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An Inventory Model for Non-Instantaneous Decaying Items under Trade Credit Policy and Volume Flexibility

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I. Introduction

This paper considers the Economic Production Quantity (EPQ) for non-instantaneous deteriorating item allowing price discount in which production and demand rate are constant. The holding cost varies with time. Completely deteriorated units are discarded. Partially deteriorated items are allowed to carry discount, no shortage is allowed. Essentially, this chapter focuses on the conditions of the retailer receiving the supplier trade credit and providing the customer trade credit simultaneously so as to minimize the average total cost. The main contribution to literature is the inclusion of the facility of permissible delay in payments.

An economical production quantity (EPQ) model is an inventory control model that determines the amount of product to be produced on a single facility so as to meet a deterministic demand over an infinite planning horizon. In all inventory models a general assumption is that products generated have indefinitely long lives. In general, almost all items deteriorate over time. Often the rate of deterioration is low and there is little need to consider the deterioration in the determination of economic lot size. Nevertheless, there are many products in the real world that are subject to a significant rate of deterioration. Hence, the impact of product deterioration should not be neglected in the decision process of production lot size. Deterioration can be classified as age dependent on-going deterioration and age-independent on-going deterioration. Blood, fish and strawberries are some examples of commodities facing age-dependent on-going deterioration.

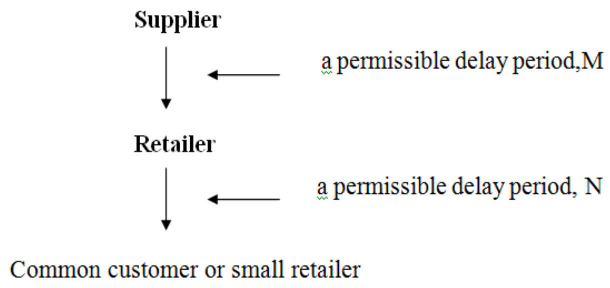
Volatile liquids such as alcohol and gasoline, radioactive chemicals, and grain products are examples of age-independent on-going deteriorating items. Legally these products do not have an expiry date; they can be stored indefinitely, though they suffer natural attrition while being held in inventory. In general, perishableness or deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of a commodity that results in decreasing usefulness from the original. The decrease or loss of utility due to decay is usually a function of the on-hand inventory. It is reasonable to note that a product may be understood to have a lifetime which ends when utility reaches zero.

Schweitzer and **Seidmann** adopted, for the first time, the concept of flexibility in the machine production rate and discussed optimization of processing rates for a FMS (flexible manufacturing system). Obviously, the machine production rate is a decision variable in the case of a FMS and then the unit production cost becomes a function of the production rate. **Khouja** extended the EPLS model to an imperfect production process with a flexible production rate. **Silver**, **Gallego** discussed the effects of slowing down production in the context of a manufacturing equipment of a family of items, assuming a common cycle for all the items. **Gallego** extended this model by removing the stipulation of a common cycle for all the items. But the above studies did not consider the demand rate to be variable. It is a common belief that large piles of goods displayed in a supermarket lead the customers to buy more. Volume flexibility is a major component in a FMS. The manufacturing flexibility which is capable of adjusting the production rate with the variability in the market demand is known as volume flexibility. **S. Sana** and **K. S. Chaudhuri**, consider a volume flexible manufacturing system for a deteriorating item with an inventory-level-dependent demand rate.

More recently, the supplier offers the retailer a trade credit period in a competitive market environment, to pay the cost of the supplied material. Usually, there is no charge if the outstanding amount is settled within the permitted fixed settlement period. Beyond this period, interest is charged. In recent research, the extensive use of trade credit as an alternative has been addressed by **Goyal [1985]** who developed an EOQ model under the conditions of permissible delay in payments. **Chung [1998]** then developed an alternative approach to the problem. Next, **Aggarwal et. al.[1995]** extended Goyal's model to allow for deteriorating items **Choi et. al. (1986),** developed a model determining the production rate for deteriorating items to minimize the total cost function over a finite planning period. **Raafat (1985),** extended the model, given in **Park (1983),** to deal with a case in which the finished product is also subject to a constant rate of decay. **Yang et. al. (2003),** considered a multi-lot-size production-inventory system for deteriorating items with constant production and demand rates. **Aggarwal et. al. (2001),** studied a model assuming that items are deteriorating at a constant rate, the demand rate is known and decreases negative exponentially, no shortages are allowed, and the production rate is known but can vary from one period to another over a finite planning period. **Yadav etl.(2013),(2014)** and **(2015)** study onVolume Flexibility in Production Model with Cubic Demand Rate Weibull Deterioration with partial backlogging , repairable items.

This paper incorporates the trade credit policy in the EPQ model for deteriorating products. In all the above developed models, it was assumed that the products obtained were from an outside supplier and the entire lot size was delivered at the same time. In fact, when a product can be produced in-house, the replenishment rate is also the production rate, and is hence finite.

A retailer for purchase from a supplier is that the supplier provides a delay period to the retailer, likewise, the retailer adopts the trade credit policy as well to stimulate the demand of the customer. The path of the trade credit policy is as follows:



Keeping this scenario in mind and this viewpoint can be found in **Huang (2003)**. Under these conditions, we want to investigate the retailer's replenishment policy under the conditions of the retailer receiving the supplier trade credit and providing the customer trade credit simultaneously.

In this paper an EPQ model for single product subject to non-instantaneous deterioration under a production-inventory policy in which holding cost varies with time is considered and production rate is variable. Each unit of the item is provided with price discount decayed units. In the next section, assumptions and notations that are employed for the development of the model are given. The optimal cycle time is derived.

II. Notation And Assumptions

The following notation and assumptions are used throughout the chapter.

S	Cost of Set up of a production run for the product.
D	Actual demand of the product given in the number of units per unit time.
K	Production rate per unit time. (K>D)
$h = \alpha + \beta t$	Inventory carrying cost per unit time. Where α and β are positive constants.
θ	A constant deterioration rate (unit time)
$\eta(K)$	Unit production cost $\left[\eta(K) = R + \frac{G}{K} + HK\right]$ is depends upon the production rate, where R, G and H are the material cost, labour cost and tool or dye cost respectively .
r	price discount per unit cost.
M	the retailer's trade credit period by the supplier.
N	the retailer's trade credit period by the retailer.
I_E	the annual interest that can be earned per unit.
I_P	the annual interest charge that payable per unit when $I_P > I_E$.
T	optimal cycle time.
T_1	Production period.
T_2	Time during there is no production of the product i.e. $T_1 = T - T_2$.
$I_1(t)$	the inventory level at time t $(0 \le t \le T_1)$ in which the product has no deterioration.
$I_2(t)$	the inventory level at time t $(0 \le t \le T_2)$ in which there is no production
I (M)	Maximum inventory level of the product.

TVC(T) the total relevant inventory cost per unit time of inventory system.

In addition, the inventory model is developed under the following assumptions:

(1) A single non-instantaneous deteriorating item is modeled and the demand rate for the product is known and finite.

(2) An infinite planning horizon is assumed.

(3) Shortages are not allowed.

(4) Inventory holding cost is charged only to the amount of undecayed stock.

(5) The production rate of product is variable. The machine has large enough capacity to produce all the items to meet the demand of all products.
(6) Once a unit of a product is produced, it is available to meet the demand.
(7) Once the production is terminated, the product starts deterioration and the price discount is considered.
(9) The time to deterioration of the product follows an exponential distribution.
(10) There is no replacement or repair for a deteriorated item.
(11) The production lot size is unknown but it will not vary from one cycle to another.
(12) The cost of deterioration unit is known and includes any disposal cost orsalvage value.
(13) Customer is given credit period and the credit period is less than or equal to production cycle time.

III. Model Formulation

In this chapter, at start t = 0, the inventory level is zero. The production and supply start simultaneously and the production stops at t =T_1 at which the maximum inventory $I(T_1)$ is reached. The inventory built up at a rate is $K-D$ in the interval [0, T_1] and there is no deterioration. After the time T_1, the maximum inventory is reached, production is terminated and the deterioration starts. From this point, the on-hand inventory diminishes to extend of the demand plus the loss due to the deterioration. Production will be resumed when all on hand inventories are depleted at time T. Then an identical production run will begin. Since an exponential deterioration process is assumed, the inventory level of the system for the product at time t over period [0,T] can be the represented by the differential equations: when the inventory reduces to zero level and production run begins.

$$\frac{dI_1(t)}{dt} = K - D, \qquad for \quad 0 \le t \le T_1 \qquad \text{……(1)}$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -D, \qquad for \quad 0 \le t \le T_2 \qquad \text{……(2)}$$

Boundary conditions, $I_1(0) = 0$ and $I_2(T_2) = 0$.
Solve equation (1) and (2),

$$I_1(t) = (K-D)t \qquad for \quad 0 \le t \le T_1 \qquad \text{……(3)}$$

and

$$I_2(t) = \frac{(-D)}{\theta} + C_2\, e^{-t}\theta$$

$$Apply\, the\, boundary\, Condition\, I_2(T_2) = 0\, then$$

$$we\, get, \qquad C_2 = \frac{De^{\theta T_2}}{\theta}\theta$$

$$Hence$$

$$I_2(t) = \frac{D}{\theta}\left(e^{\theta T_2} - e^{\theta t}\right)e^{-\theta t}$$

$$= \frac{D}{\theta}\left(e^{\theta(T_2 - t)} - 1\right), \qquad for \quad 0 \le t \le T_2 \qquad \text{……(4)}$$

The total relevant inventory cost per unit time consists of the following five elements.

(1) **Holding Cost:** The holding cost per unit time is given by

$$HC = \frac{1}{T}\left[\int_0^{T_1} (\alpha+\beta t) I_1(t)\,dt + \int_0^{T_2} (\alpha+\beta t) I_2(t)\; dt\right]$$

$$= \frac{1}{T}\left[\int_0^{T_1} (\alpha+\beta t)(\mathrm{K}-D)t\,dt + \int_0^{T_2} (\alpha+\beta t)\frac{D}{\theta}(e^{\theta(T_2-t)}-1)\;\; dt\right]$$

Assuming tθ < 1, an approximate value is got by neglecting those items of degree greater than or equal to 2 in t θ.

$$= \frac{1}{T}\left[(\mathrm{K}-D)\int_0^{T_1} (\alpha t+\beta t^2)\,dt + D\int_0^{T_2} (\alpha+\beta t)(T_2-t)\;\; dt\right]$$

$$= \alpha(\mathrm{K}-D)\frac{T_1^2}{2T} + \beta(\mathrm{K}-D)\frac{T_1^3}{3T} + \frac{\alpha D T_2^2}{2T} + \frac{\beta D T_2^3}{6T} \qquad \ldots\ldots(5)$$

(2) **Production Cost:** The production cost per unit time is given by

PC $=\mathrm{K}\,\eta(\mathrm{K})\,\frac{T_1}{T}$

$=\mathrm{K}\,(R+\frac{G}{K}+HK)\frac{T_1}{T}$ ……(6)

(3) **Setup Cost:** The setup cost per unit time is given by

SC $= \frac{S}{T}$ ……(7)

(4) **Deterioration Cost:** The number of units that deteriorate in a cycle is the difference between the maximum inventory and the number of units used to meet the demand, Hence the deterioration cost per unit time is given as

$$DC = \frac{\eta(\mathrm{K})}{T}\left[I_2(0) - \int_0^{T_2} D dt\right]$$

$$DC = \frac{(R+\frac{G}{K}+HK)}{T}\left[\frac{D}{\theta}\left(e^{\theta(T_2)}-1\right) - \int_0^{T_2} D dt\right]$$

Assuming tθ < 1, an approximate value is got by neglecting those items of degree greater than or equal to 2 in tθ in Tailors expansion of the exponential functions.

$$DC = \frac{(R+\frac{G}{K}+HK)}{T}\left[\frac{D}{\theta}\left(1+\theta T_2+\frac{(\theta T_2)^2}{2}+\ldots\ldots\ldots\ldots\ldots-1\right) - DT_2\right]$$

$$DC = \frac{(RK+G+HK^2)D\theta T_2^{\,2}}{2KT}$$

……(8)

(5) **Price discount:** Price discount is offered as a fraction of production cost for the units in the period [0, T_2]

PD $= \frac{\eta(K)\mathrm{r}}{T}\int_0^{T_2} D\,dt \quad = \frac{(RK+G+HK^2)rDT_2}{KT}$ ……(9)

The total average cost per unit time can be formulated as

TVC = HC+PC +SC +DC+PD

$$= \alpha(\mathrm{K}-D)\frac{T_1^2}{2T} + \beta(\mathrm{K}-D)\frac{T_1^3}{3T} + \frac{\alpha D T_2^2}{2T} + \frac{\beta D T_2^3}{6T}$$

$$+ K(R + \frac{G}{K} + HK)\frac{T}{T_1} + \frac{S}{T} + \frac{(RK+G+HK^2)D\theta T_2^2}{2KT} + \frac{(RK+G+HK^2)rDT_2}{KT}$$

……(10) To minimize the total cost per unit time TVC to express in terms of T,K in equation (3.10) so that there is only two variable T,K in the equation. At the moment when production run is terminated during a cycle.

$$I_1(T_1) = I_2(0)$$

$$(\text{K-D})\ T_1 = \frac{D}{\theta}\left[e^{(\theta T_2)} - 1\right]$$

$$(\text{K-D})\ \mathrm{T}_1 = D\left(T_2 + \frac{\theta T_2^2}{2} + ...\right)$$

$$T_1 = \frac{\mathrm{D}T_2}{(\text{K-D})}$$

$$and\ \ T = \mathrm{T}_1 + T_2$$

$$= \frac{\mathrm{D}T_2}{(\text{K-D})} + T_2$$

$$T_2 = \frac{(\text{K-D})\mathrm{T}}{K} \quad ……(11)$$

$$T_1 = \frac{\mathrm{DT}}{K} \quad ……(12)$$

Substitute the value of T_1 and T_2 in equation (10)

$$TVC = \left[\begin{array}{l} \alpha(\mathrm{K}-D)\frac{D^2T}{2K^2} + \beta(\mathrm{K}-D)\frac{D^3T^2}{3K^3} + \frac{\alpha DT(\mathrm{K}-D)^2}{2K^2} + \frac{\beta DT^2(\mathrm{K}-D)^3}{6K^3} \\ +\ (RK+G+HK^2)\frac{D}{K} + \frac{S}{T} + \frac{(RK+G+HK^2)D\theta T(\mathrm{K}-D)^2}{2K^3} + \\ \frac{(RK+G+HK^2)rD(\mathrm{K}-D)}{K^2} \end{array}\right]$$

……(13)

There are three cases to discuss the annual capital opportunity Cost.

Case 1: M $\leq$ T

The optimal production cycle time is greater than the credit period, the loss of revenue to the supplier by way of interest.

$$I_{E1} = \frac{kI_E}{T}\int_N^M Dtdt$$

$$= \frac{kI_E D}{2T}(M^2 - N^2) \quad …….(14)$$

The interest payable by the customer to the supplier per unit time on the assumption that the customer has to pay the entire amount before the next production.

$$IP = \frac{\eta(K)I_P}{T}\int_M^{T_2} I_2(t)\ dt$$

$$= \frac{(RK+G+HK^2)I_P}{KT}\int_M^{T_2}\frac{D}{a}\left(e^{\theta(T_2-t)}-1\right)\quad dt$$

$$= \frac{(RK+G+HK^2)I_P D}{\theta KT}\int_M^{T}\left(1+\theta(T-t)+............-1\right)dt$$

$$= \frac{(RK+G+HK^2)I_P D}{KT}\int_M^T (T-t)dt$$

$$= \frac{(RK+G+HK^2)I_P D}{KT}\left(Tt-\frac{t^2}{2}\right)_M^T = \frac{(RK+G+HK^2)I_P D}{KT}\left(\frac{T^2}{2}+\frac{M^2}{2}-TM\right)$$

$$= \frac{(RK+G+HK^2)I_P D}{2K}\left(T-2M+\frac{M^2}{T}\right) \qquad ……(15)$$

Case 2: N $\leq$ T $\leq$ M

Customer need not pay interest. The annual opportunity cost:

$$I_{E2} = \frac{(RK+G+HK^2)I_E}{KT}\left[\int_N^T Dtdt+\int_T^M DTdt\right]$$

$$= \frac{(RK+G+HK^2)I_E D}{2KT}\left[T^2-N^2+2TM-2T^2\right]$$

$$= \frac{(RK+G+HK^2)I_E D}{2KT}\left[-N^2+2TM-T^2\right]$$

$$= \frac{kI_E D}{2}\left[-\frac{N^2}{T}+2M-T\right] \qquad ……(16)$$

Case 3: T $\leq$ N

The annual opportunity cost:

$$I_{E3} = \frac{\eta(K)I_E}{T}\left[\int_N^M DTdt\right]$$

$$= (R+\frac{G}{K}+HK\ \ )I_E D\left[M-N\right]$$

……(17)

Therefore the total variable cost function per unit TVC is

$$TVC = \begin{cases} TVC_1 & if\ \ M\leq T \\ TVC_2 & if\ \ N\leq T\leq M \\ TVC_3 & if\ N\geq T \end{cases}$$

Where

$TVC_1 =$ HC+PC +SC +DC+PD + IP-I_{E1}

$$TVC_1 = \left[\begin{array}{l} \alpha(K-D)\dfrac{D^2T}{2K^2}+\beta(K-D)\dfrac{D^3T^2}{3K^3}+\dfrac{\alpha DT(K-D)^2}{2K^2} \\ +\dfrac{\beta DT^2(K-D)^3}{6K^3}+\left(R+\dfrac{G}{K}+HK\right)D+\dfrac{S}{T}+ \\ \dfrac{(RK+G+HK^2)D\theta T(K-D)^2}{2K^3}+\dfrac{(RK+G+HK^2)rD(K-D)}{K^2} \\ +\dfrac{(RK+G+HK^2)I_PD}{2K}\left(T-2M+\dfrac{M^2}{T}\right)- \\ \dfrac{(RK+G+HK^2)I_ED}{2KT}(M^2-N^2) \end{array}\right]$$

And

TVC_2 = HC+PC +SC +DC+PD -I_{E2}

$$= \left[\begin{array}{l} \alpha(K-D)\dfrac{D^2T}{2K^2}+\beta(K-D)\dfrac{D^3T^2}{3K^3}+\dfrac{\alpha DT(K-D)^2}{2K^2} \\ +\dfrac{\beta DT^2(K-D)^3}{6K^3}+\dfrac{(RK+G+HK^2)}{K}\;D+\dfrac{S}{T}+\dfrac{(RK+G+HK^2)D\theta T(K-D)^2}{2K^3} \\ +\dfrac{(RK+G+HK^2)rD(K-D)}{K^2}-\dfrac{(RK+G+HK^2)I_ED}{2K}\left[-\dfrac{N^2}{T}+2M-T\right] \end{array}\right]$$

And again

TVC_3 = HC+PC +SC +DC+PD -I_{E3}

=

$$\left[\begin{array}{l} \alpha(K-D)\dfrac{D^2T}{2K^2}+\beta(K-D)\dfrac{D^3T^2}{3K^3}+\dfrac{\alpha DT(K-D)^2}{2K^2} \\ +\dfrac{\beta DT^2(K-D)^3}{6K^3}+(RK+G+HK^2)\;\dfrac{D}{K}+\dfrac{S}{T}+\dfrac{(RK+G+HK^2)D\theta T(K-D)^2}{2K^3} \\ +\dfrac{(RK+G+HK^2)rD(K-D)}{K^2}-\left(R+\dfrac{G}{K}+HK\right)I_ED\left[M-N\right] \end{array}\right]$$

The objective of the study to determine the optimal production policy for minimizing the total variable cost per unit time.

For this the first derivative of TVC_1 with respect to T,K equating equal to zero :

$$\frac{\partial TVC_1}{\partial T}=\begin{bmatrix}-\frac{S}{T^2}+\alpha(K-D)\frac{D^2}{2K^2}+2\beta(K-D)\frac{D^3T}{3K^3}+\frac{\alpha D(K-D)^2}{2K^2}\\ +\frac{\beta DT(K-D)^3}{3K^3}+\frac{(RK+G+HK^2)D\theta(K-D)^2}{2K^3}+\\ \frac{(RK+G+HK^2)I_PD}{2K}\left(1-\frac{M^2}{T^2}\right)\\ +\frac{(RK+G+HK^2)I_ED}{2KT^2}(M^2-N^2)\end{bmatrix}=0 \quad \ldots(18)$$

$$\frac{\partial TVC_1}{\partial K}=\begin{bmatrix}\alpha(-K+2D)\frac{D^2T}{2K^3}+\beta(-2K+3D)\frac{D^3T^2}{3K^4}+(-G+HK^2)\ \frac{D}{K^2}+\\ \frac{\alpha TD^2(K-D)}{K^3}+\frac{\beta D^2T^2(K-D)^2}{2K^4}+\frac{\{HK^3-(G-DR)K^2+2DG\}rD}{K^3}\\ \frac{D\theta\{HK^4-(G-2DR+HD^2)K^2-2(D^2R-2DG)K-3GD^2\}}{2K^4}+\\ \frac{(K^2H-G)I_PD}{2K^2}\left(T-2M+\frac{M^2}{T}\right)-\frac{(K^2H-G)I_ED}{2TK^2}(M^2-N^2)\end{bmatrix}=0. \quad \ldots(19)$$

and the second derivative of TVC_1 with respect to T,K is given by:

$$\frac{\partial^2 TVC_1}{\partial T^2}=\begin{bmatrix}\frac{2S}{T^3}+2\beta(K-D)\frac{D^3}{3K^3}++\frac{\beta D(K-D)^3}{3K^3}\\ \frac{M^2(RK+G+HK^2)I_PD}{KT^3}-\frac{(RK+G+HK^2)I_ED}{KT^3}(M^2-N^2)\end{bmatrix}$$

$$=\begin{bmatrix}\frac{2S}{T^3}+2\beta(K-D)\frac{D^3}{3K^3}++\frac{\beta D(K-D)^3}{3K^3}\\ \frac{M^2(RK+G+HK^2)D}{KT^3}(I_P-I_E)+\frac{(RK+G+HK^2)I_ED}{KT^3}N^2\end{bmatrix}>0$$

$$\frac{\partial^2 TVC_1}{\partial K^2}=\begin{bmatrix}\alpha(K-3D)\frac{D^2T}{K^4}+2\beta(K-2D)\frac{D^3T^2}{K^5}+\\ \frac{\alpha TD^2(-2K+3D)}{K^4}+\frac{\beta D^2T^2(3DK-K^2-2D^2)}{K^5}+\frac{2G}{K^3}D\\ \frac{D\theta\{(G-2DR+HD^2)K^2+4(D^2R-2DG)K+6GD^2\}}{K^5}+\\ \frac{\{(G-DR)K^2-6DG\}rD}{K^4}+\frac{GI_PD}{K^3}\left(T-2M+\frac{M^2}{T}\right)-\frac{GI_ED}{TK^3}(M^2-N^2)\end{bmatrix}$$

For optimal cost

$\frac{\partial^2 TVC_1}{\partial K^2} > 0$, $\frac{\partial^2 TVC_1}{\partial T^2} > 0$ and $\frac{\partial^2 TVC_1}{\partial K^2} . \frac{\partial^2 TVC_1}{\partial T^2} - \frac{\partial^2 TVC_1}{\partial K \, \partial T} > 0$.

and again first derivative of TVC_2 with respect to T,K equating equal to zero:

$$\frac{\partial TVC_2}{\partial T} = \begin{bmatrix} -\frac{S}{T^2} + \alpha(K-D)\frac{D^2}{2K^2} + 2\beta(K-D)\frac{D^3 T}{3K^3} + \frac{\alpha D(K-D)^2}{2K^2} \\ +\frac{\beta DT(K-D)^3}{3K^3} + \frac{(RK+G+HK^2)D\theta(K-D)^2}{2K^3} - \\ \frac{(RK+G+HK^2)I_E D}{2K}(\frac{N^2}{T^2}-1) \end{bmatrix} = 0. \quad \ldots(20)$$

$$\frac{\partial TVC_2}{\partial K} = \begin{bmatrix} \alpha(-K+2D)\frac{D^2 T}{2K^3} + \beta(-2K+3D)\frac{D^3 T^2}{3K^4} + \\ \frac{\alpha TD^2(K-D)}{K^3} + \frac{\beta D^2 T^2 (K-D)^2}{2K^4} + (-G+HK^2)\frac{D}{K^2} + \\ \frac{D\theta\{HK^4 - (G-2DR+HD^2)K^2 - 2(D^2R - 2DG)K - 3GD^2\}}{2K^4} \\ + \frac{\{HK^3 - (G-DR)K^2 + 2DG\}rD}{K^3} - \frac{(K^2H-G)I_E D}{2K^2}\left[-\frac{N^2}{T} + 2M - T\right] \end{bmatrix} = 0. \quad \ldots(21)$$

and the second derivative of TVC_2 with respect to T,K is given by:

$$\frac{\partial^2 TVC_2(T)}{\partial T^2} = \left[\frac{2S}{T^3} + 2\beta(K-D)\frac{D^3}{3K^3} + \frac{\beta D(K-D)^3}{3K^3} + \frac{(RK+G+HK^2)I_E DN^2}{KT^3}\right]$$

$$\frac{\partial^2 TVC_2(T)}{\partial K^2} = \begin{bmatrix} \alpha(K-3D)\frac{D^2 T}{K^4} + 2\beta(K-2D)\frac{D^3 T^2}{K^5} + \frac{2G}{K^3}D \\ \frac{\alpha TD^2(-2K+3D)}{K^4} + \frac{\beta D^2 T^2(3DK - K^2 - 2D^2)}{K^5} + \\ \frac{D\theta\{(G-2DR+HD^2)K^2 + 4(D^2R-2DG)K + 6GD^2\}}{K^5} + \\ \frac{\{(G-DR)K^2 - 6DG\}rD}{K^4} - \frac{GI_E D}{TK^3}\left[-\frac{N^2}{T} + 2M - T\right] \end{bmatrix}$$

$\frac{\partial^2 TVC_2}{\partial K^2} > 0$, $\frac{\partial^2 TVC_2}{\partial T^2} > 0$ and $\frac{\partial^2 TVC_2}{\partial K^2} . \frac{\partial^2 TVC_2}{\partial T^2} - \frac{\partial^2 TVC_2}{\partial K \, \partial T} > 0$.

Taking the first derivative of $TVC_3(T)$ with respect to T,K equal to zero :

$$\frac{\partial TVC_3}{\partial T} = \begin{bmatrix} -\frac{S}{T^2} + \alpha(\mathrm{K}-D)\frac{D^2}{2K^2} + 2\beta(\mathrm{K}-D)\frac{D^3T}{3K^3} + \frac{\alpha D(\mathrm{K}-D)^2}{2K^2} \\ + \frac{\beta DT(\mathrm{K}-D)^3}{3K^3} + \frac{(RK+G+HK^2)D\theta(\mathrm{K}-D)^2}{2K^3} \end{bmatrix} = 0.$$

…(22)

$$\frac{\partial TVC_3}{\partial K} = \begin{bmatrix} \alpha(-\mathrm{K}+2D)\frac{D^2T}{2K^3} + \beta(-2\mathrm{K}+3D)\frac{D^3T^2}{3K^4} + \\ \frac{\alpha TD^2(\mathrm{K}-D)}{K^3} + \frac{\beta D^2T^2(\mathrm{K}-D)^2}{2K^4} + (-G+HK^2)\ \frac{D}{K^2} + \\ \frac{D\theta\{\mathrm{HK}^4-(G-2DR+HD^2)\mathrm{K}^2-2(\mathrm{D}^2\mathrm{R}-2\mathrm{DG})\mathrm{K}-3\mathrm{GD}^2\}}{2K^4} \\ + \frac{\{HK^3-(\mathrm{G}-\mathrm{DR})\mathrm{K}^2+2DG\}rD}{K^3} - \frac{(K^2H-G)I_ED}{2K^2}\left[M-N\right] \end{bmatrix} = 0.$$

…(23)

and the second derivative of $TVC_3(T)$ with respect to T is given by:

$$\frac{\partial^2 TVC_3}{\partial T^2} = \frac{2S}{T^3} + 2\beta(\mathrm{K}-D)\frac{D^3}{3K^3} + \frac{\beta D(\mathrm{K}-D)^3}{3K^3}\ .$$

$$\frac{\partial^2 TVC_3}{\partial K^2} = \begin{bmatrix} \alpha(\mathrm{K}-3D)\frac{D^2T}{K^4} + 2\beta(\mathrm{K}-2D)\frac{D^3T^2}{K^5} + \frac{2G}{K^3}D \\ \frac{\alpha TD^2(-2\mathrm{K}+3D)}{K^4} + \frac{\beta D^2T^2(3\mathrm{DK}-K^2-2D^2)}{K^5} + \\ \frac{D\theta\{(G-2DR+HD^2)\mathrm{K}^2+4(\mathrm{D}^2\mathrm{R}-2\mathrm{DG})\mathrm{K}+6\mathrm{GD}^2\}}{K^5} + \\ \frac{\{(\mathrm{G}-\mathrm{DR})\mathrm{K}^2-6DG\}rD}{K^4} - \frac{GI_ED}{TK^3}\left[M-N\right] \end{bmatrix}$$

$\frac{\partial^2 TVC_3}{\partial K^2} > 0$, $\frac{\partial^2 TVC_3}{\partial T^2} > 0$ and $\frac{\partial^2 TVC_3}{\partial K^2}.\frac{\partial^2 TVC_3}{\partial T^2} - \frac{\partial^2 TVC_3}{\partial K \partial T} > 0$

IV. Conclusion

Optimal production cycle and production rate can be find out by solving non- linear equation separately in three cases .For case 1 we can solve (18),(19) by suitable numerical method and can determined optimal value of variable associated in total cost in order to minimize total cost. For case 2 we can solve (20),(21) by suitable numerical method and can determined optimal value of variable associated in total cost in order to minimize total cost. And for case 3 we can solve (22),(23) by suitable numerical method and can determined optimal value of variable associated in total cost in order to minimize total cost.

In last studying the three cases we can find out the optimal production policies for minimum total average cost. In this chapter, an EPQ model for a single machine single product system in which the product deteriorates non-instantaneously receives the price discount, the purchaser receives credit period from the supplier has been developed. In this chapter we reduces the production cycle time and maximize the total profit and it also presents a volume flexible inventory model that incorporates some realistic features. First, an item is deteriorated over time and follows an exponential distribution that makes a broader application scope. Second, the retailer receives the supplier trade credit and provides the customer traded credit simultaneously. In the proposed model, we explore the fact that there exists a unique optimal replenishment time to minimize the total variable cost per unit time. This paper helps to reduce the total cost for non-instantaneous deterioration.

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