O₂ flow. Several authors Singh [10, 11], Sharan and Singh [7, 8, 9] have developed the various mathematical models for process of the gas exchange to the blood in pulmonary capillaries by taking into account the transport mechanism of molecular diffusion due to the presence of hemoglobin as a carrier of the gases. However these studies are limited to the pulmonary capillaries either considering the effect of permeability, facilitated diffusion, convection or resistance offered by the membrane. The most important mechanism is the study of gaseous diffusion in the red blood cell for clinical purpose. Nicholsson and Roughton [6] have successfully modeled the initial stage of O₂ uptake by red blood cells by starting with a flat layer model of a cell.

Some additional numerical calculations by Froster [1] have indicated, that the shape of the red blood cell has only a minor effect on gas uptake. i.e. a flat disc of uniform thickness has essentially the same uptake rate as a biconcave cell having the same volume. Thus the model suggested by earlier author is not a bad approximation, since red blood cell is much larger in diameter than in thickness. Therefore such a model, which neglects the curvature of the cells surface, turns out to be surprisingly good. Moll [5] and Kutchal [3] have obtained the solution for oxygen uptake in the red blood cell by taking unsteady general differential equation without neglecting the back reaction.

A mathematical model is developed in this paper for the transport of O₂ in the red blood cell. The model takes into account the transport of the species due to molecular diffusion and the facilitated diffusion.

The A-function of one variable is defined by Gautam [2] and we will represent here in the following manner:

$$A_{p,q}^{m,n} [x |_{(b_j, \beta_j)_{1,q}}^{(a_j, \alpha_j)_{1,p}}] = \frac{1}{2\pi i} \int_L \theta(s) x^s ds \quad (1)$$

where $i = \sqrt{-1}$ and

(i)

$$\theta(s) = \frac{\prod_{j=1}^m \Gamma(a_j + \alpha_j s) \prod_{j=1}^n \Gamma(1 - b_j - \beta_j s)}{\prod_{j=m+1}^p \Gamma(1 - a_j - \alpha_j s) \prod_{j=n+1}^q \Gamma(b_j + \beta_j s)}$$

(ii) m, n, p and q are non-negative numbers in which $m \leq p$, $n \leq q$.

(iii) $x \neq 0$ and parameters a_j , α_j , b_k and β_k ($j = 1$ to p and $k = 1$ to q) are all complex.

The integral in the right hand side of is convergent if

$$(i) \quad x \neq 0, k = 0, h > 0, |\arg(ux)| < \pi h/2$$

$$(ii) \quad x > 0, k = 0 = h, (v - \sigma\omega) < -1$$

where

$$\begin{aligned} k &= \text{Im} \left(\sum_{j=1}^p \alpha_j - \sum_{j=1}^q \beta_j \right) \\ h &= \text{Re} \left(\sum_{j=1}^m \alpha_j - \sum_{j=m+1}^p \alpha_j + \sum_{j=1}^n \beta_j - \sum_{j=n+1}^q \beta_j \right) \end{aligned} \quad (2)$$

$$\begin{aligned}
 u &= \prod_1^p \alpha_j^{\alpha_j} \prod_1^q \beta_j^{\beta_j} \\
 v &= Re(\sum_1^p a_j - \sum_1^q b_j) \\
 w &= Re(\sum_1^q \beta_j - \sum_1^p \alpha_j)
 \end{aligned}$$

and $s = \sigma + it$ is on path L when $|t| \rightarrow \infty$.

II. Formulation of the Problem:

This section considers the transport of O₂ due to their pure diffusional flux as well as the facilitated diffusion and reaction rates in the red blood cell. O₂ is known to undergo a number of reactions with hemoglobin in its various states. Some of these reactions have been subjected to considerable quantitative study and their roles in gaseous transport are well understood (Ulanovier and Frazier, 1970). The O₂ combine with hemoglobin in the blood according to the following reversible reactions:



where k' is the forward rate coefficients and k'' is the backward rate coefficients.

A planner model of red blood cell has been given for diffusion of the O₂ in Fig. 1. The equations describing gaseous transport through the membrane is:

$$\partial c' / \partial t = D_m (\partial^2 c' / \partial x^2), \tag{4}$$

Where c' is the concentration of O₂ in the membrane. D_m represents the diffusion coefficient membrane.

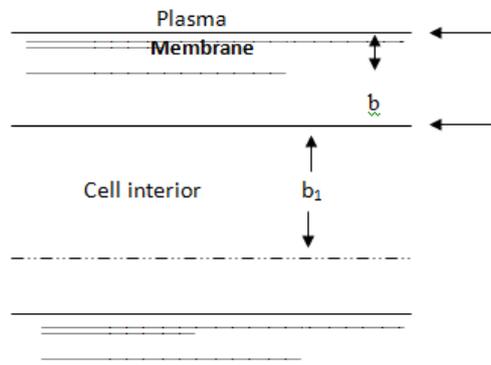


Fig. 1 Planner model of red cell for modeling diffusion and reaction

III. Result in terms of A-function:

Choose concentration $c'(x, t)$ in terms of A-function as

$$c'(x, t) = A_{p,q}^{m,n} [z x^\sigma t^\mu]_{(b_j, \beta_j)_{1,q}}^{(a_j, \alpha_j)_{1,p}} \tag{5}$$

provided that $\sigma > 0, \mu > 0, |\arg z| < \frac{1}{2} \pi h$, where h is given in (2).

Now differentiate it with respect to x and t partially, we get

$$\partial c' / \partial t = (1/t) A_{p+1,q+1}^{m+1,n} [z x^\sigma t^\mu]_{(b_j, \beta_j)_{1,q}, (0, \mu)}^{(1, \mu), (a_j, \alpha_j)_{1,p}} \tag{6}$$

and

$$(\partial^2 c' / \partial x^2) = (1/x^2) A_{p+1,q+1}^{m+1,n} [z x^\sigma t^\mu]_{(b_j, \beta_j)_{1,q}, (-1, \sigma)}^{(1, \sigma), (a_j, \alpha_j)_{1,p}} \tag{7}$$

Now after using (6) and (7) in (4), we get following result

$$\begin{aligned}
 (1/t) A_{p+1,q+1}^{m+1,n} [z x^\sigma t^\mu]_{(b_j, \beta_j)_{1,q}, (0, \mu)}^{(1, \mu), (a_j, \alpha_j)_{1,p}} \\
 = D_m (1/x^2) A_{p+1,q+1}^{m+1,n} [z x^\sigma t^\mu]_{(b_j, \beta_j)_{1,q}, (-1, \sigma)}^{(1, \sigma), (a_j, \alpha_j)_{1,p}} \tag{8}
 \end{aligned}$$

provided that $\sigma > 0, \mu > 0, |\arg z| < \frac{1}{2} \pi h$, where h is given in (2).

IV. Special Cases

On the other hand, the A-function is the generalized form of the hypergeometric function in one argument and it contains an important class of symmetrical Fourier kernel of a very general nature and a vast number of well known analytic functions as special case like Meijer's G-function which itself is a generalization of many higher transcendental functions [4]. Therefore the result (8) given here is useful in obtaining many new results involving commonly used functions appearing in biological models.

References

- [1] Forester, R. E.: Rate of Gas uptake by Red cells, Chapter 32 in Handbook of Physiology, Section 3, Respiration 1 (W. O. Fenn and H. Rohdeds) Am. Physiol. Soc. Washington D. C., (1964 b).
- [2] Gautam, G. P. and Goyal, A. N.: On Fourier Kernels and Asymptotic Expansion, Ind. J. Pure and Appl. Math. ,1981, 12(9), 1094-1105.
- [3] Kutchai, H.: Numerical study of oxygen uptake by layers of hemoglobin solution, Respir, physiol. 10 273 (1970).
- [4] Luke, Y. L.: The Special Functions and their Approximations, Vol.II, Academic Press, New York (1969).
- [5] Moll W.: The influence of hemoglobin diffusion on oxygen uptake and release by red cells, respire Physiol, 6 1 (1968).
- [6] Nicholson, P. and Roughton, F. J. W. A.: A theoretical study of the influence of diffusion and chemical reaction velocity on the rate of exchange of carbon dioxide and surrounding fluid, Proc. Roy. Soc. London, Ser. B. 138 241 (1951).
- [7] Sharan Maithili and Singh, M. P.: Equivalence between one step kinetics and Hills equation, J. Biomed. Engg. (1984).
- [8] Sharan Maithili and Singh, M. P.: Numerical simulation of Pulmonary O₂ and CO₂ exchange, Int. J. Biomed. Comput. 16 59 (1985).
- [9] Sharan Maithili, Singh, M. P. and Aminataer: A mathematical Formulation for the oxygen dissociation curve based on kinetics of the hemoglobin generalization of Hills equation, J. Bioscience (1987).
- [10] Singh, M. P., Khetarpal, K. and Sharan Maithili: A theoretical model for studying the rate of oxygenation of blood in pulmonary capillaries, J. Math Bio. 9, 305 (1980).
- [11] Singh, M. P., Sharan Maithili and Verma, S. P.: Transport of gases in in pulmonary capillaries as computation of equilibrium constants arising in the oxygen dissociation curve, J. Biomed. Engg. 4 279 (1982a).