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Acharya Polynomial of Thorn Graphs

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Abstract : *Let G be a connected graph of order n and degree d, the* ***Acharya Polynomial AP(G, λ)*** *is defined as* $\sum_{\substack{1\le d\le n-1\\1\le k\le p}} \mu(d,G)\cdot\lambda^k$ *of graph G and* λ *is a parameter. In the present work we determine Acharya polynomial for thorn graph, thorn trees, thorn rods, rings and stars, which are the special cases of thorn graphs.*
Keywords: *Acharya index, Acharya Polynomial, Thorn Tree Thorn rod, Thorn cycle.*

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I. Introduction

In the present paper we shall confine ourselves to the study of finite graphs . The graph will always mean a finite graph. Let G be a connected graph with vertex set $V(G)= \{v_1, v_2, v_3, \ldots\ldots v_n\}$ and edge set $E(G)= \{e_1, e_2, e_3, \ldots\ldots e_n\}$. The distance between the vertices v_i and v_j , is equal to the length (=number of edges) of the shortest path starting at v_i and ending at v_j (or vice versa) and it is denoted by $d(v_i, v_j \mid G)$ .The diameter of a graph is the maximum distance between any pair of vertices of G, and it is denoted by $diam(G)$ [1].

Definition 1. Let G a connected n-vertex graph with vertex set $V(G)=\{v_1, v_2, v_3, \ldots\ldots v_n\}$, let $b=(b_1, b_2, b_3, \ldots\ldots, b_n)$ be an *n-tuple* of non negative integers . The thorn tree G^* is the graph obtained by attaching b_i pendent vertices to the vertex v_i of G for $i=1,2,\ldots\ldots, n$. The b_i pendent vertices attached to the vertex v_i will be called the thorn of v_i [2-3].

The thorn graph of the graph G will be denoted by G^*, or if the respective parameter need to be speciefied, by $G^*(b_1, b_2, b_3, \ldots\ldots, b_n)$.

In this work we find the Acharya Polynomial for thorn graphs and thorn tree.

Definition 2 [4]. Let G be a connected graph of order n and degree d, the ***Acharya Index*** $AI_\lambda(G)$ of a graph G as the sum of the distance between all pair of degree d vertices, i.e.,

$$AI_\lambda(G) = \sum_{\substack{1\le d\le n-1\\1\le k\le p}} \mu(d,G)\cdot k$$

where $\mu(d,G)$ denotes pair of vertices of degree d at distance k, $p=diam(G)$.

Definition 3 [5]. Let G be a connected graph of order n and degree d, the ***Acharya Polynomial AP(G, λ)*** is defined as $\sum_{\substack{1\le d\le n-1\\1\le k\le p}} \mu(d,G)\cdot\lambda^k$ of graph G and λ is a parameter.

The relation between Acharya index and polynomial is $AI_\lambda(G) = AP'(G,1)$.

In this work we compute the Acharya polynomial for thorn graphs, special cases of thorn graphs, thorn trees, caterpillar, thorn rod, star and cycle.

Theorem 3.1. Let G^* be the thorn graph obtained by attaching b_i pendent vertices to the vertex v_i of the non regular connected graph G, $i=1,2,3\ldots\ldots,n$. If $b_i>0$ for all $i=1,2,3\ldots\ldots,n$. then

$$i)\ AP(G^*,\lambda) = \binom{b_i}{2}\lambda^2 + \sum_{1\le i\le j\le n} b_i b_j \lambda^{d(v_i,v_j)+2}$$

$$for\ \ \deg(v_i)+b_i \ne \deg(v_j)+b_j\ \ \forall\, i,\, j = 1,2\ldots\ldots,\ \ n$$

$$ii)\, AP(G^*,\lambda) = \binom{b_i}{2}\lambda^2 + \sum_{1}^{diam(G)} \lambda^{d(v_i,v_j)} + \sum_{1\le i\le j\le n} b_i b_j \lambda^{d(v_i,v_j)+2}$$

$$for \quad \deg(v_i) + b_i = \deg(v_j) + b_j \ \forall\, i, j = 1,2 \ldots\ldots, \quad n$$

Proof: *i*) Considering thorn attached to v_i and v_j, if deg v_i+ $b_i \neq$ deg v_j+ b_j , i,j=1,2…..n, then the pendent vertices are only vertices of same degree. Hence finding the distance between pendent vertices.The Acharya polynomial reduces to Terminal Hosoya Polynomial which given by expression (i).

ii) If for some vertices if degv_i+b_i=degv_j+ b_j, i,j=1,2…..n,then the there are other set of vertices with same degree other than pendent vertices .Finding the distance between those vertices, and the distance between pendent vertices we have the expression (2).

Corollarary 3.2. Let G be connected graph on n vertices. If b_i=b>0, i=1,2,3……,n then

$$AP(G^*,\lambda) = AP(G,\lambda) + TW(G,\lambda)$$

Corollarary 3.3. If G be complete graph on n vertices . If $b_i \neq b_j$, i,j=1,2,3………, n.then

$$AP(G^*,\lambda) = n\binom{b_i}{2}\lambda^2 + \sum_{1\le i\le j\le n} \frac{n(n-1)}{2} b_i b_j \lambda^3$$

Corollarary 3.4. If G be complete graph on n vertices . If b_i=b>0, i,j=1,2,3………, n.then

$$AP(G^*,\lambda) = n\lambda + n\binom{b_i}{2}\lambda^2 + \sum_{1\le i\le j\le n} \frac{n(n-1)}{2} b_i b_j \lambda^3$$

Proof: If G is a completed then $d(v_i,v_j)$=1, i,j=1,2,3……..,n. $i\neq j$. Therefore the distance between n vertices of complete graph are distance is 1, which is the first term in the expression . And the vertices at distance 2 and 3 are pendent vertices. By the corollarary 3.3. we have the second and third term.

Bonchev and Klein [7] defined thorn trees, if the parent graph is a tree. If the parent graph is a path then particular thorn trees is called caterpillar, thorn star if parent graph is star and thorn cycle if parent graph is a cycle.

Let u_1 , u_2 , u_3 ,………., u_l, u_{l+1}, u_{l+2} are the vertices of path l+2 and T= $T(b_1, b_2, \ldots\ldots b_l)$ is caterpillar obtained by attaching b_i pendent vertices to the vertex u_{i+1}=1,2,…….,l

Theorem 3.5. For a thorn tree T= $T(b_1, b_2, \ldots\ldots b_l)$ of order $n \geq 3$,the Acharya polynomial is of the form

i) $AP(T, \lambda)=TW(T, \lambda)$, if $b_i \neq b_j$

ii) $AP(T, \lambda)=a_1\lambda+ a_2\lambda^2+ a_3\lambda^3+ \ldots\ldots\ldots + a_{l+1}\lambda^{l+1}$, if $b_i = b_j=b$

where

$$a_1 = l-1$$

$$a_2 = \sum_{i=1}^{l}\binom{b_i}{2} + b_1 + b_l + (l-2)$$

$$a_3 = \sum_{i=1}^{l-1}(b_i b_{i+1}) + (b_2 + b_{l-1}) + (l-3)$$

$$\vdots$$

$$\vdots$$

$$a_k = \sum_{i=1}^{l-k+2}(b_i b_{i+k-2}) + b_{k-1} + b_{l-k+2} + (l-k)$$

$$\vdots$$

$$\vdots$$

$$a_l = \sum_{i=1}^{2l}(b_i b_{i+l-2}) + b_l + b_1$$

$a_{l+1} = b_1 b_{l-1}+1$

Proof: *i*) Obvious, as all vertices are of different degrees except terminal vertices.

ii) Let A= $\{u_1, u_2, u_3, \ldots\ldots, u_l, u_{l+1}, u_{l+2}\}$, B= $\{ b_i / i=1,2,\ldots\ldots n\}$ for $i=2,3,\ldots\ldots l+1$

and $B = \bigcup_{i}^{l+1} B_i$

Let $d_A(G, b_i+2, k)$ = Number of pair of vertices in the set A of degree $b_i + 2$ at distance k,

$d_B(G, 1, k)$ = Number of pair of pendent vertices in the set B of at distance k.

Then it is clear that

$a_k = d_A(G, b_i+2, k) + d_B(G, 1, k)$

Computing in terms of these two terms in the above expression we get the coefficient a_i's given the statement of the theorem.

A thorn rod is caterpillar obtained by attaching b_i pendent vertices at each of the two rod ends.By taking path $l+2$ as in the above theorem we have

Theorem 3.6. For a thorn rod $T= T(b_1, 0, 0 \ldots\ldots b_l)$, the Acharya polynomial is of the form

$$i)\ AP(T,\lambda) = (l-1)\lambda + \left[\binom{b_1}{2} + \binom{b_l}{2}\right](l-2)\lambda^2 + (l-3)\lambda^3 \ldots\ldots + 3\lambda^{l-3} + 2\lambda^{l-2} + \lambda^{l-1} + b_1 b_l \lambda^{l+3} \qquad \text{if } b_1 \neq b_l$$

$$ii)\ AP(T,\lambda) = (l-1)\lambda + \left[\binom{b_1}{2} + \binom{b_l}{2}\right](l-2)\lambda^2 + (l-3)\lambda^3 \ldots\ldots + 3\lambda^{l-3} + 2\lambda^{l-2} + \lambda^{l-1} + \lambda^{l+1} + b_1 b_l \lambda^{l+3} \qquad \text{if } b_1 = b_l$$

Proof: *i*) Consider a path $l+2$, if $b_1 \neq b_l$ then except end vertices all other vertices are same degree 2.Finding the pair vertices at various distances we have the expression (*i*).

ii) If $b_1 = b_l = b$ then the end vertices are of same degrees ,computing distance between end vertices and internal vertices as in (*i*) we have the expression (*ii*).

The graph obtained by attaching b_i pendent vertices to *i*-th pendent vertex of the star graph $K_{1,n}$ is called as thorn star.

Theorem 3.7. For thorn star $K^*_{1,n}$ the Acharya polynomial with code $(b_1, b_2, \ldots\ldots b_l)$ is

i) $AP(K^*_{1,n},\lambda) = TW(K^*_{1,n},\lambda)$ if $b_i \neq b_j$

ii) $AP(K^*_{1,n},\lambda) = =a_1\lambda + a_2\lambda^2 + a_3\lambda^3 + a_4\lambda^4$ if $b_i = b_j = b$

where

$a_1 = 0$

$a_2 = \sum_{i=1}^{n} \binom{b_i}{2} + \binom{n}{2}$

$a_3 = 0$

$a_4 = \sum_{i=1, j=2}^{n} b_i b_j$

Proof: *i*) Obvious, as all vertices are of different degrees except terminal vertices.

ii) There is no pair of pendent vertices at distance 1 and 3 in $K^*_{1,n}$. . Therefore $a_1 = a_3 = 0$. And the distance between pendent vertices and vertices of parent graph is 2. Therefore

$a_2 = \sum_{i=1}^{n} \binom{b_i}{2} + \binom{n}{2}$

The distance between pendent vertices attached to v_i and v_j is 4. There are $b_i b_j$ such pair of vertices at distance 4.Therefore

$$a_4 = \sum_{i=1, j=2}^{n} b_i b_j$$

The graph obtained by attaching b_i pendent vebrtices to *i-th* vertex of n vertex cycle C_n ,called thorn ring

Theorem 3.8. For a thorn ring C_n^* , $n \geq 3$with code ($b_1, b_2, \ldots \ldots b_l$) the Acharya polynomial is of the form

$$i)\ AP(C_n^*, \lambda) = TW(C_n^*, \lambda) \quad \text{if} \quad b_i \neq b_j$$

$$ii)\ AP(C_n^*, \lambda) = a_1\lambda + a_2\lambda^2 + a_3\lambda^3 + \ldots\ldots\ldots + a_{\lfloor n/2 \rfloor + 2}\lambda^{\lfloor n/2 \rfloor + 2} \quad \text{if} \quad b_i = b_j = b$$

Where, if n is odd, then

$a_1 = n$

$$a_2 = \sum_{1}^{n} \binom{b_i}{2} + n$$

for $3 \leq k \leq \lfloor n/2 \rfloor + 2$

$$a_k = \sum_{i=1}^{n-k+2} b_i b_{i+k-2} + \sum_{i=n-k+3}^{n} b_i b_{i-n+k-2} + n$$

Where, if n is even, then

$a_1 = n$

$$a_2 = \sum_{1}^{n} \binom{b_i}{2} + n$$

for $3 \leq k \leq \lfloor n/2 \rfloor + 1$

$$a_k = \sum_{i=1}^{n-k+2} b_i b_{i+k-2} + \sum_{i=n-k+3}^{n} b_i b_{i-n+k-2} + n$$

Proof: *i*) Obvious, as all vertices are of different degrees except terminal vertices.

ii) The proof is similar to Theorem 3.7.

Corollarary 3.9. For a thorn ring C_n^* , $n \geq 3$ with code ($b_1, b_2, \ldots \ldots b_l$) the Acharya polynomial is of the form

$$AP(C_n^*, \lambda) = TW(C_n^*, \lambda) + H(G, \lambda) \quad \text{if} \quad b_i = b_j = b$$

II. Conclusion

In this paper we have worked on Acharya Polynomial of Thorn, Thorn Graph, Thorn Trees, Thorn Cycle and Thorn star for when all thorns attached to vertex are all equal and for all are unequal. Obtaining the Acharya Polynomial for different b_i is interesting to compute.

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