IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 13, Issue 5 Ver. I1 (Sep. - Oct. 2017), PP 12-20
www.iosrjournals.org

On Fuzzy Pseudo- Continuous Functions

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Abstract: *This paper introduces the concepts of fuzzy pseudo-open sets in fuzzy topological spaces and fuzzy pseudo-continuous functions between fuzzy topological spaces. Several characterizations of fuzzy pseudo-open sets are established. A condition under which fuzzy hyper-connected spaces become fuzzy Baire spaces, is obtained by means of fuzzy pseudo-open sets. Several examples are given to illustrate the concepts introduced in this paper.*

Keywords: *Fuzzy simply open set, fuzzy simply* open set, fuzzy resolvable set, fuzzy hyper-connected space, fuzzy Baire space, fuzzy resolvable space.*

Date of Submission: 02-09-2017 Date of acceptance: 20-09-2017

I. Introduction

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by **L.A. Zadeh** [19] in 1965. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. In 1968 **C. L. Chang** [4], applied basic concepts of general topology to fuzzy sets and introduced fuzzy topology. .The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In the recent years, there has been a growing trend among many fuzzy topologists to introduce and study different forms of fuzzy sets and a considerable amount of research has been done on many types of fuzzy continuity in fuzzy topology.

In 1899, **Rene Louis Baire [3]** introduced the concepts of first and second category sets in his doctoral thesis. By means of first category sets, the notion of pseudo-open sets was introduced and studied by **A. Neubrunnova [6]** in classical topology. The purpose of this paper is to introduce and study fuzzy pseudo-open sets in fuzzy topological spaces. A new class of functions, called fuzzy pseudo-continuous functions between fuzzy topological spaces, is introduced and studied. It is observed that the fuzzy pseudo –open sets in fuzzy hyper-connected spaces are fuzzy simply open sets and fuzzy resolvable sets. A condition under which fuzzy hyper-connected spaces become fuzzy Baire spaces, is also obtained by means of fuzzy pseudo-open sets. Several examples are given to illustrate the concepts introduced in this paper.

II. Preliminaries

In order to make the exposition self- contained, some basic notions and results used in the sequel are given. In this work (X,T) or simply by X, we will denote a fuzzy topological space due to **Chang** (1968). Let X be a non- empty set and I, the unit interval [0,1]. A fuzzy set λ in X is a function from X into I. The null set 0 is the function from X into I which assumes only the value 0 and the whole fuzzy set 1 is the function from X into I which takes 1 only.

Definition 2.1 [4]: Let (X,T) be a fuzzy topological space and λ be any fuzzy set defined on X. The interior and the closure of λ are defined respectively as follows: (i). $\mathrm{int}(\lambda) = \vee\{\mu / \mu \leq \lambda, \mu \in T\}$ (ii). $\mathrm{cl}(\lambda) = \wedge\{\mu / \lambda \leq \mu, 1 - \mu \in T\}$.

Lemma 2.1 [1]: For a fuzzy set λ of a fuzzy topological space X,
(i). $1 - \mathrm{int}(\lambda) = \mathrm{cl}(1 - \lambda)$, (ii). $1 - \mathrm{cl}(\lambda) = \mathrm{int}(1 - \lambda)$.

Definition 2.2 [8]: A fuzzy set λ in a fuzzy topological space (X,T), is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $\mathrm{cl}(\lambda) = 1$, in (X,T).

Definition 2.3 [8]: A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < \mathrm{cl}(\lambda)$. That is, $\mathrm{int}\,\mathrm{cl}(\lambda) = 0$, in (X,T).

Definition 2.4 [2] : A fuzzy set λ in a fuzzy topological space (X,T) is called

(i). a fuzzy G_δ - set in (X, T) if $\lambda = \bigwedge_{i=1}^{\infty} (\lambda_i)$, where $\lambda_i \in T$ for $i \in I$.
(ii). a fuzzy F_σ - set in (X, T) if $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$, where $1 - \lambda_i \in T$ for $i \in I$.

Definition 2.5 [7]: Let λ be a fuzzy set in a fuzzy topological space (X,T). The fuzzy boundary of λ is defined as $Bd(\lambda) = cl(\lambda) \wedge cl(1 - \lambda)$.

Definition 2.6 [16] : A fuzzy set λ in a fuzzy topological space (X,T), is called a fuzzy simply open set if Bd(λ) is a fuzzy nowhere dense set in (X,T).

Definition 2.7 [8]: A fuzzy set λ in a topological space (X,T) is called a fuzzy first category set if $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)'$s are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category.

Definition 2.8 [10]: Let λ be a fuzzy first category set in a fuzzy topological space in (X,T). Then, $1 - \lambda$ is called a fuzzy residual set in (X,T).

Definition 2.9 [17]: A fuzzy set λ in a fuzzy topological space (X,T), is called a fuzzy simply* open set if $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X,T) and $1 - \lambda$ is called a fuzzy simply* closed set in (X,T).

Definition 2.10 [12]: A fuzzy set λ in a fuzzy topological space (X,T), is called a fuzzy σ-nowhere dense set if λ is a fuzzy F_σ-set in (X,T) with $int(\lambda) = 0$.

Definition 2.11 [12]: A fuzzy set λ in a topological space (X,T) is called a fuzzy σ -first category set if $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)'$s are fuzzy σ-nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy σ -second category.

Definition 2.12 [18]: Let (X,T) be a fuzzy topological space. A fuzzy set λ is called a fuzzy resolvable set if for each fuzzy closed set μ in (X,T), $cl(\mu \wedge \lambda) \wedge cl(\mu \wedge (1 - \lambda))$ is a fuzzy nowhere dense in (X,T).

III. Fuzzy Pseudo –Open Sets

Definition 3.1: Let (X,T) be a fuzzy topological space. A fuzzy set λ defined on X is called a fuzzy pseudo-open set in (X,T) if $\lambda = \mu \vee \delta$, where μ is a non-zero fuzzy open set in (X,T) and δ is a fuzzy first category set in (X,T).

Example 3.1 : Let $X = \{a, b, c\}$. Consider the fuzzy sets α, β, δ, γ, σ, ω, μ and θ defined on X as follows:

$\alpha : X \rightarrow [0, 1]$ is defined as $\alpha(a) = 0.5$; $\alpha(b) = 1$; $\alpha(c) = 0.4$,
$\beta : X \rightarrow [0, 1]$ is defined as $\beta(a) = 1$; $\beta(b) = 0.6$; $\beta(c) = 0.1$,
$\delta : X \rightarrow [0, 1]$ is defined as $\delta(a) = 0.6$; $\delta(b) = 0.4$; $\delta(c) = 1$,
$\gamma : X \rightarrow [0, 1]$ is defined as $\gamma(a) = 1$; $\gamma(b) = 0.6$; $\gamma(c) = 0.9$,
$\mu : X \rightarrow [0, 1]$ is defined as $\mu(a) = 0.6$; $\mu(b) = 0.6$; $\mu(c) = 0.9$,
$\sigma : X \rightarrow [0, 1]$ is defined as $\sigma(a) = 0.5$; $\sigma(b) = 1$; $\sigma(c) = 0.9$,
$\omega : X \rightarrow [0, 1]$ is defined as $\omega(a) = 0.6$; $\omega(b) = 1$; $\omega(c) = 0.9$,
$\theta : X \rightarrow [0, 1]$ is defined as $\theta(a) = 1$; $\theta(b) = 0.6$; $\theta(c) = 0.4$.

Then, $T = \{ 0, \alpha, \beta, \delta, \alpha \vee \beta, \alpha \vee \delta, \beta \vee \delta, \alpha \wedge \beta, \alpha \wedge \delta, \beta \wedge \delta, (\alpha \vee [\beta \wedge \delta]), (\beta \vee [\alpha \wedge \delta]), (\delta \vee [\alpha \wedge \beta]), \alpha \wedge [\beta \vee \delta], \beta \wedge [\alpha \vee \delta], \delta \wedge [\alpha \vee \beta], [\alpha \wedge \beta \wedge \delta] ,1\}$ is a fuzzy topology on X. On computation, the fuzzy nowhere dense sets in (X,T), are $1 - \alpha$, $1 - \beta$, $1 - \delta$, $1 - [\alpha \vee \beta]$, $1 - [\alpha \vee \delta]$, $1 - [\beta \vee \delta]$, $1 - [\beta \wedge \delta]$, $1 - (\alpha \vee [\beta \wedge \delta])$, $1 - (\beta \vee [\alpha \wedge \delta])$, $1 - (\delta \vee [\alpha \wedge \beta])$, $1 - (\beta \wedge [\alpha \vee \delta])$, $1 - (\delta \wedge [\alpha \vee \beta])$, $1 - \gamma$, $1 - \sigma$, $1 - \omega$, $1 - \mu$ and $1 - \theta$ and $1 - (\alpha \wedge \beta \wedge \delta)$ is a fuzzy first category set in (X,T). The fuzzy pseudo-open sets in (X,T), are $1 - (\alpha \wedge \beta \wedge \delta)$, $\alpha \vee \delta$, $\beta \vee \delta$, $(\delta \vee [\alpha \wedge \beta])$, μ , γ , θ, σ and ω.

Remark 3.1 : A fuzzy pseudo- open set need not be a fuzzy open set in a fuzzy topological space. For, in example 3.1, μ , γ , θ, σ and ω are fuzzy pseudo-open sets in (X,T), but μ , γ , θ, σ and ω are not fuzzy open sets in (X,T).

Proposition 3.1: Let (X,T) be a fuzzy topological space. If λ is a fuzzy pseudo-open set in (X,T) ,then

(i). $Int(\lambda) \neq 0$, in (X,T).
(ii). λ is not a fuzzy nowhere dense set in (X,T).
(iii). $Cl(1 - \lambda) \neq 1$, in (X,T).

Proof : (i). Let λ be a fuzzy pseudo-open set in (X,T).Then $\lambda = \mu \vee \delta$, where μ is a non-zero fuzzy open set in (X,T) and δ is a fuzzy first category set in (X,T). Then, $int(\lambda) = int(\mu \vee \delta) \geq int(\mu) \vee int(\delta) \geq \mu \vee int(\delta) \geq \mu \neq 0$ and hence $int(\lambda) \neq 0$, in (X,T).

(ii). Since $int(\lambda) \leq int\, cl(\lambda)$ and $int(\lambda) \neq 0$, for a fuzzy pseudo-open set in (X,T) (by (i)), $int\, cl(\lambda) \neq 0$ and hence λ is not a fuzzy nowhere dense set in (X,T).

(iii). Now $cl(1-\lambda) = 1 - int(\lambda) \neq 1$, in (X,T) (by (i)) and hence $cl(1-\lambda) \neq 1$, in (X,T).

Proposition 3.2 : If the fuzzy set λ on X is a fuzzy pseudo-open set in a fuzzy topological space (X,T) , then $\lambda = \vee_{i=1}^{\infty}(\alpha_i)$, where (α_i)'s are fuzzy simply* open sets in (X,T).

Proof : Let λ be a fuzzy pseudo-open set in (X,T). Then, $\lambda = \mu \vee \delta$, where μ is a non-zero fuzzy open set in (X,T) and δ is a fuzzy first category set in (X,T). Now $\lambda = \mu \vee \delta = \mu \vee [\vee_{i=1}^{\infty}(\delta_i)]$,where $(\delta_i)'s$ are fuzzy nowhere dense sets in (X,T). Then, $\lambda = [\vee_{i=1}^{\infty}(\mu \vee \delta_i)]$. Since μ is a non-zero fuzzy open set in (X,T) and $(\delta_i)'s$ are fuzzy nowhere dense sets in (X,T), $(\mu \vee \delta_i)'s$ are fuzzy simply* open sets in (X,T). Let $\alpha_i = \mu \vee \delta_i$. Then $\lambda = \vee_{i=1}^{\infty}(\alpha_i)$, where (α_i)'s are fuzzy simply* open sets in (X,T).

Theorem 3.1 [16]: If $\lambda = \mu \vee \delta$ where μ is a fuzzy open and fuzzy dense set and δ is a fuzzy nowhere dense set in a fuzzy topological space (X,T) , then λ is a fuzzy simply open set in (X,T).

Definition 3.2 [5] : A fuzzy topological space (X,T) is said to be fuzzy hyper-connected if every non-null fuzzy open subset of (X,T) is fuzzy dense in (X,T).

Proposition 3.3 : If the fuzzy set λ on X is a fuzzy pseudo-open set in a fuzzy hyper-connected space (X,T) , then $\lambda = \vee_{i=1}^{\infty}(\alpha_i)$, where (α_i)'s are fuzzy simply open sets in (X,T).

Proof : Let λ be a fuzzy pseudo-open set in (X,T). Then, $\lambda = \mu \vee \delta$, where μ is a non-zero fuzzy open set in (X,T) and δ is a fuzzy first category set in (X,T). Now $\lambda = \mu \vee \delta = \mu \vee [\vee_{i=1}^{\infty}(\delta_i)]$,where $(\delta_i)'s$ are fuzzy nowhere dense sets in (X,T). Then, $\lambda = [\vee_{i=1}^{\infty}(\mu \vee \delta_i)]$. Since μ is a fuzzy open set in the fuzzy hyper-connected space (X,T) , μ is a fuzzy dense set in (X,T). Thus, μ is a fuzzy open and fuzzy dense set in (X,T). Then, by theorem 3.1, $(\mu \vee \delta_i)'s$ are fuzzy simply open sets in (X,T). Let $\alpha_i = \mu \vee \delta_i$. Hence $\lambda = \vee_{i=1}^{\infty}(\alpha_i)$, where (α_i)'s are fuzzy simply open sets in (X,T).

Proposition 3.4 : If λ is a fuzzy pseudo-open set in a fuzzy topological space (X,T) , then $1 - \lambda = \beta \wedge \gamma$, where β is a fuzzy closed set in (X,T) and γ is a fuzzy residual set in (X,T).

Proof : Let λ be a fuzzy pseudo-open set in (X,T). Then, $\lambda = \mu \vee \delta$, where μ is a non-zero fuzzy open set in (X,T) and δ is a fuzzy first category set in (X,T). Now $1-\lambda = 1-(\mu \vee \delta) = (1-\mu) \wedge (1-\delta)$.Let $\beta = 1-\mu$ and $\gamma = 1-\delta$. Since μ is a fuzzy open set in (X,T) , $1-\mu$ is a fuzzy closed set in (X,T). Also since δ is a fuzzy first category set in (X,T), $(1-\delta)$ is a fuzzy residual set in (X,T). Thus, $1-\lambda = \beta \wedge \gamma$, where β is a fuzzy closed set in (X,T) and γ is a fuzzy residual set in (X,T).

Proposition 3.5 : If $cl(\delta) = 1$, for each fuzzy first category set δ in a fuzzy topological space (X,T) and if λ is a fuzzy pseudo-open set in (X,T), then λ is a fuzzy dense set in (X,T).

Proof : Let λ be a fuzzy pseudo-open set in (X,T). Then, $\lambda = \mu \vee \delta$, where μ is a non-zero fuzzy open set in (X,T) and δ is a fuzzy first category set in (X,T). Then, $cl(\lambda) = cl(\mu \vee \delta) = cl(\mu) \vee cl(\delta) = cl(\mu) \vee 1 = 1$ and hence λ is a fuzzy dense set in (X,T).

Proposition 3.6 : If $int(\gamma) = 0$, for each fuzzy residual set γ in a fuzzy topological space (X,T) and if λ is a fuzzy pseudo-open set in (X,T), then λ is a fuzzy dense set in (X,T).

Proof : Let λ be a fuzzy pseudo-open set in (X,T). Then, $1-\lambda = \beta \wedge \gamma$, where β is a fuzzy closed set in (X,T) and γ is a fuzzy residual set in (X,T). By hypothesis, $int(\gamma) = 0$, for the fuzzy residual set γ in (X,T). Now $int(1-\lambda) = int(\beta \wedge \gamma) = int(\beta) \wedge int(\gamma) = int(\beta) \wedge 0 = 0$. Then $1 - cl(\lambda) = int(1-\lambda) = 0$. Thus $cl(\lambda) = 1$ and hence λ is a fuzzy dense set in (X,T).

Proposition 3.7 : If λ is a fuzzy pseudo-open set in a fuzzy topological space (X,T) such that $cl\, int(\lambda) = 1$, then λ is a fuzzy simply open set in (X,T).

Proof : Let λ be a fuzzy pseudo-open set in (X,T). Now $cl\, int(\lambda) \leq cl(\lambda)$ and $cl\, int(\lambda) = 1$, implies that $1 \leq cl(\lambda)$ and hence $cl(\lambda) = 1$, in (X,T).Then, $int\, cl[Bd(\lambda)] = int\, cl[cl(\lambda) \wedge cl(1-\lambda)] = int\, cl[1 \wedge cl(1-\lambda)] = int\, cl[cl(1-\lambda)] = int\, cl(1-\lambda) = 1 - cl\, int(\lambda) = 1 - 1 = 0$, in (X,T) .Hence λ is a fuzzy simply open set in (X,T).

Proposition 3.8 : If λ is a fuzzy pseudo-open set in a fuzzy hyper-connected space (X,T), then λ is a fuzzy simply open set in (X,T).

Proof : Let λ be a fuzzy pseudo-open set in (X,T). Then, $\lambda = \mu \vee \delta$, where μ is a non-zero fuzzy open set in (X,T) and δ is a fuzzy first category set in (X,T). Then $cl\, int(\lambda) = cl\, int(\mu \vee \delta) \geq cl[int(\mu) \vee int(\delta)] = cl\, int(\mu) \vee cl\, int(\delta) = cl(\mu) \vee clint(\delta)$. Since μ is a fuzzy open set in the fuzzy hyper-connected space (X,T) , μ is a fuzzy dense set and thus, cl (μ) = 1 in (X,T). Then, cl int (λ) = cl (μ) ∨ clint(δ) = 1 ∨ clint(δ) = 1. Then, by propostion 3.7, λ is a fuzzy simply open set in (X,T).

Definition 3.3 [14] : A fuzzy topological space (X,T) is called a fuzzy P-space if countable intersection of fuzzy open sets in (X,T) is fuzzy open. That is ,every non- zero fuzzy G_δ -set in (X,T), is fuzzy open in (X,T).

Theorem 3.2 [13]: If λ is a fuzzy σ-nowhere dense set in a fuzzy topological space (X,T), then λ is a fuzzy first category set in (X,T).

Theorem 3.3 [13]: If λ is a fuzzy σ- first category set in a fuzzy hyper-connected and fuzzy P-space (X,T), then λ is a fuzzy first category set in (X,T).

Proposition 3.9 : Let λ = μ ∨ δ , where μ is a fuzzy open and δ is a fuzzy σ-nowhere dense set in a fuzzy topological space (X,T) , then λ is a fuzzy pseudo- open set in (X,T).

Proof : Suppose that λ = μ ∨ δ , where μ is a fuzzy open and δ is a fuzzy σ-nowhere dense set in (X,T). By theorem 3.2, the fuzzy σ-nowhere dense set δ is a fuzzy first category set in (X,T) and hence λ = μ ∨ δ , is a fuzzy pseudo- open set in (X,T).

Proposition 3.10 : If λ = μ ∨ δ , where μ is a fuzzy open and δ is a fuzzy σ- first category set in a fuzzy hyper-connected and fuzzy P-space (X,T), then λ is a fuzzy pseudo- open set in (X,T).

Proof : Suppose that λ = μ ∨ δ , where μ is a fuzzy open and δ is a fuzzy σ- first category set in (X,T). Since (X,T) is a fuzzy hyper-connected and fuzzy P-space ,by theorem 3.3, the fuzzy σ- first category set δ is a fuzzy first category set in (X,T) and hence λ = μ ∨ δ , is a fuzzy pseudo- open set in (X,T).

Proposition 3.11 : If λ = μ ∨ δ , where μ is a fuzzy open and δ is a fuzzy σ- first category set in a fuzzy topological space (X,T), then $\lambda = \bigvee_{i=1}^{\infty}(\mu \vee \delta_i)$, where $(\mu \vee \delta_i)'s$ are fuzzy pseudo-open sets in (X,T).

Proof : Suppose that λ = μ ∨ δ , where μ is a fuzzy open and δ is a fuzzy σ- first category set in (X,T). Then, $\lambda = \mu \vee \delta = \mu \vee [\bigvee_{i=1}^{\infty}(\delta_i)]$, where $(\delta_i)'s$ are fuzzy σ- nowhere dense sets in (X,T). This implies that $\lambda = \bigvee_{i=1}^{\infty}(\mu \vee \delta_i)$. By proposition 3.9, $(\mu \vee \delta_i)'s$ are fuzzy pseudo-open sets in (X,T). Thus, $\lambda = \bigvee_{i=1}^{\infty}(\mu \vee \delta_i)$, where $(\mu \vee \delta_i)'s$ are fuzzy pseudo- open sets in (X,T).

Proposition 3.12 : If α = γ ∧ β , where γ is a fuzzy closed set in (X,T) and β is a fuzzy dense and fuzzy G_δ set in a fuzzy topological space (X,T), then α is a fuzzy pseudo-closed set in (X,T).

Proof : Suppose that α = γ ∧ β , where γ is a fuzzy closed set in (X,T) and β is a fuzzy dense and fuzzy G_δ -set in (X,T). Then, 1 − α = 1 − (γ ∧ β) = (1 − γ) ∨ (1 − β). Now cl (β) =1, implies that 1 − cl(β) = 0 and then , by lemma 2.1, int (1 – β) = 0, in (X,T). Since γ is a fuzzy closed set in (X,T), 1 − γ is a fuzzy open set in (X,T). Also since β is a fuzzy G_δ - set in (X,T), 1− β is a fuzzy F_σ–set in (X,T). Thus 1 – β, being a fuzzy F_σ–set with int (1 – β) = *0*, is a fuzzy σ-nowhere dense set in (X,T). Let μ =1 – γ and δ = 1 – β. Then, 1 – α = μ ∨ δ , where μ is a fuzzy open and δ is a fuzzy σ-nowhere dense set in (X,T). Hence, by proposition 3.9, 1 − α is a fuzzy pseudo- open set in (X,T).Therefore α is a fuzzy pseudo-closed set in (X,T).

Theorem 3.4 [18]: If λ is a fuzzy simply open set in a fuzzy topological space (X,T), then λ is a fuzzy resolvable set in (X,T).

Proposition 3.13 : If λ is a fuzzy pseudo-open set in a fuzzy hyper-connected space (X,T), then λ is a fuzzy resolvable set in (X,T).

Proof : Let λ be a fuzzy pseudo-open set in (X,T). Since (X,T) is a fuzzy hyper-connected space , by theorem 3.8, the fuzzy pseudo-open set λ is a fuzzy simply open set in (X,T). Then, by theorem 3.4, fuzzy simply open set λ is a fuzzy resolvable set in (X,T).

Definition 3.4 [10]: Let (X,T) be a fuzzy topological space. Then (X,T) is called a fuzzy Baire space if $\text{int}(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T).

Theorem 3.5 [18] : If $\text{int}(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy resolvable sets in a fuzzy topological space (X,T), then (X,T) is a fuzzy Baire space.

Proposition 3.14 : If $\text{int}(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy pseudo-open sets in a fuzzy hyper-connected space (X,T), then (X,T) is a fuzzy Baire space.

Proof : Let (λ_i)'s be fuzzy pseudo-open sets in the fuzzy hyper-connected space (X,T). Then, by proposition 3.13, (λ_i)'s are fuzzy resolvable sets in (X,T). By hypothesis, $int(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy resolvable sets in (X,T). Then, by theorem 3.5, (X,T) is a fuzzy Baire space.

Theorem 3.6 [18] : If λ is a fuzzy resolvable set in a fuzzy topological space (X,T), then there exists a fuzzy regular open set δ in (X,T) such that $\delta \leq cl[\lambda \vee (1-\lambda)]$.

Proposition 3.15 : If λ is a fuzzy pseudo-open set in a fuzzy hyper-connected space (X,T), then there exists a fuzzy regular open set δ in (X,T) such that $\delta \leq cl[\lambda \vee (1-\lambda)]$.

Proof : Let λ be a fuzzy pseudo-open set in (X,T). Since (X,T) is a fuzzy hyper-connected space , by theorem 3.13 , the fuzzy pseudo-open set λ is a a fuzzy resolvable set in (X,T). Then, by theorem 3.6, there exists a fuzzy regular open set δ in (X,T) such that $\delta \leq cl[\lambda \vee (1-\lambda)]$.

Definition 3.5 [2] : A fuzzy topological space (X,T) is called a fuzzy submaximal space if for each fuzzy set λ in (X,T) such that $cl(\lambda) = 1$, then $\lambda \in T$ in (X,T).

Theorem 3.7 [15] : If λ is a fuzzy residual set in a fuzzy submaximal space (X, T) , then λ is a fuzzy G_δ-set in (X, T).

Proposition 3.16 : If λ is a fuzzy pseudo-open set in a fuzzy submaximal space (X,T) , then $\lambda = \mu \vee \delta$, where μ is a fuzzy open and δ is a fuzzy F_σ set in (X,T).

Proof : Let λ be a fuzzy pseudo-open set in (X,T). Then, by proposition 3.4, $1-\lambda = \beta \wedge \gamma$, where β is a fuzzy closed set in (X,T) and γ is a fuzzy residual set in (X,T). Since (X,T) is a fuzzy submaximal space , by theorem 3.7, the fuzzy residual set γ is a fuzzy G_δ-set in (X,T) and hence $1-\lambda = \beta \wedge \gamma$, where β is a fuzzy closed set in (X,T) and γ is a fuzzy G_δ-set in (X,T). Then, $1-[1-\lambda] = 1-[\beta \wedge \gamma] = (1-\beta) \vee (1-\gamma)$. Let $\mu = (1-\beta)$ and $\delta = 1-\gamma$. Thus $\lambda = \mu \vee \delta$, where μ is a fuzzy open and δ is a fuzzy F_σ set in (X,T).

Proposition 3.17 : If λ is a fuzzy pseudo-open set in a fuzzy submaximal space (X,T) , then $1-\lambda = \gamma \wedge \beta$, where γ is a fuzzy closed and β is a fuzzy G_δ - set in (X,T).

Proof : The proof follows from proposition 3.17, by taking the complements of fuzzy sets in (X,T).

IV. Fuzzy Pseud0- Continuous Functions

Definition 4.1: Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function $f : (X,T) \to (Y,S)$ is called a fuzzy pseudo-continuous function if for each fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy pseudo- open set in (X,T).

Example 4.1 : Let $X = \{a, b, c\}$. Consider the fuzzy sets α, β, δ, γ, σ, ω, μ and θ defined on X as follows:

$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.5$; $\alpha(b) = 1$; $\alpha(c) = 0.4$,
$\beta : X \to [0, 1]$ is defined as $\beta(a) = 1$; $\beta(b) = 0.6$; $\beta(c) = 0.1$,
$\delta : X \to [0, 1]$ is defined as $\delta(a) = 0.6$; $\delta(b) = 0.4$; $\delta(c) = 1$,
$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 1$; $\gamma(b) = 0.6$; $\gamma(c) = 0.9$,
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.6$; $\mu(b) = 0.6$; $\mu(c) = 0.9$,
$\sigma : X \to [0, 1]$ is defined as $\sigma(a) = 0.5$; $\sigma(b) = 1$; $\sigma(c) = 0.9$,
$\omega : X \to [0, 1]$ is defined as $\omega(a) = 0.6$; $\omega(b) = 1$; $\omega(c) = 0.9$,
$\theta : X \to [0, 1]$ is defined as $\theta(a) = 1$; $\theta(b) = 0.6$; $\theta(c) = 0.4$.

Then, $T = \{0, \alpha, \beta, \delta, \alpha \vee \beta, \alpha \vee \delta, \beta \vee \delta, \alpha \wedge \beta, \alpha \wedge \delta, \beta \wedge \delta, (\alpha \vee [\beta \wedge \delta]), (\beta \vee [\alpha \wedge \delta]), (\delta \vee [\alpha \wedge \beta]), \alpha \wedge [\beta \vee \delta], \beta \wedge [\alpha \vee \delta], \delta \wedge [\alpha \vee \beta], [\alpha \wedge \beta \wedge \delta], 1\}$ and $S = \{0, \gamma, 1\}$ are fuzzy topologies on X. On computation , the fuzzy nowhere dense sets in (X,T), are $1-\alpha$, $1-\beta$, $1-\delta$, $1-[\alpha \vee \beta]$, $1-[\alpha \vee \delta]$, $1-[\beta \vee \delta]$, $1-[\beta \wedge \delta]$, $1-(\alpha \vee [\beta \wedge \delta])$, $1-(\beta \vee [\alpha \wedge \delta])$, $1-(\delta \vee [\alpha \wedge \beta])$, $1-(\beta \wedge [\alpha \vee \delta])$, $1-(\delta \wedge [\alpha \vee \beta])$, $1-\gamma$, $1-\sigma$, $1-\omega$, $1-\mu$ and $1-\theta$ and $1-[\alpha \wedge \beta \wedge \delta]$ is a fuzzy first category set in (X,T). The fuzzy pseudo-open sets in (X,T), are $1-(\alpha \wedge \beta \wedge \delta)$, $\alpha \vee \delta$, $\beta \vee \delta$, $(\delta \vee [\alpha \wedge \beta])$, μ , γ , θ, σ and ω. Define a function $f : (X, T) \to (X, S)$ by $f(a) = b$; $f(b) = a$; $f(c) = c$. Now $f^{-1}(\gamma)(a) = \gamma[f(a)] = \gamma[b] = 0.6$; $f^{-1}(\gamma)(b) = \gamma[f(b)] = \gamma[a] = 1$; $f^{-1}(\gamma)(c) = \gamma[f(c)] = \gamma[c] = 0.9$ and thus $f^{-1}(\gamma) = \omega$ and ω is a fuzzy pseudo- open set in (X,T). Thus, for the non-zero fuzzy open set γ in (Y,S), $f^{-1}(\gamma)$ is a fuzzy pseudo- open set in (X,T), implies *that f is a fuzzy pseudo-continuous function from* (X,T) *into* (Y,S).

Proposition 4.1 : If $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) and if λ is a fuzzy open set in (Y,S), then $int\, f^{-1}(\lambda) \neq 0$ and $cl(f^{-1}(1-\lambda)) \neq 1$, in (X,T).

Proof : Let λ be a fuzzy open set in (Y,S). Since $f: (X,T) \rightarrow (Y,S)$ is a fuzzy pseudo-continuous function, $f^{-1}(\lambda)$ is a fuzzy pseudo- open set in (X,T). Then by proposition 3.1, int $[f^{-1}(\lambda)] \neq 0$, in (X,T). Also $cl(1 - f^{-1}(\lambda)) \neq 1$, in (X,T), implies that cl $(f^{-1}(1 - \lambda)) \neq 1$, in (X,T).

Definition 4.2 [8]: Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function $f: (X,T) \rightarrow (Y,S)$ is called a somewhat fuzzy continuous function if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$, there exists a non-zero fuzzy open set μ in (X,T) such that $\mu \leq f^{-1}(\lambda)$. That is, int $f^{-1}(\lambda) \neq 0$, in (X,T).

Proposition 4.2 : If $f: (X,T) \rightarrow (Y,S)$ is a fuzzy pseudo-continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then f is a somewhat fuzzy continuous function.

Proof : Let $f: (X,T) \rightarrow (Y,S)$ be a fuzzy pseudo-continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S). Then, by proposition 4.1, for a fuzzy open set λ in (Y,S), int $[f^{-1}(\lambda)] \neq 0$, in (X,T). Hence f is a fuzzy somewhat fuzzy continuous function from (X,T) into (Y,S).

Proposition 4.3 : If $f: (X,T) \rightarrow (Y,S)$ is a fuzzy pseudo-continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) and if λ is a fuzzy open set in (Y,S), then $f^{-1}(\lambda) = \vee_{i=1}^{\infty}(\alpha_i)$, where (α_i)'s are fuzzy simply* open sets in (X,T).

Proof : Let $f: (X,T) \rightarrow (Y,S)$ be a fuzzy pseudo-continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S). Then, for the fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy pseudo-open set in (X,T). Then, by proposition 3.2, $f^{-1}(\lambda) = \vee_{i=1}^{\infty}(\alpha_i)$, where (α_i)'s are fuzzy simply* open sets in (X,T).

Proposition 4.4 : If $f: (X,T) \rightarrow (Y,S)$ is a fuzzy pseudo-continuous function from a fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S) and if λ is a fuzzy open set in (Y,S), then $f^{-1}(\lambda) = \vee_{i=1}^{\infty}(\alpha_i)$, where (α_i)'s are fuzzy simply open sets in (X,T).

Proof : Let $f: (X,T) \rightarrow (Y,S)$ be a fuzzy pseudo-continuous function from a fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S). Then, for the fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy pseudo-open set in (X,T). Since (X,T) is a fuzzy hyper-connected space, for the fuzzy pseudo-open set $f^{-1}(\lambda)$ in (X,T), by proposition 3.3, we have $f^{-1}(\lambda) = \vee_{i=1}^{\infty}(\alpha_i)$, where (α_i)'s are fuzzy simply open sets in (X,T).

Proposition 4.5 : If $f: (X,T) \rightarrow (Y,S)$ is a fuzzy pseudo-continuous function from (X,T) in which each fuzzy first residual set has zero interior, into a fuzzy topological space (Y,S), then, for a fuzzy open set λ in (Y,S), cl $[f^{-1}(\lambda)] = 1$, in (X,T).

Proof : Let $f: (X,T) \rightarrow (Y,S)$ be a fuzzy pseudo-continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S). Then, for the fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy pseudo-open set in (X,T). By hypothesis, int $(\gamma) = 0$, for each fuzzy residual set γ in (X,T). Then, for the pseudo-open set $f^{-1}(\lambda)$ in (X,T), by proposition 3.6, $f^{-1}(\lambda)$ is a fuzzy dense set in (X,T) and hence cl $[f^{-1}(\lambda)] = 1$ in (X,T).

Proposition 4.6 : If $f: (X,T) \rightarrow (Y,S)$ is a fuzzy pseudo-continuous function from (X,T) in which each fuzzy first category set is a fuzzy dense set, into a fuzzy topological space (Y,S), then, for a fuzzy open set λ in (Y,S), cl $[f^{-1}(\lambda)] = 1$, in (X,T).

Proof : Let $f: (X,T) \rightarrow (Y,S)$ be a fuzzy pseudo-continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S). Then, for the fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy pseudo-open set in (X,T). By hypothesis, cl $(\delta) = 1$, for each fuzzy first category set δ in (X,T). Then, for the pseudo-open set $f^{-1}(\lambda)$ in (X,T), by proposition 3.5, $f^{-1}(\lambda)$ is a fuzzy dense set in (X,T) and hence cl $[f^{-1}(\lambda)] = 1$ in (X,T).

Definition 4. 3 [9]: A fuzzy topological space (X,T) is called a fuzzy resolvable space if there exists a fuzzy dense set λ in (X,T) such that cl $(1 - \lambda) = 1$. Otherwise (X,T) is called a fuzzy irresolvable space .

Proposition 4.7 : If $f: (X,T) \rightarrow (Y,S)$ is a fuzzy pseudo-continuous function from (X,T) in which each fuzzy first category set is a fuzzy dense set, into a fuzzy topological space (Y,S), then (X,T) is a fuzzy irresolvable space.

Proof : Let $f: (X,T) \rightarrow (Y,S)$ be a fuzzy pseudo-continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S). Then, for the fuzzy open set λ in (Y,S), by proposition 4.1, int $[f^{-1}(\lambda)] \neq 0$ in (X,T). Now cl $[1 - f^{-1}(\lambda)] = 1 -$ int$[f^{-1}(\lambda)] \neq 1$ and thus cl $[1 - f^{-1}(\lambda)] \neq 1$, in (X,T). Since cl $(\delta) = 1$, for each fuzzy first category set δ in (X,T), by proposition 4.5, $\mathrm{cl}\,[\,f^{-1}(\lambda)\,] = 1$ in (X,T).Thus, $\mathrm{cl}\,[f^{-1}(\lambda)\,] = 1$ and $\mathrm{cl}\,[1 - f^{-1}(\lambda)] \neq 1$ in (X,T), implies that (X,T) is a fuzzy irresolvable space.

Proposition 4.8 : If $f : (X,T) \to (Y,S)$ is a function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) and if $f^{-1}(\lambda) = \mu \vee \delta$, where μ is a fuzzy open and δ is a fuzzy σ-nowhere dense set in (X,T), for a fuzzy open set λ in (Y,S), then f is a fuzzy pseudo-continuous function from (X,T) into (Y,S).

Proof : Let $f : (X,T) \to (Y,S)$ be a function from (X,T) into (Y,S). Suppose that for a fuzzy open set λ in (Y,S), $f^{-1}(\lambda) = \mu \vee \delta$, where $\mu \in T$ and δ is a fuzzy σ-nowhere dense set in (X,T).Then, by proposition 3.9, $f^{-1}(\lambda)$ is a fuzzy pseudo- open set in (X,T) and hence f is a fuzzy pseudo-continuous function from (X,T) into (Y,S).

Proposition 4.9 : If $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-continuous function from a fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S) and if λ is a fuzzy open set in (Y,S), then $f^{-1}(\lambda)$ is a fuzzy simply open set in (X,T).

Proof : Let $f : (X,T) \to (Y,S)$ be a fuzzy pseudo-continuous function from a fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S). Then, for the fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy pseudo-open set in (X,T). Since (X,T) is a fuzzy hyper-connected space , by proposition 3.8 the pseudo-open set $f^{-1}(\lambda)$ is a fuzzy simply open set in (X,T).

Proposition 4.10 : If $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-continuous function from a fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S), then f is a fuzzy simply continuous function from (X,T) into (Y,S).

Proof : Let $f : (X,T) \to (Y,S)$ be a fuzzy pseudo-continuous function from a fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S). Then, for the fuzzy open set λ in (Y,S), by proposition 4.9, $f^{-1}(\lambda)$ is a fuzzy simply open set in (X,T). Hence f is a fuzzy simply continuous function from (X,T) into (Y,S).

Theorem 4.1 [16]: If λ is a fuzzy simply open set in a fuzzy topological space (X,T), then $\lambda \wedge (1 - \lambda)$ is a fuzzy nowhere dense set in (X,T).

Proposition 4.11: If $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-continuous function from a fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S) and if λ is a fuzzy open set in (Y,S), then $f^{-1}\,[\lambda \wedge (1 - \lambda)]$ is a fuzzy nowhere dense set in (X,T).

Proof : Let $f : (X,T) \to (Y,S)$ be a fuzzy pseudo-continuous function from a fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S). Then, for the fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy simply open set in (X,T). Now , by theorem 4.1, $[f^{-1}(\lambda)\,] \wedge [\,1 - (f-1\,(\lambda))\,]$ is a fuzzy nowhere dense set in (X,T). Then, $[\,f-1\,(\lambda)\,] \wedge [f-1\,(1-\lambda)\,]$ is a fuzzy nowhere dense set in (X,T). Since $[\,f^{-1}(\lambda)\,] \wedge [f^{-1}(1-\lambda)\,] = f^{-1}\,[\lambda \wedge (1-\lambda)]$ in (X,T), $f^{-1}\,[\lambda \wedge (1-\lambda)]$ is a fuzzy nowhere dense set in (X,T).

Proposition 4.12: If $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-continuous function from a fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S) and if λ is a fuzzy open set in (Y,S), then $\mathrm{int}\,(\,f^{-1}\,[\lambda \wedge (1-\lambda)]) = 0$, in (X,T).

Proof : Let $f : (X,T) \to (Y,S)$ be a fuzzy pseudo-continuous function from a fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S). Then, for the fuzzy open set λ in (Y,S), by proposition 4.10, $f^{-1}\,[\lambda \wedge (1-\lambda)]$ is a fuzzy nowhere dense set in (X,T). Then, $\mathrm{int\,cl}\,(\,f^{-1}\,[\lambda \wedge (1-\lambda)]) = 0$, in (X,T). Since $\mathrm{int}\,(\,f^{-1}[\,\lambda \wedge (1-\lambda)]) \leq \mathrm{int\,cl}\,(\,f^{-1}[\lambda \wedge (1-\lambda)])$ in (X,T) , $\mathrm{int}\,(\,f^{-1}\,[\lambda \wedge (1-\lambda)]\,) = 0$.

Proposition 4.13: If $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then $\mathrm{int}\,[\,\bigvee_{i=1}^{\infty} f^{-1}(\lambda_i)\,] \neq 0$, in (X,T), where $(\lambda_i)'s$ are fuzzy open sets in (Y,S).

Proof : Let $f : (X,T) \to (Y,S)$ be a fuzzy pseudo-continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S). Then , for the fuzzy open sets λ_i (i = 1 to ∞) in (Y,S), $[\,f^{-1}(\,\lambda_i)\,]'s$ are fuzzy pseudo-open sets in (X,T). Then, by proposition 4.1, $\mathrm{int}\,[\,f^{-1}(\lambda_i)\,] \neq 0$, in (X,T). Suppose that $\mathrm{int}\,[\,\bigvee_{i=1}^{\infty} f^{-1}(\lambda_i)] = 0$, in (X,T). Then, $\bigvee_{i=1}^{\infty}\,(\,\mathrm{int}\,[f^{-1}(\lambda_i)]\,) \leq \mathrm{int}\,[\,\bigvee_{i=1}^{\infty} f^{-1}(\lambda_i)]$, will imply that $\bigvee_{i=1}^{\infty}[\,\mathrm{int}\,f^{-1}(\lambda_i)] = 0$ and hence will imply that $\mathrm{int}\,[\,f^{-1}(\lambda_i)\,] = 0$, a contradiction. Hence, $\mathrm{int}\,[\,\bigvee_{i=1}^{\infty} f^{-1}(\lambda_i)\,] \neq 0$, in (X,T).

Proposition 4.14 : If $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-continuous function from a fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S) and if $(\lambda_i)'s$ are fuzzy open sets in (Y,S), then $\bigvee_{i=1}^{\infty}(\,f^{-1}\,[\,\lambda_i \wedge (1 - \lambda_i)]\,)$ is a fuzzy first category set in (X,T).

Proof : Let $f : (X, T) \to (Y, S)$ be a fuzzy pseudo-continuous function from a fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S). Then, for the fuzzy open sets λ_i (i = 1 to ∞) in (Y,S), by proposition 4.11, the fuzzy sets $(f^{-1} [\lambda_i \wedge (1 - \lambda_i)])$'s are fuzzy nowhere dense sets in (X,T). Hence $\bigvee_{i=1}^{\infty}(f^{-1} [\lambda_i \wedge (1 - \lambda_i)])$ is a fuzzy first category set in (X,T).

Definition 4.4 [12]: Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called a somewhat fuzzy nearly continuous function if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$, there exists a non-zero fuzzy open set μ in (X,T) such that $\mu \leq cl[f^{-1}(\lambda)]$. That is, $\text{int cl } f^{-1}(\lambda) \neq 0$, in (X,T).

Proposition 4.15 : If $f : (X, T) \to (Y, S)$ is a fuzzy pseudo-continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then f is a fuzzy somewhat fuzzy nearly continuous function from (X,T) into (Y,S).

Proof : Let λ be a fuzzy open set in (Y,S). Since $f: (X, T) \to (Y, S)$ is a fuzzy pseudo-continuous function, $f^{-1}(\lambda)$ is a fuzzy pseudo- open set in (X,T). Then by proposition 3.1, $[f^{-1}(\lambda)]$ is not a fuzzy nowhere dense set in (X,T) and hence $\text{int cl } [f^{-1}(\lambda)] \neq 0$, in (X,T). Thus, for the fuzzy open set λ in (Y,S), $\text{int cl } [f^{-1}(\lambda)] \neq 0$, in (X,T), implies that f is a fuzzy somewhat fuzzy nearly continuous function from (X,T) into (Y,S).

Remark 4.1: A somewhat fuzzy nearly continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) need not be a fuzzy pseudo-continuous function. For, consider the following example:

Example 4.2 : Let X = { a, b, c}. Consider the fuzzy sets α, β, δ and γ defined on X as follows:

$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.7$; $\alpha(b) = 0.8$; $\alpha(c) = 0.6$,
$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.9$; $\beta(b) = 0.6$; $\beta(c) = 0.5$,
$\delta : X \to [0, 1]$ is defined as $\delta(a) = 0.8$; $\delta(b) = 0.7$; $\delta(c) = 0.6$,
$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.6$; $\gamma(b) = 0.9$; $\gamma(c) = 0.5$.

Let $T = \{ 0, \alpha, \beta, \alpha \vee \beta, \alpha \wedge \beta, 1\}$ and $S = \{0, \delta, \gamma, \delta \vee \gamma, \delta \wedge \gamma, 1\}$. Clearly T and S are fuzzy topologies on X . Define $f : (X,T) \to (Y,S)$ by f (a) = b, f (b) = a, f (c) = c. Then, for the non-zero fuzzy open sets δ, γ, $\delta \vee \gamma$, $\delta \wedge \gamma$ in (Y, S), $\text{int cl } [f^{-1}(\delta)] \neq 0$, $\text{int cl } [f^{-1}(\gamma)] \neq 0$, $\text{int cl } [f^{-1}(\delta \vee \gamma)] \neq 0$, $\text{int cl } [f^{-1}(\delta \wedge \gamma)] \neq 0$, in (X,T). Hence f is a somewhat fuzzy nearly continuous functions but not a fuzzy pseudo- continuous function from (X,T) into (Y,S).

Proposition 4.16 : If $f : (X, T) \to (Y, S)$ is a fuzzy pseudo-continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) and if λ is a fuzzy open and fuzzy dense set in (X,T), then f (λ) is a fuzzy dense set in (Y,S).

Proof : Let $f : (X, T) \to (Y, S)$ be a fuzzy pseudo-continuous function from (X,T) into (Y,S) and λ be a fuzzy open and fuzzy dense set in (X, T). It has to be proved that $cl [f (\lambda)] = 1$. Assume the contrary. Then, $cl [f (\lambda)] \neq 1$, in (Y,S). Then, there exists a non- zero fuzzy closed set δ of (Y, S) such that $f (\lambda) < \delta < 1$ and then $f^{-1}(f (\lambda)) < f^{-1} (\delta)$. Now $\lambda \leq f^{-1} f (\lambda)$ implies that $\lambda < f^{-1} (\delta)$. Then $1 - \lambda > 1 - f^{-1} (\delta)$ and hence $1 - \lambda > f^{-1} (1 - \delta)$. Since δ (≠ 1) is a non-zero fuzzy closed set δ of (Y, S), $1 - \delta$ is a non-zero fuzzy open set in (Y, S). Since f is a fuzzy pseudo-continuous function from (X,T) into (Y,S), for the fuzzy open set $1 - \delta$ in (Y,S), $f^{-1} (1 - \delta)$ is a fuzzy pseudo-open set in (X,T). Then, by proposition 3.1, $f^{-1} (1 - \delta)$ is not a fuzzy nowhere dense set in (X,T) and hence $\text{int cl } [f^{-1} (1 - \delta)] \neq 0$, in (X,T). Then, $\text{int cl}[1 - f^{-1}(\delta)] \neq 0$ implies that $1 - \text{cl int } [f^{-1}(\delta)] \neq 0$ in (X,T). That is, $\text{cl int } [f^{-1} (\delta)] \neq 1$. Now $\lambda < f^{-1} (\delta)$ implies that $\text{cl int } (\lambda) < \text{cl int } [f^{-1} (\delta)]$. Then $\text{cl int}(\lambda) \neq 1$. Since λ is a fuzzy open set in (X, T), $\text{int}(\lambda) = \lambda$ and hence $\text{cl}(\lambda) \neq 1$, which is a contradiction to λ being a fuzzy dense set in (X, T). Hence it must be that $cl[f (\lambda)] = 1$ in (Y,S) and therefore f (λ) is a fuzzy dense set in (Y,S).

Proposition 4.17 : If $f : (X, T) \to (Y, S)$ is a fuzzy pseudo-continuous and one to one function from a fuzzy topological space (X,T) onto a fuzzy topological space (Y,S) and if λ is a fuzzy nowhere dense set in (X,T), then $\text{int } [f (\lambda)] = 0$ in (Y,S).

Proof : Let λ be a fuzzy nowhere dense set in (X, T). Then, $\text{int cl } (\lambda) = 0$, in (X, T). Now $1 - \text{int cl}(\lambda) = 1 - 0 = 1$, implies that $\text{cl int } (1 - \lambda) = 1$. Hence $\text{int } (1 - \lambda)$ is a fuzzy open and fuzzy dense set in (X,T). Then, by proposition 4.16 $f (\text{int } (1 - \lambda))$ is a fuzzy dense set in (Y,S). That is, $cl[f (\text{int}(1 - \lambda))] = 1$ in (Y,S). Now $\text{int } (1 - \lambda) \leq (1 - \lambda)$, implies that $cl [f (\text{int}(1 - \lambda))] \leq cl [f (1 - \lambda)]$.

Then ,we have $1 \leq$ cl[f(1 − λ)] in(Y,S). That is, cl[f (1 − λ)] = 1. Since the function f is one to one and onto, f (1 − λ) = 1 – f (λ). Then cl (1 − f(λ) = 1. This implies that 1 – int [f (λ)] = 1 and therefore int [f (λ)] = 0 in (Y, S).

Proposition 4.18 : If $f : (X, T) \to (Y, S)$ is a fuzzy pseudo-continuous function from a fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S) and if λ is a fuzzy open set in (Y,S), then there exists a fuzzy regular open set δ in (X,T) such that $\delta \leq$ cl [$f^{-1}(\lambda \vee (1-\lambda))$].

Proof : Let $f : (X, T) \to (Y, S)$ be a fuzzy pseudo-continuous function from (X,T) into (Y,S) and λ be a fuzzy open set in (Y,S). Then, $f^{-1}(\lambda)$ is a fuzzy pseudo- open set in (X,T). Since Since (X,T) is a fuzzy hyper-connected space , by theorem 3.15 , for the fuzzy pseudo-open set $f^{-1}(\lambda)$, there exists a fuzzy regular open set δ in (X,T) such that $\delta \leq$ cl [$f^{-1}(\lambda) \vee f^{-1}(1-\lambda)$] and hence $\delta \leq$ cl [$f^{-1}(\lambda \vee (1-\lambda))$], in (X,T).

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