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A New Modification Method for Partial Differential Equations with Boundary Integral Conditions

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Abstract: *This paper presents an initial value problem with boundary integral conditions for the partial differential equation. Modification of adomian decomposition method is introduced for solving problem. Finally, simulation results for some examples illustrate the comparison of the analytical and numerical solutions. The results are presented using the MathCAD 12 and Matlab software package.*
Key words: *Modified decomposition method, partial differential equation, boundary integral condition problem.*

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I. Introduction

In recent years a semi-analytical method named Adomian decomposition method proposed by G. Adomian has been attracting the attention of many mathematicians, physicist and engineers. The method has the adventage of converging to the exact solution and can easily handle a wide class of both linear and nonlinear differential and integral equations [1]. There are many literatures developed concerning Adomian decomposition method [2, 8, 9] and the related modification to investigate various sciencetific model [3, 4, 5, 6]. The theoretical treatment of the convergence of Adomian decomposition method has been considered in [7].
In this paper, we present computationally efficient numerical method for solving the partial differential equation with boundary integral conditions:

$$D_t u(x,t) - D_{xx} u(x,t) + u(x,t) = q(x,t) \tag{1}$$

with the initial condition

$$u(x,0) = f(x), 0 \le x \le T$$

and the integral conditions

$$\int_0^1 u(x,t)dx = g_1(t), 0 < t \le T$$

$$\int_0^1 \psi(x)u(x,t)dx = g_2(t), 0 < t \le T$$

Where f, g_1, g_2, ψ and q are known functions, T is given constant. In the present work, we apply the modified Adomian's decomposition method for solving eq.(1) and compare the results with exact solution. The paper is organized as follows: In section 2 the partial differential equations with boundary integral conditions and its solution by Adomian's decomposition and modified decomposition method is presented. In section 3 an example is solved numerically using the modified decomposition method. Finally, we present conclusion about solution of the fractional partial differential equation.

Solution Partial Differential Equations with Boundary Integral Conditions by Modified Adomian's Decomposition Method
The aim of this section is to discuss the use of modified decomposition method for solving of partial differential equations with boundary integral given in eq.(1). In this method we assume that:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$$

Can be rewritten eq.(1):

$$L_t u(x,t) = L_{xx} u(x,t) - u(x,t) + q(x,t) \quad (2)$$

Where

$$L_t(\cdot) = \frac{\partial}{\partial t}(\cdot)$$

and $L_{xx} = \dfrac{\partial^2}{\partial x^2}$

The inverse L^{-1} is assumed an integral operator given by

$$L^{-1} = \int_0^t (\cdot)dt \quad (3)$$

Take the operator L^{-1} on both sides of eq (2) we have

$$L^{-1}(L_t u((x,t))) = L^{-1}(L_{xx}(u(x,t))) - L^{-1}(u(x,t)) + L^{-1}(q(x,t))$$

Therefore, we can write,

$$u(x,t) = u(x,0) + L_t^{-1}\left(L_{xx}\left(\sum_{n=0}^{\infty} u_n \right)\right) - L_t^{-1}(u(x,t)) + L_t^{-1}\big(q(x,t)\big) \quad (4)$$

The modified decomposition method was introduced by Wazwaz [4]. This method is based on the assumption that the function $K(x)$ can be divided into two parts, namely $K_1(x)$ and $K_2(x)$. Under this assumption we set

$$K(x) = K_1(x) + K_2(x)$$

Then the modification

$$u_0 = K_1$$

$$u_1 = K_2 + L_t^{-1}(L_{xx}u_0) - L_t^{-1}(u_0)$$

$$u_{n+1} = L_t^{-1}\left(L_{xx}\left(\sum_{n=0}^{\infty} u_n \right)\right) - L_t^{-1}\left(\sum_{n=0}^{\infty} u_n \right), \text{ n>1}$$

Numerical Illustration:
Three examples are presented in this part to to illustrate the proposed method.

Example 1:
Consider partial differential equation with boundary integral condition for the equation (1), as taken in [10]:

$$D_t u - D_{xx} u + u = 2t + t^2 + x$$

$$u(x,0) = x, \qquad x \in (0,1), \quad 0 \le t \le T$$

$$\int_0^1 u(x,t)dx = 0.5 + t^2, \qquad 0 \le t \le T$$

$$\int_0^1 xu(x,t)dx = \frac{1}{3} + \frac{1}{2}t^2, \qquad 0 \le t \le T$$

We apply the above proposed method; we obtain:

$$u_0(x,t) = x + t^2$$
$$u_1(x,t) = 0$$
$$u_2(x,t) = 0$$
$$u_3(x,t) = 0$$

Then the series form is given by:

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t)$$

$$= x + t^2$$

This is the exact solution $u(x,t) = x + t^2$.

Table 1 shows the analytical solutions for partial differential equation with boundary integral condition obtained for different values and comparison between exact solution and analytical solution.

Table1. Comparison between exact solution and analytical solution For example 1

x	**t**	**Exact Solution**	**Modified Adomian Decomposition Method**	$\lvert u_{ex} - u_{MADM} \rvert$
0	1	1.0	1.0	0.0000
0.1	1	1.1	1.1	0.0000
0.2	1	1.2	1.2	0.0000
0.3	1	1.3	1.3	0.0000
0.4	1	1.4	1.4	0.0000
0.5	1	1.5	1.5	0.0000
0.6	1	1.6	1.6	0.0000
0.7	1	1.7	1.7	0.0000
0.8	1	1.8	1.8	0.0000
0.9	1	1.9	1.9	0.0000
1	1	2.0	2.0	0.0000

Example 2:
Consider the problem (1) with the following conditions, as taken in [10]:

$$D_t u - D_{xx} u + u = (10 - 2x)e^t$$

$$u(x,0) = 5 - x, \qquad x \in (0,1), \quad 0 \le t \le T$$

$$\int_0^1 u(x,t)dx = \frac{9}{2}e^t, \qquad 0 \le t \le T$$

$$\int_0^1 xu(x,t)dx = \frac{13}{6}e^t, \qquad 0 \le t \le T$$

Now after modified decomposition method, we obtain:

$$u_0(x,t) = (5 - x)e^t$$
$$u_1(x,t) = 0$$
$$u_2(x,t) = 0$$
$$u_3(x,t) = 0$$

Then the series form is given by:

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t)$$

$$= (5-x)e^t$$

Which gives the exact solution $u(x,t) = (5-x)e^t$.

Table 2 shows the analytical solutions for partial differential equation with boundary integral condition obtained for different values and comparison between exact solution and analytical solution.

Table2. Comparison between exact solution and analytical solution
For example 2 when t=0.4

x	t	Exact Solution	Modified Adomian Decomposition Method	$\lvert u_{ex}-u_{MADM}\rvert$
0	0.4	5.00000	5.00000	0.000
0.1	0.4	7.30994	7.30994	0.000
0.2	0.4	7.16076	7.16076	0.000
0.3	0.4	7.01158	7.01158	0.000
0.4	0.4	6.86239	6.86239	0.000
0.5	0.4	6.71321	6.71321	0.000
0.6	0.4	6.56403	6.56403	0.000
0.7	0.4	6.41485	6.41485	0.000
0.8	0.4	6.26566	6.26566	0.000
0.9	0.4	6.11648	6.11648	0.000
1	0.4	5.96730	5.96730	0.000

Example 3:
Consider the problem (1) with the following boundary integral and initial conditions, as taken in [10]

$$D_t u - D_{xx} u + u = 11 + 6t + 11t^2 - x - 2xt - xt^2 - 4x^2 - 8tx^2 - 4x^2t^2$$

$$u(x,0) = 3 - x - 4x^2, \qquad x \in (0,1), \quad 0 \le t \le T$$

$$\int_0^1 u(x,t)dx = \frac{7}{6}(1+t^2), \qquad 0 \le t \le T$$

$$\int_0^1 (1+2x)u(x,t)dx = \frac{3}{2} + \frac{3}{2}t^2, \qquad 0 \le t \le T$$

Now we apply the above modified decomposition method, we obtain:

$$u_0(x,t) = (1+t^2)(3-x-4x^2)$$
$$u_1(x,t) = 0$$
$$u_2(x,t) = 0$$
$$u_3(x,t) = 0$$

Then the series form is given by:

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t)$$

$$= (1+t^2)(3-x-4x^2)$$

This is the exact solution $u(x,t)=(1+t^2)(3-x-4x^2)$.

Table 3 shows the analytical solutions for partial differential equation with boundary integral condition obtained for different values and comparison between exact solution and analytical solution.

Table3. Comparison between exact solution and analytical solution
For example 3 when t =2

x	**t**	**Exact Solution**	**Modified Adomian Decomposition Method**	$\lvert u_{ex}-u_{MADM}\rvert$
0	2	15.00	15.00	0.0000
0.1	2	14.30	14.30	0.0000
0.2	2	13.20	13.20	0.0000
0.3	2	11.70	11.70	0.0000
0.4	2	09.80	09.80	0.0000
0.5	2	07.50	07.50	0.0000
0.6	2	04.80	04.80	0.0000
0.7	2	01.70	01.70	0.0000
0.8	2	-01.80	-01.80	0.0000
0.9	2	-05.70	-05.70	0.0000
1	2	-10.00	-10.00	0.0000

II. Conclusion

In this paper, we have presented the modified decomposition method for the solution of the partial differential equation with boundary integral condition. This algorithm is simple and easy to implement. Furthermore numerical example is presented to show that good agreement between the numerical solution and exact solution has been noted.

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