IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 13, Issue 4 Ver. I (Jul. – Aug. 2017), PP 46-63
www.iosrjournals.org

Development and Analysis of an Inventory System for Exponential Demad Items with Perishability and Life Time

*Ravish Kumar Yadav
Associate Professor, Department of Mathematics, Hindu College, Moradabad
*Corresponding Author: *Ravish Kumar Yadav*

Abstract: *In this paper an inventory model is developed and analyzed with exponential demand under the condition of permissible delay in payments. Deterioration rate is taken to be a constant fraction of the on hand inventory. In deterioration the realistic assumption of life period is taken into account. Shortages are allowed and they are taken as completely backlogged. Interest is earned on the unit cost of generated sales revenue which is to be deposited in an interest bearing account. Interest charges are paid on the stock held beyond the permissible period, Three different cases depending upon the permissible period are discussed.*
Keywords: *inventory, permissible, perishability. Exponential*

Date of Submission: 13-07-2017 Date of acceptance: 03-08-2017

I. Introduction

In most of the literature dealing with inventory problems, either in deterministic or probabilistic model, it is often assumed that payments will be made to the supplier for the goods immediately after receiving the consignment. However under most market behaviours, a vendor often provided buyers with a credit period to stimulate the demand, boost market share or decrease inventories of certain items. Vendor allows the buyers to pay at the end of specified period of time rather than immediately after receipt of the goods. **Goyal (1985)** first studied the EOQ model under conditions of permissiable delay in payments. He assumed that the unit purchase cost is the same as the selling price per unit and concluded that the economic replenishment interval and order quality generally increases marginally under the permissiable delay in payments. To accommodate more practical features of the real inventory system, **Aggarwal** and **Jaggi (1995)** extended the previous authors work.

Shahs N.H (1993) present a lot size inventory model for exponentially decaying inventory when delay in payments is permissiable..

Chung K.J. (2000) explores the inventory replenishment policy for deteriorating items under permissiable delay in payments. The inventory system discussed by this paper is the same as that of **Shah (1993)**. This paper shows that total variable cost per unit time is convex. A simple optimization procedure is developed to improve that described by **Shah.**

Dye (2002) presented a model in which demand is taken stock dependent and shortages are permitted and partially backlogged. Teng ammeded the Goyals model by considering the difference between unit price and unit cost. In Chung`s model account is settled at the end of the credit period while in Dye`s model account is settled when the shortages starts.

Chang (2004) developed a deterministic inventory model for deteriorating items with stock dependent demand and shortages. The replenishment number and fraction of each cycle in which there is no shortage are both determined so as to minimize the present value of inventory cost over a finite planning horizon.

Chandand Werd (1987) reinvestigated the Goyal`s model (1985) within the framework of the classical EOQ model. In the model considered by 'Goyal' it is assumed that no shortage of the stock is allowed to occur, replenishment of stock is instantaneous, the sales revenue is invested to earn interest only during the permissiable settelement period and beyond this period, if stock remains it has to be financed at some interest rate, which in general differ from the rate of interest earned from the sales revenue. If the settlement period is less than the length of inventory cycle, than from remaining beyond settlement period, the sales revenue should also earn interest as it earned during the settlement period.

In this paper an inventory model is developed with exponential demand rates. Deterioration starts after a fix time. Such deterioration is termed as non instantaneous deterioration. Some credit period is provided in payments. Expressions are obtained for different parameters. Cost minimization technique is used to obtain the solution.

NOTATIONS:
C: Ordering cost of inventory per order.
C_1: Holding cost excluding interest charge per unit per unit time.
C_2: Shortage cost per unit per unit time.
C_3: Unit purchase cost.
Ir : Interest paid per rupee invested in stock s per year Ir>Ie.
Ie : Interest which can be earned per rupee per year.
q(t) : Inventory level at time t.
M : Permissible delay period for settling accounts in time 0<M<1.
t_1 : Time at which shortages starts.
T : Length of replenishment cycle.
: Life period of item at the end of which deterioration starts.
Q : Total amount of inventory produced or purchased at the beginning of each production cycle.
S(S<Q) : Initial amount of inventory after fulfilling back orders.
$TC(t_1,T)$: The total average cost of the inventory system per unit time.
$TC_{1(a)}(t_1,T)$: The total average cost of the inventory system per unit time for $M \le t_1$ and $M \le \mu$.
$TC_{1(b)}(t_1,T)$: The total average cost of the inventory system for $M \le t_1$ and $M \ge \mu$.
$TC_2(t_1,T)$: The total average cost of the inventory system per unit time for $M>t_1$.
d: Total demand during the production cycle T.

ASSUMPTIONS:
The inventory system consists of single item only.
There is no repair or replacement of the deteriorated unit.
The replenishment occurs instantaneously at an infinite rate.
A constant fraction θ of the on hand inventory deteriorates per unit time only after the expiry of the life period μ of the item.
Hence the deterioration function θ can be taken in the following form-
$= \theta_0 H(t - \mu)$, $0 < \theta_0 << 1$ t , $\mu > 0$

Where H(t-μ) is heaviside unit function defined as follows

$$H(t-\mu)\begin{cases} =1, & t \ge \mu \\ =0, & t < \mu \end{cases}$$

The demand rate function D(t) is deterministic and is exponential function of time. Demand rate is given by-

$$D(t) = \frac{d}{(e-1)} e^{t/T}$$

Shortages are allowed and they are completely backlogged.
During the fixed credit period M the unit cost of generated sales revenue is deposited in an interest bearing account. The difference between sales price and unit cost is retained by the system to meet the day to day expenses of the system. At the end of the credit period account to settled. Then interest is again earned during the period (M, t_1). If $M \le t$, interest charges are paid on the stock held beyond the permissible period.

II. Mathematical Model And Analysis Of The System:

Let Q be the total amount of inventory produced or purchased at the beginning of each production cycle. Let S(<Q) be the initial inventory after fulfilling back orders. During the period [μ, t_1] the inventory level decreases due to the market demand only. After this during the period [μ,t_1] the inventory level further decreases due to the combined effect of market demand and deterioration. At time t_1 the inventory level falls to zero and shortages starts. Demand is backlogged in the interval [t_1, T] . At time T when Q amount inventory is remaining amount S of inventory is left for the next replenishment cycle.
The differential equations showing variations of inventory level during the period [0, T] are as follows-

$$\frac{d\,q(t)}{dt} = \frac{-d}{(e-1)\,T} e^{t/T} \text{ , } 0 \le t \le \mu \qquad \text{……(1)}$$

$$\frac{dq(t)}{dt}+\theta_0 q(t)=\frac{-d}{(e-1)T}e^{t/T} \ , \mu \le t \le t_1 \qquad ..(2)$$

$$\frac{d\,q(t)}{dt}=\frac{-d}{(e-1)\,T}e^{t/T} \ , t_1 \le t \le T \qquad ...(3)$$

The boundary conditions are-

$q(t)=S$ at $t=0,$
$q(t)=q(S),$ at $t=\mu$
and
$q(t)=0$ at $t=t_1$
The solution of equation (1) is given by

$$q(t)=\frac{-d}{(e-1)}e^{t/T}+A$$

When A is a constant of integration. Using boundary condition at t=0, q (t) = s, we get-

$$A=S+\frac{d}{(e-1)}$$

$$q(t)=S+\frac{d}{(e-1)}[-e^{t/T}+1], \quad 0 \le t \le \mu \qquad ......(4)$$

Apply another boundary condition at t = μ

$$q(\mu)=S+\frac{d}{(e-1)}(1-e^{\mu/T}) \qquad ......(5)$$

The solution of equation (2) is given by-

$$q(t)e^{\theta_0 t}=\left[\frac{-d}{(e-1)(1+\theta_0 T)}e^{(1+\theta_0 T)\, t/T}\right]+B$$

here B is constant of integration.
Using boundary condition at t = μ, we get

$$B=\frac{d}{(e-1)}e^{\theta_0 \mu}(1-e^{\mu/T})+Se^{\theta_0 \mu}$$

Therefore

$$q(t)e^{\theta_0 t} = \frac{-d}{(e-1)(1+\theta_0 T)}\left[e^{(1+\theta_0 T)\,t/T} - e^{(1+\theta_0 T)\mu/T} \right]$$

$$+\frac{d}{(e-1)}e^{\theta_0\mu}(1-e^{\mu/T}) + Se^{\theta_0\mu} \qquad \text{……(6)}$$

Now applying the boundary condition at $t = t_1$, $q(t) = 0$ in equation (6), we get

$$S = \frac{d\ e^{-\theta_0\mu}}{(e-1)(1+\theta_0 T)}\left(e^{(1+\theta_0 T)\,t_1/T} - e^{(1+\theta_0 T)\mu/T} \right)$$

$$-\frac{d}{(e-1)}(1-e^{\mu/T}) \qquad \text{….. (7)}$$

By the substitution of the value of S in the equation (6) reduces to-

$$q(t)e^{\theta_0 t} = \frac{d}{(e-1)(1+\theta_0 T)}\left(e^{(1+\theta_0 T)\,t_1/T} - e^{(1+\theta_0 T)\,t/T} \right)$$

Hence

$$q(t) = \frac{d}{(e-1)(1+\theta_0 T)}\left[e^{(1+\theta_0 T)\,t_1/T - \theta_0 t} - e^{t/T} \right], \quad \mu \le t \le t_1 \qquad \text{…….(8)}$$

Solution of equation (3) using boundary condition at $t = t_1$, $q(t) = 0$ is given by

$$q(t) = \frac{d}{(e-1)}\left(e^{t_1/T} - e^{t/T} \right), t_1 \le t \le T \qquad \text{……(9)}$$

Now the total amount of carrying units (q_H) during the period $(0, t_1)$ is given by-

$$q_H = \int_0^{\mu} q(t)dt + \int_{\mu}^{t_1} q(t)dt$$

$$= \int_0^{\mu}\left\{ \frac{d}{(e-1)}(1-e^{t/T}) + S \right\}dt +$$

$$\int_{\mu}^{t_1}\left[\frac{d}{(e-1)(1+\theta_0 T)}(e^{(1+\theta_0 T)\,t_1/T - \theta_0 T} - e^{(1+\theta_0 T)\,t/T - \theta_0 T}) \right]dt$$

or

$$q_H = \frac{d}{(e-1)}(T + e^{\mu/T}(\mu - T)) + \frac{d}{(e-1)(1+\theta_0 T)}$$
$$\left[e^{\left\{(1+\theta_0 T)^{t_1/T} - \theta_0 \mu\right\}}(\mu + \frac{1}{\theta_0}) + e^{\mu/T}(T-\mu) - e^{t_1/T}(T + \frac{1}{\theta_0}) \right]$$
…..(10)

The total amount of deterioration units (q_D) during the period $(0,t_1)$ is given by-

$$q_D = q(\mu) - \int_{\mu}^{t_1} \frac{d}{(e-1)T} e^{t/T} dt$$

$$= \frac{d}{(e-1)}\left(1 - e^{\mu/T}\right) + S - \frac{d}{(e-1)}\left(e^{t_1/T} - e^{\mu/T}\right)$$

Using the value of S, one can get

$$q_D = \frac{d}{(e-1)}\left[\frac{e^{-\theta_0 \mu}}{(1+\theta_0 T)}\left\{ e^{(1+\theta_0 T)\frac{t_1}{T}} - e^{(1+\theta_0 T)\frac{\mu}{T}} \right\} \right.$$
$$\left. - e^{t_1/T} + e^{\mu/T} \right] \qquad ......(11)$$

Total Amount of the shortage units q_S during the period (t_1, T) can be obtained as

$$q_S = -\int_{t_1}^{T} q(t)\ d(t)$$

$$= -\int_{t_1}^{T} \frac{d}{(e-1)}\left[e^{t_1/T} - e^{t/T} \right] d(t)$$

$$= \frac{d}{(e-1)}\left[T\left(e - e^{t_1/T}\right) - e^{t_1/T}\left(T - t_1\right) \right] \qquad ……(12)$$

Now there are two possibilities regarding the period M of permissible delay in payments

CASE(I) If $M \le t_1$

CASE(II) If $M > t_1$

We first discuss case(I)

CASE(I) $M \le t_1$

In this case there are further two possibilities as follows

CASE I(A) $M \le \mu \le t_1$

CASE I(B) $\mu \le M \le t_1$

We first discuss case I (A)

CASE I(A) :

Since here the length of period with positive inventory stock is larger than the credit period M, the buyer can use sale revenue to earn the interest with an annual rate Ie during the period [0,M]. The unit cost of the generated sales revenue is deposited in an interest bearing account. The difference between sales price and unit cost is retained by the system to meet the day to day expenses of the system. At the end of the credit period, the account is settled. After setting the account at time M again the unit cost of generated sales revenue is deposited in an interest bearing account to earn interest with an annual rate Ie during the period $[M,t_1]$. Beyond the fixed credit period product still in stock is assumed to be financed with an annual rate Ir. Now the total interest earned $IE_{1(a)}$ during the period $[0,t_1]$ is given by-

$$IE_{1(a)} = C_3 I_e \left[\int_0^M (M-t) \frac{d}{(e-1)T} e^{t/T} dt + \int_M^{t_1} (t_1 - t) \frac{d}{(e-1)T} e^{t/T} dt \right]$$

$$= C_3 I_e \frac{d}{(e-1)T} \left[\left\{ MTe^{t/T} - Tte^{t/T} + T^2 e^{t/T} \right\}_0^M + \right.$$

$$\left. \left\{ t_1 Te^{t/T} - Tte^{t/T} + T^2 e^{t/T} \right\}_M^{t_1} \right]$$

$$= \frac{C_3 Ie\, d}{(e-1)T} \left[\left\{ T(e^{M/T} - 1) - M \right\} + T(e^{t_1/T} - e^{M/T}) \right]$$

$$= \frac{C_3 Ie\, d}{(e-1)} \left[e^{t_1/T} - M - 1 \right] \qquad \ldots\ldots(13)$$

Total interest payable $IP_{1(a)\_}$ is given by

$$IP_{1(a)} = C_3 I_r \int_M^{t_1} q(t)\, dt$$

$$= C_3 I_r \left[\int_M^{\mu} q(t)\, dt + \int_{\mu}^{t_1} q(t)\, dt \right]$$

$$= C_3 I_r \left[\int_M^{\mu} \left\{ \frac{d}{(e-1)} (1 - e^{t/T}) + S \right\} dt + \right.$$

$$\left.\int_{\mu}^{t_1} \frac{d\ e^{-\theta_0 T}}{(e-1)(1+\theta_0 T)} \left(e^{(1+\theta_0 T)^{t_1}/T} - e^{(1+\theta_0 T)^{t}/T}\right)dt \right]$$

$$= C_3 I_r \left[\left\{ \frac{d}{(e-1)} \left((\mu - M) - T\left(e^{\mu/T} - e^{M/T} \right) + S(\mu - M) \right) \right\} + \right.$$

$$\left. \frac{d}{(e\ \ -1)(1\ \ +\theta_0 T)} \left\{ Te^{\mu/T} - e^{t_1/T}\left(\frac{1}{\theta_0} + T\right) + \frac{e^{(1+\theta_0 T)^{t_1}/T} - \theta_0 \mu}{\theta_0} \right\} \right]$$

The substitution of the value of S it reduces to-

$$IP_{1(a)} = C_3 I_r \left[\left\{ \frac{d}{(e-1)} \left((\mu - M) - T\left(e^{\mu/T} - e^{M/T} \right) \right) \right\} + (\mu - M) \right.$$

$$\left\{ \frac{d\ e^{-\theta_0 \mu}}{(e-1)(1\ +\theta_0 T)} \left(e^{(1+\theta_0 T)^{t_1}/T} - e^{(1+\theta_0 T)^{\mu}/T} - \frac{d}{(e-1)}\left(1 - e^{\mu/T}\right) \right) \right\}$$

$$\left. + \frac{d}{(e-1)(1\ +\theta_0 T)} \left\{ T\ e^{\mu/T} - e^{t_1/T}\left(\frac{1}{\theta_0} + T\right) + \frac{e^{(1+\theta_0 T)^{t_1}/T} - \theta_0 \mu}{\theta_0} \right\} \right]$$

......(14)

Now the total average cost of the system in consideration is given by

$$TC_{1(a)}(t_1, T) = \frac{C + C_1 q_H + C_3 q_D + C_2 q_S + IP_{1(a)} - IE_{1(a)}}{T}$$

$$= \frac{C}{T} + \frac{C_1}{T}\left[\frac{d}{(e-1)}\left(T + e^{\mu/T}(\mu - T) \right) + \frac{d}{(e-1)(1+\theta_0 T)} \right.$$

$$\left\{ e^{(1+\theta_0 T)^{t_1}/T - \theta_0 \mu} \left(\mu + \frac{1}{\theta_0} \right) + e^{\mu/T}(T - \mu) - e^{t_1/T} \right.$$

$$\left(T+\frac{1}{Q_0}\right)\Bigg\}\Bigg]+\frac{C_3}{T}\left[\frac{d}{(e-1)}\right.$$

$$\left\{\frac{e^{-\theta_0\mu}}{(1+\theta_0 T)}\left(e^{(1+\theta_0 T)\,{}^{t_1}\!/_T}-e^{(1+\theta_0 T)\,{}^{\mu}\!/_T}\right)-e^{{}^{t_1}\!/_T}+e^{{}^{\mu}\!/_T}\right\}\Bigg]+$$

$$\frac{C_2}{T}\left[\frac{d}{(e-1)}\left\{T\left(e-e^{{}^{t_1}\!/_T}\right)-e^{{}^{t_1}\!/_T}(T-t_1)\right\}\right]+$$

$$\frac{C_3 Ir}{T}\quad\frac{d}{(e-1)}\left[\left\{(\mu-M)-T\left(e^{{}^{\mu}\!/_T}-e^{{}^{M}\!/_T}\right)\right\}+(\mu-M)\right.$$

$$\left\{\frac{e^{-\theta_0\mu}}{(1+\theta_0 T)}\left(e^{(1+\theta_0 T)\,{}^{t_1}\!/_T}-e^{(1+\theta_0 T)\,{}^{\mu}\!/_T}\right)-\left(1-e^{{}^{\mu}\!/_T}\right)\right\}$$

$$\left.+\frac{1}{(1+\theta_0 T)}\left\{Te^{{}^{\mu}\!/_T}-e^{{}^{t_1}\!/_T}\left(\frac{1}{\theta_0}+T\right)+\frac{1}{\theta_0}e^{(1+\theta_0 T)-\theta_0\mu}\right\}\right]$$

$$-\frac{C_3 Ied}{T(e-1)}\left[e^{{}^{t_1}\!/_T}-M-1\right]$$

……(15)

To minimize the total average cost per unit time $TC_{1(a)}$ (t_1,T) the optiomal values of t_1 and T (say t_1* and T*) can be obtained by solving the following two equations simultaneously-

$$\frac{\partial TC_{1(a)}(t_1,T)}{\partial t_1}=0 \qquad \text{…...(16)}$$

and

$$\frac{\partial TC_{1(a)}(t_1,T)}{\partial T}=0 \qquad \text{……(6.17)}$$

provided they satisfy the sufficient conditions-

$$\frac{\partial^2 TC_{1(a)}(t_1, T)}{\partial t_1^2} > 0$$

$$\frac{\partial^2 TC_{1(a)}(t_1, T)}{\partial T^2} > 0$$

and

$$\left(\frac{\partial^2 TC_{1(a)}(t_1, T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC_{1(a)}(t_1, T)}{\partial T^2}\right) - \left(\frac{\partial^2 TC_{1(a)}(t_1, T)}{\partial t\,\partial T}\right)^2 > 0$$

Equation (6.16) and (6.17) are equivalent to-

$$\frac{C_1}{T}\frac{d}{(e-1)}\left[\frac{1}{T}e^{(1+\theta_0 T)\ t_1/T - \theta_0\mu}\left(\mu + \frac{1}{\theta_0}\right) - \frac{1}{T}e^{t_1/T}\left(T + \frac{1}{\theta_0}\right)\right]$$

$$\left.\left.\frac{(t_1 - T)}{T}e^{t_1/T}\right]\right] + \frac{C_3 I_r d}{T(e-1)}\left[(\mu - M)\left\{\frac{e^{(1+\theta_0 T)\ t_1/T - \theta_0 M}}{T}\right\}\right.$$

$$\left.-\frac{1}{(1+\theta_0 T)}\right]\left\{e^{t_1/T}\frac{(1+\theta_0 T)}{\theta_0\, T}\right\} - \frac{C_3 I_e d}{T^2(e-1)}e^{t_1/T} = 0$$

…..(18)

and

$$= \frac{-C}{T^2} - \frac{C}{T^3}\frac{d}{(e-1)}\mu^2 e^{\mu/T} + \frac{C_1 d}{(e-1)(1+\theta_0 T)}$$

$$\left(\mu + \frac{1}{\theta_0}\right)\left[\frac{-T - 2\theta_0 T^2 - t(1+\theta_0 T)}{T^3(1+\theta_0 T)}\right]e^{(1+\theta_0)\,t_1/T - \theta_0\mu}$$

$$\left[\frac{-\theta_0 T^2 + 2\mu\theta_0 T + T^2 - \mu T + \mu + \theta_0 T^3 - \mu\theta_0 T^2}{(1+\theta_0 T)T^2}\right]e^{\mu/T}$$

$$+\left[\frac{\theta_0{}^2T^4+\theta_0T^3+T+\theta_0T^2+t_1T+\theta_0T^2t_1+1+\theta_0T}{(\theta_0T^3)(1+\theta_0T)}\right]e^{t_1/T}$$

$$-\frac{C_3}{T^3}\frac{d}{(e-1)}\left[\frac{e^{-\theta_0\mu}}{(1+\theta_0\mu)}\left\{\frac{T+2\theta_0T^2+t_1(1+\theta_0T)}{(1+\theta_0T)T^2}\right\}e^{(1+\theta_0T)\ t_1/T}\right.$$

$$\left.-\left(T+2\theta_0T^2+\mu(1+\theta_0T)\right)e^{(1+\theta_0T)\ \mu/T}\right\}+(t-T)e^{t_1/T}$$

$$\left.+(T-\mu)e^{\mu/T}\right]+\frac{C_2d}{(e-1)}\frac{t}{T^2}(T-1)e^{t_1/T}+\frac{C_3I_rd}{(e-1)}$$

$$\left[\frac{1}{T^2}\left(\mu e^{\mu/T}-Me^{M/T}\right)+(\mu-M)\left[\frac{e^{-\theta_0\mu}}{T^3(1+\theta_0T)^2}\right.\right.$$

$$\left\{\left(-2\theta_0T^2-T-t_1(1+\theta_0T)\right)e^{(1+\theta_0T)\ t_1/T}+\left(2\theta_0T^2+T+\mu(1+\theta_0T)\right)\right.$$

$$\left.e^{(1+\theta_0T)\mu/T}\right\}+\frac{1}{T^2(1+\theta_0T)^2}\left\{(T-\mu)(1+\theta_0T)^2-\theta_0T-\mu(1+\theta_0T)\right\}e^{\mu/T}$$

$$+\frac{1}{\theta_0T^3}\left\{T(\theta_0T+1)+t_1(1+\theta_0T)\right\}e^{t_1/T}+\left(\theta_0{}^2T^2-1-\theta_0T\right)$$

$$\left.e^{(1+\theta_0T-\theta_0\mu)}\right]+\frac{C_3I_ed}{T^2(e-1)}\left[e^{t_1/T}(T-t)e^{t_1/T}-M-1\right]=0$$

….(19)

To obtain the optimal values of t_1 and T that minimizes $TC_{1(a)}(t_1, T)$ one have to develop the following algorithm-

Algorithm 1(A)

STEP 1-

(I) Start with $t_1 = M$

(II) Substitute $t_{1(1)}$ in equation (18) to obtain $T_{(1)}$

(III) Using $T_{(1)}$ determines $t_{1(2)}$ from equation (19)

(IV) Repeat (II) and (III) until no change occurs in the value of t_1 and T.

STEP 2-To compare t_1 and M

(I) If $M \le t_1$, t_1 is feasible than go to step (3).

(II) If $M > t_1$, t_1 is not feasible set $t_1 = M$ and evaluate the corresponding values of T from equation (19) and then go to the step (3).

STEP 3-Calculate the corresponding.
$TC_{1(a)}$ (t_1* , T*)

CASE 1(B): ($\mu \le M \le t_1$)
When $M \ge \mu$ and $M \le t_1$
This case is similar to case 1(a). But as $M > \mu$ the interest earned during $IE_{1(a)}$ [0, t_1] is given by-

$$IE_{1(b)} = C_3 I_e \left[\int_0^M (M-t) \frac{d}{(e-1)T} e^{t/T} dt + \int_M^{t_1} (t_1 - t) \frac{d}{(e-1)T} e^{t/T} dt \right]$$

$$= \frac{C_3 Ie\, d}{(e-1)} \left[e^{t_1/T} - M - 1 \right] \qquad \text{……(20)}$$

Also the interest payable $IP_{1(a)\_}$ for the period (M, t_1) is given by

$$IP_{1(b)} = C_3 I_r \int_M^{t_1} q(t)\, dt$$

$$= C_3 I_r \int_M^{t_1} \frac{d}{(e-1)(1+\theta_0 T)} \left\{ e^{(1+\theta_0 T)^{t_1}/T - \theta_0 T} - e^{t/T} \right\} dt$$

$$= C_3 I_r \left[\frac{d}{(e-1)(1+\theta_0 T)} \left\{ \frac{e^{(1+\theta_0 T)^{t_1}/T - \theta_0 T}}{-\theta_0} - T e^{t/T} \right\} \right]_M^{t_1}$$

$$= C_3 I_r \left[\frac{d}{(e-1)(1+\theta_0 T)} \left[\frac{1}{(-\theta_0)} \left(e^{t_1/T} - e^{t_1/T + \theta_0 (t_1 - M)} \right) \right.\right.$$

$$\left.\left. - T\left(e^{t_1/T} - e^{M/T} \right) \right]\right] \qquad \text{……(21)}$$

Hence total average cost $TC_{1(b)}$ (t_1, T) in this case is given by-

$$TC_{1(b)}(t_1, T) = \frac{C + C_1 q_H + C_3 q_D + C_2 q_S + IP_{1(b)} - IE_{1(b)}}{T}$$

$$= \frac{C}{T} + \frac{C_1}{T} \frac{d}{(e-1)} \left[\left(T + e^{\mu/T} (\mu - T) \right) + \frac{1}{(1+\theta_0 T)} \left\{ \left(\mu + \frac{1}{\theta_0} \right) \right. \right.$$

$$e^{(1+\theta_0 T)^{t_1}/_T - \theta_0\mu} + e^{\mu/_T}(T-\mu) - e^{t_1/_T}\left(T+\frac{1}{\theta_0}\right)\Bigg\}\Bigg]$$

$$+\frac{C_3 d}{T(e-1)}\left[\frac{e^{-\theta_0\mu}}{(1+\theta_0 T)}\left\{e^{(1+\theta_0 T)\,t_1/_T} - e^{(1+\theta_0 T)\,\mu/_T}\right\}\right.$$

$$\left.-e^{t_1/_T} + e^{\mu/_T}\right] + \frac{C_2 d}{T(e-1)}\left[T\left(e-e^{t_1/_T}\right) - \left(e^{t_1/_T}(T-t_1)\right)\right]$$

$$+\frac{C_3 I_r}{T}\frac{d}{(e-1)(1+\theta_0 T)}\left[\frac{\left(e^{t_1/_T + \theta_0(T-M)} - e^{t_1/_T}\right)}{\theta_0}\right.$$

$$\left.-T\left(e^{t_1/_T} - e^{M/_T}\right)\right] - \frac{C_3 I_e d}{T(e-1)}\left[e^{t_1/_T} - M - 1\right] \qquad \text{……(22)}$$

Now to minimize the total average cost per year per unit time $TC_{1(b)}(t_1, T)$ the optimal values of t_1 and T (say t_1* and T) can be obtained by solving the following two equations simultaneously.

$$\frac{\partial TC_{1(b)}(t_1, T)}{\partial t_1} = 0 \qquad \text{…...(23)}$$

and

$$\frac{\partial TC_{1(b)}(t_1, T)}{\partial T} = 0 \qquad \text{……(24)}$$

Provided they satisfy the sufficient conditions-

$$\frac{\partial^2 TC_{1b}(t_1, T)}{\partial t_1^2} > 0 \quad ,$$

$$\frac{\partial^2 TC_{1b}(t_1, T)}{\partial T^2} > 0$$

and

$$\left(\frac{\partial^2 TC_{1(b)}(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC_{1(b)}(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 TC_{1(b)}(t_1,T)}{\partial t_1\,\partial T}\right)^2>0$$

Again equation (6.23) and (6.24) are equivalent to

$$\frac{d}{T(e-1)}e^{t_1/T}\left[\frac{C_1}{\theta_0 T}\left\{T(1+\theta_0\mu)e^{\theta_0(t_1-\mu)}-1\right\}\right.$$

$$\left.+\frac{C_3}{T}\frac{(\theta_0+I_r)}{\theta_0}\left\{e^{\theta_0(t_1-\mu)}-1\right\}+\frac{C_2(t_1-T)}{T}-\frac{C_3I_e}{T}\right]=0$$

……(25)

And

$$\frac{-C}{T_2}+\frac{dC_1}{T(e-1)}\left[\frac{(1+\theta_0\mu)}{(1+\theta_0T)}\;e^{(1+\theta_0T)\;t_1/T-\theta_0\mu}\right.$$

$$\left\{\frac{-(1+\theta_0T)T-\theta_0T^2}{(1+\theta_0T)T.\theta_0T^2}-t_1(1+\theta_0T)\right\}+e^{t_1/T}\left\{\frac{(T+t_1)}{\theta_0T^2}\right\}$$

$$\left.+\frac{\theta_0\,e^{\mu/T}}{T(1+\theta_0T)^2}\Big[T(M-T)-\mu^2(1+\theta_0T)\Big]\right]+\frac{C_3d}{T(e-1)}\left[\frac{-e^{-\theta_0\mu}}{T^2(1+\theta_0T)^2}\right.$$

$$e^{(1+\theta_0T)\,t_1/T}\left\{(1+\theta_0T)(T+t_1)+\theta_0T^2\right\}+\frac{e^{-\theta_0\mu}}{T^2(1+\theta_0T)^2}$$

$$e^{(1+\theta_0T)\,\mu/T}\left\{(1+\theta_0T)(\mu+T)+\theta_0T^2\right\}+\frac{e^{t_1/T}}{T^2}(T+t_1)$$

$$\left.-\frac{e^{\mu/T}}{T^2}(\mu+T)\right]+\frac{C_2d}{T_3(e-1)}e^{t_1/T}(T-t_1)+\frac{C_3I_rd}{T(e-1)(1+\theta_0T)}$$

$$\left[\frac{-e^{t_1/T}\theta_0(t_1-M)}{\theta_0T}\left\{\left(\frac{1+2\theta_0T}{(1+\theta_0T)}\right)+\frac{t_1}{T}\right\}+\frac{(1+2\theta_0T)}{T\theta_0}e^{t_1/T}\right.$$

$$-\frac{(1+2\theta_0 T)}{(1+\theta_0 T)}e^{M/T}+\frac{(1+\theta_0 T)t_1}{\theta_0 T^2}e^{t_1/T}+\frac{e^{M/T}}{T}(T-M)\Bigg]$$

$$+\frac{C_3 I_e d}{T(e-1)}\left[\frac{e^{t_1/T}}{T^2}(T-t_1)-\frac{(M+1)}{T}\right]=0 \quad \text{......(26)}$$

To obtain the optimal values of t_1 and T we develop the following algorithms.

Algorithm 1(B):
STEP 1
(I) Start with $t_1 = M$
(II) Substitute $t_{1(1)}$ into equation (25) to evaluate $T_{(1)}$
(III) Using $T_{(1)}$ to determine $t_{1(2)}$ from equation (26)
(IV) Repeat (II) and (III) until no change occurs in the value of t_1 and T.
STEP 2
(I) To compare t_1 and M
(I) If $M \le t_1$, then t_1 is feasible than go to step (3).
(II) If $M > t_1$, then t_1 is not feasible. Set $t_1 = M$ and evaluate the corresponding values of T from equation (26) and then go to the step (3).
STEP 3
Compute the corresponding.
$TC_{1(b)}(t_1^*, T^*)$
Now to discuss the next case when $t_1 < M$

CASE 2 $t_1 < M$
In this situation since $t_1 < M$ the buyer pays no interest and earns the interest during the period [0,M]. The interest earned in this case is denoted by IE_2 and it is given by-

$$IE_2 = C_3 I_e \int_0^{t_1}(M-t)\frac{d}{(e-1)T}e^{t/T}dt$$

$$= C_3 I_e \frac{d}{(e-1)}\left[T(M-t)e^{t/T}-e^{t/T}\right]_0^{t_1}$$

$$= \frac{C_3 Ie\, d}{(e-1)}\left[\{T(M-t)-1\}\left(e^{t_1/T}-1\right)\right] \quad \text{...... (27)}$$

Hence the total average cost per unit time TC2 (t1, T) in the present case is given by-

$$TC_2(t_1,T)=\frac{C+C_1 q_H+C_3 q_D+C_2 q_S-IE_2}{T}$$

$$= \frac{C}{T} + \frac{C_1}{T}\left[\frac{d}{(e-1)}\left\{T + e^{\mu/T}(\mu - T)\right\} + \frac{d}{(e-1)(1+\theta_0 T)}\right.$$

$$\left.\left. e^{(1+\theta_0 T)\,t_1/T - \theta_0\mu}\left(\mu + \frac{1}{\theta_0}\right) + e^{\mu/T}(T-\mu) - e^{t_1/T}\left(T + \frac{1}{\theta_0}\right)\right\}\right]$$

$$+ \frac{C_3}{T}\left[\frac{d}{(e-1)}\left\{\frac{e^{-\theta_0\mu}}{(1+\theta_0 T)}\left(e^{(1+\theta_0 T)\,t_1/T} - e^{(1+\theta_0 T)\,\mu/T}\right)\right.\right.$$

$$\left. - e^{t_1/T} + e^{\mu/T}\right] + \frac{C_2}{T}\frac{d}{(e-1)}\left[T\left(e - e^{t_1/T}\right) - e^{t_1/T}(T - t_1)\right]$$

$$-\frac{C_3 I_e d}{(e-1)T}\left[\left\{T(M-t) - 1\right\}\left\{e^{t_1/T} - 1\right\}\right] \qquad ...(28)$$

For minimizing total average cost per year per unit time $TC_2(t_1, T)$ the optimal values of t_1 and T (say t_1* and T) can be obtained by solving the following two equations simultaneously.

$$\frac{\partial TC_2(t_1, T)}{\partial t_1} = 0 \qquad ......(29)$$

and

$$\frac{\partial TC_2(t_1, T)}{\partial T} = 0 \qquad ......(30)$$

Provided they satisfy the sufficient conditions-

$$\frac{\partial^2 TC_2(t_1, T)}{\partial t_1^2} > 0 \quad ,$$

$$\frac{\partial^2 TC_2(t_1, T)}{\partial T^2} > 0$$

and

$$\left(\frac{\partial^2 TC_2(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC_2(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 TC_2(t_1,T)}{\partial t_1\,\partial T}\right)^2>0$$

Equation (29) and (30) are equivalent to-

$$\frac{C_1}{T}\frac{d}{(e-1)}\left[\frac{1}{(1+\theta_0 t)}\left\{\left(\mu+\frac{1}{\theta_0}\right)e^{(1+\theta_0 T)\ t_1/T-\theta_0\mu}-e^{t_1/T}\left(T+\frac{1}{\theta_0}\right)\frac{1}{T}\right\}\right]$$

$$+\frac{C_3}{T}\frac{d}{(e-1)}\left[\frac{e^{-\theta_0\mu}}{(1+\theta_0 T)}\left\{e^{(1+\theta_0 T)\ t_1/T}\ \frac{(1+\theta_0 T)}{T}-\frac{1}{T}e^{\ t_1/T}\right\}\right]$$

$$+\frac{C_2 d}{T(e-1)}\left[e^{t_1/T}\left(\frac{t_1}{T}-1\right)\right]-\frac{C_3 I_e d}{(e-1)T}\left[\{T(M-T)-1\}e^{t_1/T}\ \frac{1}{T}\right]=0 \qquad \ldots\ldots(31)$$

and

$$-\frac{C}{T^2}-\frac{C_1}{T^2}\left[\frac{d}{(e-1)}\left\{T+e^{\mu/T}(\mu-T)\right\}+\frac{d}{(e-1)(1+\theta_0 T)}\right.$$

$$\left.\left\{e^{(1+\theta_0 T)\ t_1/T-\theta_0\mu}\left(\mu+\frac{1}{\theta_0}\right)+e^{\mu/T}(T-\mu)-e^{t_1/T}\left(T+\frac{1}{\theta_0}\right)\right\}\right]$$

$$+\frac{C_1}{T}\left[\frac{d}{(e-1)}\left\{1-e^{\mu/T}\ \left(\mu/T^2\right)(\mu-T)-e^{\mu/T}-\frac{d\,\theta_0}{(e-1)(1+\theta_0 T)}\right.\right.$$

$$\left.\left.e^{(1+\theta_0 T)\ t_1/T-\theta_0\mu}\ \left(\mu+\frac{1}{\theta_0}\right)+e^{\mu/T}(T-\mu)-e^{t_1/T}\left(T+\frac{1}{\theta_0}\right)\right\}\right]$$

$$+\frac{d}{(e-1)(1+\theta_0 T)}\left\{\ e^{(1+\theta_0 T)\ \ t_1/T-\theta_0\mu}\ \left(\mu+\frac{1}{\theta_0}\right)\left(\frac{-t_1}{T^2}\right)+e^{\mu/T}+(T-\mu)\right.$$

$$\left.e^{\mu/T}\left(\frac{-\mu}{T^2}\right)-e^{t_1/T}-\left(T+\frac{1}{\theta_0}\right)\left(\frac{-t_1}{T^2}\right)\right\}$$

$$+\frac{C_3}{\left(-T^2\right)}\left[\frac{d}{(e-1)}\left\{\frac{e^{-\theta_0\mu}}{(1+\theta_0 T)}\left(e^{(1+\theta_0 T)\,t_1/T}-e^{(1+\theta_0 T)\,\mu/T}\right)\right.\right.$$

$$\left.-e^{t_1/T}+e^{\mu/T}\right]+\frac{C_3}{T}\left[\frac{\theta_0 e^{-\theta_0\mu}}{-(1+\theta_0 T)}2\left(e^{(1+\theta_0 T)\,t_1/T}-e^{(1+\theta_0 T)\,\mu/T}\right)\right.$$

$$+\frac{e^{-\theta_0\mu}}{(1+\theta_0 T)}\left(e^{(1+\theta_0 T)\,t_1/T}\left(-t_1/T^2\right)-e^{(1+\theta_0 T)}e^{\mu/T}\left(-\mu/T^2\right)\right)$$

$$\left.-e^{t_1/T}\left(\frac{-t_1}{T^2}\right)+e^{\mu/T}\left(\frac{-\mu}{T^2}\right)\right]+\frac{C_2}{-T^2}\frac{d}{(e-1)}\left[T\left(e-e^{t_1/T}\right)-e^{t_1/T}(T-t_1)\right]$$

$$+\frac{C_2}{T}\frac{d}{(e-1)}\left[\left(e-e^{t_1/T}\right)+e^{t_1/T}\frac{t_1}{T}-e^{t_1/T}+e^{t_1/T}(T-t_1)\frac{t_1}{T^2}\right]$$

$$-\frac{C_3 I_e d}{(e-1)\left(-T^2\right)}\left[\left\{T(M-T)-1\right\}\left\{e^{t_1/T}-1\right\}\right]$$

$$+\frac{C_3 I_e d}{(e-1)T}\left[(M-2T)\left(e^{t_1/T}-1\right)+\left\{T(M-T)-1\right\}e^{t_1/T}\left(\frac{-t_1}{T^2}\right)\right]=0$$

…...(32)

Now we use the following algorithm to obtain the optimal values.

Algorithm 2

STEP 1

(I) Start with $(t_1)_1 = M$

(II) Substituting $t_{1(1)} = M$ into equation (31) to evaluate $T_{(1)}$

(III) Using $T_{(1)}$ to determine $t_{1(2)}$ from equation (32)

(IV) Repeat (II) and (III) until no change occurs in the value of t_1 and T.

STEP 2

Compare t_1 and M

(I) If $t_1 < M$, t_1 is feasible than go to step (3).

(II) If $t_1 \geq M$, t_1 is not feasible. Set $t_1 = M$ and evaluate the corresponding values of T from equation (32) and then go to the step (3).

STEP 3

As started earlier the objective of this problem is to determine the optimal values of t_1 and T so that TC (t_1, T) is minimum. As the discussion carried out so far one can get-

$$TC\left(t_1^*,T^*\right)=\mathrm{Min}\left\{TC_{1(a)}\left(t_1^*,T^*\right),TC_{1(b)}\left(t_1^*,T^*\right),TC_2\left(t_1^*,T^*\right)\right\}$$

III. Conclusion

In the present discussion an inventory model is developed and analyzed with exponential demand under the condition of permissible delay in payments. Deterioration rate is taken to be a constant fraction of the on hand inventory. In deterioration the realistic assumption of life period is taken into account. Shortages are allowed and they are taken as completely backlogged. Interest is earned on the unit cost of generated sales revenue which is to be deposited in an interest bearing account. Interest charges are paid on the stock held beyond the permissible period, Three different cases depending upon the permissible period ($0 < M < \mu \le t_1$, $0 \le \mu \le M \le t_1$, and $M > t_1$) are discussed. In each case equations are derived for total average cost of the system per unit time. An algorithm for each case is given to obtain the value of optimal shortage time t_1* and total cycle period T*.

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*Ravish Kumar Yadav. "Development and Analysis of an Inventory System for Exponential Demad Items with Perishability and Life Time." IOSR Journal of Mathematics (IOSR-JM) 13.4 (2017): 46-63.