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Some Properties of FGSPR-Continuous Functions

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Abstract: *In this paper some properties of fgspr-continuous functions are studied. It is shown that the class of all fgspr-irresolute functions implies the class of all fgspr*-continuous functions and the class of all fgspr*-continuous functions implies the class of all fgspr-continuous functions.*
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I. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by Zadeh [15]. Subsequently, several authors have applied various basic concepts from general topology to fuzzy sets and developed the theory of fuzzy topological spaces. Fuzzy topology was introduced by Chang [3]. Wong [13] studied the product of fuzzy topological spaces in 1974. Azad [1] introduced fuzzy semi continuity in 1981. Balasubramanian and Sundaram [2] introduced generalized fuzzy continuous functions in 1997 and Thakur and Singh [8] introduced fuzzy semi pre continuity in 1998. Gnanambal and Balachandran [4] introduced the concept of gpr-continuous functions in 1998. In 2010, Govindappa Navalagi et al [5] defined the concept of generalized semi preregular closed sets and also introduced the notion of generalized semi preregular continuity and studied their properties. In 2013, Vadivel et al [11] explained the concept of fuzzy generalized preregular continuous mappings and studied their properties.

In this paper, some properties of fgspr-continuous functions and fgspr-irresolute functions are studied. Also fgspr*-continuousfunction is introduced and some of its properties are studied. It is shown that the class of all fgspr-irresolute functions implies the class of all fgspr*-continuous functions and the class of all fgspr*-continuous functions implies the class of all fgspr-continuous functions.

II. Preliminaries

Let X, Y and Z be fuzzy sets. Throughout this paper (X,τ), $(Y,\sigma)$and (Z,η) (or simply X, Y and Z) mean fuzzy topological spaces on which no separation axioms are assumed unless explicitly stated. Let $f:(X,\tau)\to(Y,\sigma)$ be function from a fuzzy topological space X to fuzzy topological space Y. Let us recall the following definitions which we shall require later.

Definition 2.1: A fuzzy set λ in a fuzzy topological space (X,τ) is called
(i) a fuzzy semi-preopen set [8] if $\lambda\le \mathrm{cl}(\mathrm{int}(\mathrm{cl}(\lambda)))$ and a fuzzy semi-preclosed set if $\mathrm{int}(\mathrm{cl}(\mathrm{int}(\lambda)))\le\lambda$.
(ii) a fuzzy regular open set [1] if $\mathrm{int}\big(\mathrm{cl}(\lambda)\big)=\lambda$ and a fuzzy regular closed set if $\mathrm{cl}\big(\mathrm{int}(\lambda)\big)=\lambda$.

Definition 2.2: A fuzzy set λ in a fuzzy topological space (X,τ) is called
(i) a fuzzy generalized closed set (briefly, fg-closed) [2] if $\mathrm{cl}(\lambda)\le\mu$, whenever $\lambda\le\mu$ and μ is a fuzzy open set in X.
(ii) a fuzzy generalized preregular closed set (briefly, fgpr-closed) [11] if $\mathrm{pcl}(\lambda)\le\mu$, whenever $\lambda\le\mu$ and μ is a fuzzy regular open set in X.
(iii) a fuzzy generalized semi preregular closed set (briefly, fgspr-closed) [9] if $\mathrm{spcl}(\lambda)\le\mu$, whenever $\lambda\le\mu$ and μ is a fuzzy regular open set in X.

Definition 2.3: Let X, Y be two fuzzy topological spaces. A function $f:(X,\tau)\to(Y,\sigma)$ is called
(i) a fuzzy continuous (briefly, f-continuous) [3] if $f^{-1}(\lambda)$ is a fuzzy open (fuzzy closed) set in X, for every fuzzy open (fuzzy closed) set λ in Y.
(ii) a fuzzy semi pre continuous (briefly, fsp-continuous) [8] if $f^{-1}(\lambda)$ is a fuzzy semi preopen (fuzzy semi preclosed) set in X,for every fuzzy open(fuzzy closed) set λ in Y.
(iii) a fuzzy generalized preregular continuous (briefly, fgpr-continuous) [12] if $f^{-1}(\lambda)$ is a fuzzy generalized preregular open (fuzzy generalized preregular closed) set in X, for every fuzzy open (fuzzy closed) set λ in Y.

(iv) a fuzzy generalized semi preregular continuous (briefly, fgspr-continuous) [10] if $f^{-1}(\lambda)$ is a fuzzy generalized semi preregular open (fuzzy generalized semi preregular closed) set in X, for every fuzzy open (fuzzy closed) set λ in Y.
(v) a fuzzy semi irresolute (briefly, fs-irresolute) [14] if $f^{-1}(\lambda)$ is a fuzzy semi open (fuzzy semi closed) set in X, for every fuzzy semi open (fuzzy semi closed) set λ in Y.
(vi) a fuzzy generalized preregular irresolute (briefly, fgpr- irresolute) [12] if $f^{-1}(\lambda)$ is a fuzzy generalized preregular open (fuzzy generalized preregular closed) set in X, for every fuzzy generalized preregular open (fuzzy generalized preregular closed) set λ in Y.
(vii) a fuzzy generalized semi preregular irresolute (briefly, fgspr-irresolute) [10] if $f^{-1}(\lambda)$ is a fuzzy generalized semi preregular open (fuzzy generalized semi preregular closed) set in X, for every fuzzy generalized semi preregular open (fuzzy generalized semi preregular closed) set λ in Y.

Definition 2.4: [9] For any fuzzy set λ in any fuzzy topological space,
(i) fgspr-cl(λ) = ∧{μ: μ is a fgspr-closed set and μ ≥ λ }
(ii) fgspr-int(λ) = ∨{μ: μ is a fgspr-open set and μ ≤ λ }

Definition 2.5: A fuzzy topological space $(X,\tau)$is said to be
(i) a fuzzy $T_{1/2}$ space [2] if every fg-closed is fuzzy closed.
(ii) a fuzzy semi preregular $T_{1/2}$ space [9] if every fgspr-closed is fuzzy semi preclosed.
(iii) a fuzzy semi preregular $T^{*}_{1/2}$space [9] if every fgspr-closed is fuzzy closed.

Definition 2.6:[6] A fuzzy point $x_p \epsilon A$ is said to be quasi-coincident with the fuzzy set A denoted by $x_p q A$ if and only if p + A(x) > 1**.** A fuzzy set A is quasi-coincident with a fuzzy set B denoted by A_qB if and only if there exists $x \epsilon X$ such that A(x) + B(x) > 1. If A and B are not quasi-coincident then we write $A_{\bar{q}}B$. Note that $A \leq B \leftrightarrow A_{\bar{q}}(1-B)$.

Lemma 2.7: [1] Let $f: X \to Y$ be a mapping and $\{\lambda_\alpha\}$ be a family of fuzzy sets of Y, then
(a) $f^{-1}(\cup \lambda_\alpha) = \cup f^{-1}(\lambda_\alpha)$
(b) $f^{-1}(\cap \lambda_\alpha) = \cap f^{-1}(\lambda_\alpha)$

Lemma 2.8: [1] For mappings $f_i: X_i \to Y_i$ and fuzzy sets λ_i of Y_i, $i = 1,2;$ we have $(f_1 \times f_2)^{-1}(\lambda_1 \times \lambda_2) = f_1^{-1}(\lambda_1) \times f_2^{-1}(\lambda_2)$.

Lemma 2.9: [1] Let $g: X \to X \times Y$ be the graph of a mapping $f: X \to Y$ then, if λ is a fuzzy set of X and μ is a fuzzy set of Y, $g^{-1}(\lambda \times \mu) = \lambda \cap f^{-1}(\mu)$.

III. Fgspr-Continuous Functions

In this section, some properties of fuzzy generalized semi preregular continuous function are studied.

Theorem 3.1: If a function$f: (X,\tau) \to (Y,\sigma)$is fgspr-continuous then f[fgspr-cl(λ)] ≤ cl[f(λ)] for every fuzzy set λ in X.

Proof: Let λ be any fuzzy set in X. Now cl [f(λ)] is a fuzzy closed set in Y. As f is fgspr-continuous, f^{-1}(cl [f(λ)]) is a fgspr-closed set in X. λ ≤ f^{-1}(cl [f(λ)]) and so fgspr-cl(λ) ≤ f^{-1}(cl [f(λ)]). Hence f[fgspr-cl(λ)] ≤ cl[f(λ)].

The following example shows that the converse of the above theorem is not true.

Example 3.2: Let X = {a, b, c}, Y = {x, y} and consider the fuzzy sets λ_1 = {(a, 0.3), (b, 0.5), (c,0.4)}, λ_2 = {(a, 0.2), (b, 0.4), (c,0.2)}, λ_3 = {(x, 0.8), (y, 0.6)}, λ_4 = {(x, 0.2), (y, 0.4)} and λ_5 = {(x, 0.1), (y, 0.5)}. Let τ = {0, λ_1, λ_2, 1} and σ = {0, λ_3, 1}. Define the function $f: (X,\tau) \to (Y,\sigma)$ by f(a) = f(c) = x and f(b) = y. Then for any fuzzy set λ, f[fgspr-cl(λ)] ≤ cl[f(λ)]. Since the fuzzy set λ_5 is not a fuzzy closed set in Y but $f^{-1}(\lambda_5)$ is a fgspr-closed set in X. Hence f is not fgspr-continuous.

Theorem 3.3: If a function $f: (X,\tau) \to (Y,\sigma)$ is fgspr-continuous then fgspr-cl[f^{-1}(μ)] ≤ f^{-1}[cl(μ)] for every fuzzy set μ in Y.

Proof: Let μ be any fuzzy set in Y. Now cl(μ) is a fuzzy closed set in Y. As f is fgspr-continuous, f^{-1}[cl(μ)] is a fgspr-closed set in X. Since f^{-1}(μ) ≤ f^{-1}[cl(μ)], it follows from Definition 2.4 that fgspr-cl[f^{-1}(μ)] ≤ f^{-1}[cl(μ)].

The following example shows that the converse of the above theorem is not true.

Example 3.4: Let X = {a, b, c}, Y = {x, y} and consider the fuzzy sets μ_1 = {(a, 0.2), (b, 0.5), (c, 0.7)}, μ_2 = {(x, 0), (y, 0.7)}, μ_3 = {(x, 1), (y, 0.3)} and μ_4 = {(x, 0), (y, 0.4)}. Let τ = {0, μ_1, 1} and σ = {0, μ_2, 1}. Define the function $f: (X,\tau) \to (Y,\sigma)$ by f(a) = f(c) = x and f(b) = y. Then for any fuzzy set μ, fgspr-cl[f^{-1}(μ)] ≤ f^{-1}[cl(μ)]. Since the fuzzy set μ_4 is not a fuzzy closed set in Y but $f^{-1}(\mu_4)$ is a fgspr-closed set in X. Hence f is not fgspr-continuous.

Theorem 3.5: A function$f: (X,\tau) \to (Y,\sigma)$ is fgspr-continuous iff f^{-1}[int(μ)] ≤ fgspr-int[f^{-1}(μ)] for every fuzzy set μ in Y,

Proof: Let μ be any fuzzy set in Y. Now $\mathrm{int}(\mu)$ is a fuzzy open set in Y. As f is fgspr-continuous, $f^{-1}[\mathrm{int}(\mu)]$ is a fgspr-open set in X and we have $f^{-1}[\mathrm{int}(\mu)]$ = fgspr-int($f^{-1}[\mathrm{int}(\mu)]$) $\leq$ fgspr-int($f^{-1}(\mu)$). Hence $f^{-1}[\mathrm{int}(\mu)] \leq$ fgspr-int[$f^{-1}(\mu)$].

Conversely, let μ be any fuzzy open set in Y. By hypothesis we have, fgspr-int[$f^{-1}(\mu)$] $\geq f^{-1}[\mathrm{int}(\mu)] = f^{-1}(\mu)$ and so fgspr-int[$f^{-1}(\mu)$] $\geq f^{-1}(\mu)$. Also we have $f^{-1}(\mu) \geq$ fgspr-int[$f^{-1}(\mu)$] we get $f^{-1}(\mu)$ = fgspr-int[$f^{-1}(\mu)$]. Hence $f^{-1}(\mu)$ is a fgspr-open set in X and f is fgspr-continuous.

Theorem 3.6: A function $f: (X, \tau) \to (Y, \sigma)$is fgspr-continuous iff int[f(λ)] ≤ f [fgspr-int(λ)] for every fuzzy set λ in X.

Proof: Let λ be any fuzzy set in X. Now int[f(λ)] is a fuzzy open set in Y. As f is fgspr-continuous, f^{-1}(int [f(λ)]) is a fgspr-open set in X. From Theorem 3.5, f^{-1}(int [f(λ)]) ≤ fgspr-int(f^{-1}[f(λ)]) ≤ fgspr-int(λ) and so f^{-1}(int [f(λ)]) ≤ fgspr-int(λ). Hence int[f(λ)] ≤ f [fgspr-int(λ)].

Conversely, let μ be any fuzzy open set in Y. By hypothesis we have, f [fgspr-int($f^{-1}(\mu)$)] ≥ int $\left[f\left(f^{-1}(\mu)\right)\right]$ = $\mathrm{int}(\mu) = \mu$ and so fgspr-int($f^{-1}(\mu)$) $\geq f^{-1}(\mu)$. Also we have $f^{-1}(\mu) \geq$ fgspr-int($f^{-1}(\mu)$) we get $f^{-1}(\mu)$ = fgspr-int($f^{-1}(\mu)$). Hence $f^{-1}(\mu)$ is a fgspr-open set in X and f is fgspr-continuous.

Theorem 3.7: If $f: (X, \tau) \to (Y, \sigma)$ is fgspr-continuous then for each fuzzy point x_p of X and each $\mu \epsilon \sigma$ such that $f(x_p) \epsilon \mu$, there exists a fgspr-open set λ of X such that $x_p \epsilon \lambda$ and $f(\lambda) \leq \mu$.

Proof: Let x_p be a fuzzy point of X and $\mu \epsilon \sigma$ such that $f(x_p) \epsilon \mu$. Put $\lambda = f^{-1}(\mu)$ then by hypothesis λ is a fgspr-open set of X such that $x_p \epsilon \lambda$ and $f(\lambda) = f[f^{-1}(\mu)] \leq \mu$.

Theorem 3.8: If $f: (X, \tau) \to (Y, \sigma)$ is fgspr-continuous then for each fuzzy point x_p of X and each $\mu \epsilon \sigma$ such that $f(x_p) q\mu$, there exists a fgspr-open set λ of X such that $x_p q\lambda$ and $f(\lambda) \leq \mu$.

Proof: Let x_p be a fuzzy point of X and $\mu \epsilon \sigma$ such that $f(x_p) q\mu$. Put $\lambda = f^{-1}(\mu)$ then by hypothesis λ is a fgspr-open set of X such that $x_p q\lambda$ and $f(\lambda) = f[f^{-1}(\mu)] \leq \mu$.

Theorem 3.9: Let $X_1, X_2, Y_1 and\ Y_2$ be fuzzy topological spaces such that X_1 is product related to X_2. Then the product $f_1 \times f_2: X_1 \times X_2 \to Y_1 \times Y_2$ is fgspr-continuous if $f_1: X_1 \to Y_1$ and $f_2: X_2 \to Y_2$ are fgspr-continuous.

Proof: Let$\lambda \equiv \cup (\lambda_1 \times \lambda_2)$, where $\lambda_1\ and\ \lambda_2$ are fuzzy open sets of $Y_1\ and\ Y_2$ respectively, $\lambda_1 \times \lambda_2$be a fuzzy open set of $Y_1 \times Y_2$. Using Lemma 2.7 (a) and 2.8, we have

$$\begin{aligned}(f_1 \times f_2)^{-1}(\lambda_1 \times \lambda_2)(x_1, x_2) &= \lambda_1 \times \lambda_2[f_1(x_1), f_2(x_2)] \\ &= \min[\lambda_1 f_1(x_1), \lambda_2 f_2(x_2)] \\ &= \min[f_1^{-1}(\lambda_1)(x_1), f_2^{-1}(\lambda_2)(x_2)] \\ &= (f_1^{-1}(\lambda_1) \times f_2^{-1}(\lambda_2))(x_1, x_2)\end{aligned}$$

(i.e) $(f_1 \times f_2)^{-1}[\cup (\lambda_1 \times \lambda_2)] = \cup [f_1^{-1}(\lambda_1) \times f_2^{-1}(\lambda_2)]$

(i.e) $(f_1 \times f_2)^{-1}(\lambda) = \cup [f_1^{-1}(\lambda_1) \times f_2^{-1}(\lambda_2)]$

Therefore $(f_1 \times f_2)^{-1}(\lambda)$ is a fgspr-open set of $X_1 \times X_2$. Hence $f_1 \times f_2: X_1 \times X_2 \to Y_1 \times Y_2$ is fgspr-continuous.

Theorem 3.10: Let $f: X \to Y$ be any function. Then if the graph $g: X \to X \times Y$ of f is fgspr-continuous, f is also fgspr-continuous.

Proof: Let μ be any fuzzy open set in Y. Using Lemma 2.9, $f^{-1}(\mu) = 1 \cap f^{-1}(\mu) = g^{-1}(1 \times \mu)$. Since g is fgspr-continuous and $1 \times \mu$ is a fuzzy open set of $X \times Y$ and $g^{-1}(1 \times \mu)$ is a fgspr-open set of X. Then $f^{-1}(\mu)$ is a fgspr-open set of X and hence f is fgspr-continuous.

IV. Fgspr-Irresolute Functions

In this section, some properties of fuzzy generalized semi preregular irresolute function are studied.

Theorem 4.1: If a function $f: (X, \tau) \to (Y, \sigma)$is fgspr-irresolute then f[fgspr-cl(λ)] ≤ fgspr-cl[f(λ)] for every fuzzy set λ in X,

Proof: Let λ be any fuzzy set in X. Now fgspr-cl[f(λ)] is a fgspr-closed set in Y. As f is fgspr-irresolute, f^{-1}(fgspr-cl[f(λ)]) is a fgspr-closed set in X. Furthermore, λ ≤ f^{-1}[f(λ)] ≤ f^{-1}(fgspr-cl[f(λ)]) and it follows from Definition 2.4 that fgspr-cl(λ) ≤ f^{-1}(fgspr-cl[f(λ)]). Hence f[fgspr-cl(λ)] ≤ fgspr-cl[f(λ)]

Theorem 4.2: If a function$f: (X, \tau) \to (Y, \sigma)$ is fgspr-irresolute then fgspr-cl[$f^{-1}(\mu)$] ≤ f^{-1}[fgspr-cl(μ)] for every fuzzy set μ in Y.

Proof: Let μ be any fuzzy set in Y. Now fgspr-cl(μ) is a fgspr-closed set in Y. As f is fgspr-irresolute, f^{-1}[fgspr-cl(μ)] is a fgspr-closed set in X. Since $f^{-1}(\mu)$ ≤ f^{-1}[fgspr-cl(μ)], it follows from the Definition 2.4 that fgspr-cl[$f^{-1}(\mu)$] ≤ f^{-1}[fgspr-cl(μ)].

Theorem 4.3: A function$f: (X, \tau) \to (Y, \sigma)$ is fgspr-irresolute iff f^{-1}[fgspr-int(μ)] ≤ fgspr-int[$f^{-1}(\mu)$] for every fuzzy set μ in Y,

Proof: Let μ be any fuzzy set in Y. Now fgspr-int(μ) is a fgspr-open set in Y. As f is fgspr-irresolute, f^{-1}[fgspr-int(μ)] is a fgspr-open set in X and we have f^{-1}[fgspr-int(μ)] = fgspr-int(f^{-1}[fgspr-int(μ)]) ≤ fgspr-int[$f^{-1}(\mu)$]. Hence f^{-1}[fgspr-int(μ)] ≤ fgspr-int[$f^{-1}(\mu)$].

Conversely, let μ be any fgspr-open set in Y. Then μ = fgspr-int(μ) and $f^{-1}(\mu) = f^{-1}$[fgspr-int(μ)] ≤ fgspr-int($f^{-1}(\mu)$) and so $f^{-1}(\mu)$ ≤ fgspr-int($f^{-1}(\mu)$). Also we have $f^{-1}(\mu)$ ≥ fgspr-int($f^{-1}(\mu)$) we get $f^{-1}(\mu)$ = fgspr-int($f^{-1}(\mu)$). Hence $f^{-1}(\mu)$ is a fgspr-open set in X and f is fgspr-irresolute.

Theorem 4.4: A function $f: (X, \tau) \to (Y, \sigma)$ is fgspr-irresoulte iff fgspr-int[f(λ)] ≤ f[fgspr-int(λ)] for every fuzzy set λ in X.

Proof: Let λ be any fuzzy set in X. Now fgspr-int[f(λ)] is a fgspr-open set in Y. As f is fgspr-irresolute, f^{-1}[fgspr-int[f(λ)] is a fgspr-open set in X. From Theorem 4.3, f^{-1}(fgspr-int[f(λ)]) ≤ fgspr-int(f^{-1}[f(λ)]) ≤ fgspr-int(λ) and so f^{-1}(fgspr-int[f(λ)]) ≤ fgspr-int(λ). Hence fgspr-int[f(λ)] ≤ f[fgspr-int(λ)].

Conversely, let μ be any fgspr-open set in Y. Then μ = fgspr-int(μ). By hypothesis we have, f[fgspr-int$\left(f^{-1}(\mu)\right)$ ≥ fgspr-int$\left[f\left(f^{-1}(\mu)\right)\right]$ = fgspr-int(μ) = μ and so fgspr-int$\left(f^{-1}(\mu)\right) \geq f^{-1}(\mu)$. Also we have $f^{-1}(\mu)$ ≥ fgspr-int$\left(f^{-1}(\mu)\right)$ we get $f^{-1}(\mu)$ = fgspr-int$\left(f^{-1}(\mu)\right)$. Hence $f^{-1}(\mu)$ is a fgspr-open set in X and f is fgspr-irresolute.

V. Fgspr*-Continuous Functions

Definition 5.1: Let X and Y be two fuzzy topological spaces. A function $f: (X, \tau) \to (Y, \sigma)$ is said to be fuzzy generalized semi preregular*-continuous (briefly, fgspr*-continuous) if the inverse image of every fuzzy semi preclosed set in Y is a fgspr-closed set in X.

Example 5.2: Let X = {a, b, c} = Y and consider the fuzzy sets λ_1 = {(a, 1), (b, 1), (c, 0)}, λ_2 = {(a, 0), (b, 1), (c, 0)}, λ_3 = {(a, 1), (b, 0), (c, 0)}, λ_4 ={(a, 0), (b, 0), (c, 1)} and λ_5 ={(a, 0), (b, 1), (c, 1)}. Let τ = {0, λ_1, λ_2, 1} and σ = {0, λ_3, 1}. Define the function $f: (X, \tau) \to (Y, \sigma)$ by f(a) = f(b) = a and f(c) = b. Then the only fuzzy semi preclosed sets in Y are λ_2, λ_4 and λ_5 and $f^{-1}(\lambda_2)$, $f^{-1}(\lambda_4)$ and $f^{-1}(\lambda_5)$ are fgspr-closed sets in X. Hence f is fgspr*-continuous.

Theorem 5.3: A function $f: (X, \tau) \to (Y, \sigma)$ is fgspr*-continuous iff the inverse image of every fuzzy semi preopen set in Y is a fgspr-open set in X.

Proof: Suppose the function $f: (X, \tau) \to (Y, \sigma)$ is fgspr*-continuous. Let λ be fuzzy semi preopen set in Y. Then $1 - \lambda$ is a fuzzy semi preclosed set in Y. Since f is fgspr*-continuous, $f^{-1}(1 - \lambda)$ is a fgspr-closed set in X. But $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$ and so $f^{-1}(\lambda)$ is a fgspr-open set in X.

Conversely, assume that the inverse image of every fuzzy semi preopen set in Y is a fgspr-open set in X. Let μ be fuzzy semi preclosed set in Y. Then $1 - \mu$ is a fuzzy semi preopen set in Y. By hypothesis, $f^{-1}(1 - \mu)$ is a fgspr-open set in X. But $f^{-1}(1 - \mu) = 1 - f^{-1}(\mu)$ and so $f^{-1}(\mu)$ is a fgspr-closed set in X. Hence f is fgspr*-continuous function.

Remark 5.4: The class of all fgspr-irresolute functions implies the class of all fgspr*-continuous functions and the class of all fgspr*-continuous functions implies the class of all fgspr-continuous functions, as seen from the following theorem.

Theorem 5.5: Let $f: (X, \tau) \to (Y, \sigma)$ be a function

(i) If f is fgspr-irresolute, then it is a fgspr*-continuous function

(ii) If f is fgspr*-continuous, then it is a fgspr-continuous function

Proof: (i)Let $f: (X, \tau) \to (Y, \sigma)$ be fgspr-irresolute. Let λ be fuzzy semi preclosed set in Y and so λ is a fgspr-closed set in Y as every fuzzy semi preclosed is fgspr-closed. Since f is fgspr-irresolute, $f^{-1}(\lambda)$ is a fgspr-closed set in X. Hence f is fgspr*-continuous function.

(ii) Let $f: (X, \tau) \to (Y, \sigma)$ be fgspr*-continuous. Let λ be fuzzy closed set in Y and so λ is a fuzzy semi preclosed set in Y as every fuzzy closed is fuzzy semi preclosed. Since f is fgspr*-continuous, $f^{-1}(\lambda)$ is a fgspr-closed set in X. Hence f is fgspr-continuous function.

Theorem 5.6: If $f: X \to Y$ is fgspr*-continuous function and $g: Y \to Z$ is fsp-continuous function, then $gof: X \to Z$ is fgspr-continuous function.

Proof: Let λ be fuzzy closed set in Z. Then $g^{-1}(\lambda)$ is a fuzzy semi preclosed set in Y as g is fsp-continuous function. Since f is fgspr*-continuous, $f^{-1}[g^{-1}(\lambda)]$ is a fgspr-closed set in X. Now $(gof)^{-1}(\lambda) = f^{-1}[g^{-1}(\lambda)]$ is a fgspr-closed set in X. Hence $gof: X \to Z$ is fgspr-continuous function.

Theorem 5.7: If $f: X \to Y$ is fgspr-irresolute function and $g: Y \to Z$ is fgspr*-continuous function, then $gof: X \to Z$ is fgspr*-continuous function.

Proof: Let λ be fuzzy semi preclosed set in Z. Then $g^{-1}(\lambda)$ is a fgspr-closed set in Y as g is fgspr*-continuous function. Since f is fgspr-irresolute, $f^{-1}[g^{-1}(\lambda)]$ is a fgspr-closed set in X. Now $(gof)^{-1}(\lambda) = f^{-1}[g^{-1}(\lambda)]$ is a fgspr-closed set in X. Hence $gof: X \to Z$ is fgspr*-continuous function.

Theorem 5.8: If $f: X \to Y$ is fgspr*-continuous function, $g: Y \to Z$ is fgspr-continuous function and Y is fuzzy semi preregular $T_{1/2}$space, then $gof: X \to Z$ is fgspr-continuousfunction.

Proof: Let λ be fuzzy closed set in Z. Then $g^{-1}(\lambda)$ is a fgspr-closed set in Y as g is fgspr-continuous function. Since Y is fuzzy semi preregular $T_{1/2}$space, $g^{-1}(\lambda)$ is a fuzzy semi preclosed set in Y. Since f is fgspr*-continuous, $f^{-1}[g^{-1}(\lambda)]$ is a fgspr-closed set in X. Now $(gof)^{-1}(\lambda) = f^{-1}[g^{-1}(\lambda)]$ is a fgspr-closed set in X. Hence $gof: X \to Z$ is fgspr-continuous function.

Theorem 5.9: If a function $f: (X,\tau) \to (Y,\sigma)$ is fgspr*-continuous then for every fuzzy set μ in Y,

(i) fgspr-cl$[f^{-1}(\mu)] \leq f^{-1}[cl(\mu)]$

(ii) $f^{-1}[int(\mu)] \leq$ fgspr-int$[f^{-1}(\mu)]$

Proof: (i) Let μ be any fuzzy set in Y. Now $cl(\mu)$ is a fuzzy semi preclosed set in Y. As f is fgspr*-continuous, $f^{-1}[cl(\mu)]$ is a fgspr-closed set in X. Since $f^{-1}(\mu) \leq f^{-1}[cl(\mu)]$, it follows from Definition 2.4 that fgspr-cl$[f^{-1}(\mu)] \leq f^{-1}[cl(\mu)]$. .

(ii)Let μ be any fuzzy set in Y. Now $int(\mu)$ is a fuzzy semi preopen set in Y. As f is fgspr*-continuous, $f^{-1}[int(\mu)]$ is a fgspr-open set in X and we have $f^{-1}[int(\mu)]$ = fgspr-int$(f^{-1}[int(\mu)]) \leq$ fgspr-int$[f^{-1}(\mu)]$. Hence $f^{-1}[int(\mu)] \leq$ fgspr-int$[f^{-1}(\mu)]$.

Theorem 5.10: If a function $f: (X,\tau) \to (Y,\sigma)$ is fgspr*-continuous then for every fuzzy set λ in X,

(i) f[fgspr-cl(λ)] $\leq$ cl [f(λ)]

(ii) int [f(λ)] $\leq$ f[fgspr-int(λ)]

Proof: (i) Let λ be any fuzzy set in X. Now cl [f(λ)] is a fuzzy semi preclosed set in Y. As f is fgspr*-continuous $f^{-1}(cl\,[f(\lambda)])$ is a fgspr-closed set in X. $\lambda \leq f^{-1}(cl\,[f(\lambda)])$ and so fgspr-cl(λ) $\leq$ $f{-}1(cl\, f\lambda)$. Hence f[fgspr-cl(λ)] ≤cl[fλ].

(ii) Let λ be any fuzzy set in X. Now int[f(λ)] is a fuzzy semi preopen set in Y. As f is fgspr*-continuous, $f^{-1}(int\,[f(\lambda)])$ is a fgspr-open set in X. From Theorem 5.9 (ii), $f^{-1}(int\,[f(\lambda)]) \leq$ fgspr-int$(f^{-1}[f(\lambda)]) \leq$ fgspr-int(λ) and so $f^{-1}(int\,[f(\lambda)]) \leq$ fgspr-int(λ). Hence int [f(λ)] ≤ f[fgspr-int(λ)].

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