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Mathematical Modeling for Population Projection and Management: A Case Study Of Niger State.

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Abstract: *The purpose of this research paper is to develop a mathematical model to predict population figures of Niger state for the period of 20 years using2006 census figures. The data used were collected from National Population commission and were analyzed by using MATLAB Software. The idea behind the projection is to get an estimated figure of the population of Niger state without waiting for census data. The exponential growth model, also known as Malthus Model used in this paper simply shows that for any number of year specified as the proposed year for the growth model, the corresponding value can be obtain which importantly predicts the estimated amount of population. The parameter used for the population, can be used for any given region, which helps to check unwanted population increase or decrease. It can also be exploited to access the success of the method implemented over time.*

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I. Introduction

Projection of any country's population plays a significant role in the planning as well as in the decision making for the socio-economic and demographic development. The population projection has become one of the most important problems in the world. Population sizes and growth in a country directly influence the situation of economy, policy, culture, education and environment etc of this country, and determine exploring and cost of natural resources. No one wants to wait until those resources are exhausted because of population explosion. Every government and collective sectors always require accurate idea about the future size of various entities like population, resources, demands and consumptions for their planning activities. To obtain this information, the behavior of the connected variables is analyzed based on the previous data by the statisticians and mathematicians, and using the conclusions drawn from the analysis, they make future projections. There are enormous concerns about the consequences of human population growth for social, environment and economic development. Raising all these problems is population growth.

Mathematical modeling is a broad interdisciplinary science that uses mathematical and computational techniques to model and elucidate the phenomena arising in life sciences.It require the use of a set of formulas or equations based on quantitative description or real world phenomenon and created in the hope that the behavior it predicts will resemble the real behavior on which it is based (Glenn, 2005). It also, involves the following processes.

(1) The formulation of a real-world problem in mathematical terms: thus the construction of mathematical model.
(2) The analysis or solution of the resulting mathematical problem.
(3) The interpretation of the mathematical results in the context of the original situation.

Many people examine population growth through observation, experimentation or through Mathematical modeling. Mathematical models can take many forms, including but not limited to dynamical systems, statistical models and deferential equations. In this paper we model the population growth of Niger state by using Malthusian growth model. The use of this growth model is widely established in many fields of modeling and forecasting (Banks, 1994). First order deferential equations govern the growth of various species.It would seem be impossible to model the growth of a population by a deferential equation since the population of any species always changes by integer amounts. However if a given population is very large and it is suddenly increased by one, then the change is very small compared to the given population (Martin, 1992). Furthermore, the study of population projection has been started earlier.

In 1798 the Englishman Thomas R. Malthus proposed a mathematical model of population growth. His model, though simple, has become a basis for the most future modeling of biological populations. He successfully discussed the caveats of mathematical modeling through his paper, *"An Essay on the Principle of Population"*. Furthermore, he gave a pessimistic view over the dangers of over-population and described the dangers of over-population would eventually lead to a shortage of food on a global scale, poverty, hunger and disease. He believed that human population increases geometrically (i.e. 2, 4, 8, 16, etc.) whereas food supplies can only grow arithmetically (i.e. 2, 4, 6, 8, etc.) as it is limited by available land and technology. The geometric population growth outruns an arithmetic increase in food supply. He stated that the 'laws of nature' dictate that a population can never increase beyond the food supplies necessary to support it.

In 1840 a Belgian Mathematician Verhulst modified Malthus's Model, he thought population growth not only depends on the population size but also on how far this size is from its upper limit.

In addition to sufficient size, populations need to be imbued with enough gene diversity in the form of founders, and they need to sustain the requisite growth rate to avoid large fluctuations in size. In recent years, a number of studies, including the one on which this article is based (Lees & Wilcken 2009), have shown that regional population management programmes are not achieving the conditions for sustainability. They are too small, are based on too few founders and are not achieving the required growth rates.

OBJECTIVES OF THE STUDY

The purpose of this research paper is to project the population of Niger State using exponential growth model to predict the population for the period of 20years so as to know the population of Niger State ahead which can foretell the census figure

II. Materials And Methods

A research is best understood as a process of arriving at dependent solutions to the problems through the systematic collection, analysis and interpretation of data. In relation to this paper, secondary data of Niger state 2006 population census figures were collected from National Population Commission (NPC). MATLAB software was used to compute the predicted population values.

DEVELOPMENT OF NIGER STATE POPULATION GROWTH MODEL

We want to model the population growth of Niger state over time (t), this paper assumed that reproduction is carried out continuously by its member without regard to age or sex differences. We also, assume thatP (t) denotes the population level at time t and let (r-α) denote the difference between its birth rate and death rate. If we consider that the environment inhabited by the population to be closed, that is (Nomigration in or out). Since each member of the population reproduce at the same per capital rate, hence the total growth rate is $(r-\alpha)p$.

Similarly we have:

$$\frac{dp}{dt} = (r-\alpha)p \tag{1}$$

Where,

$r = \text{Re}\,productive\ rate$

$\alpha = Mortality\ rate$

$r\ and\ \alpha$ are variables, this is because birth rate and death rate changes every year.

That is: $\frac{dp}{dt} = (r-\alpha)p$

Using separation of variable and integrate both sides, we have,

$$\int \frac{dp}{p} = \int (r-\alpha)dt \tag{2}$$

$$\ln p = (r-\alpha)t + c \tag{3}$$

$$p(t) = e^{(r-\alpha)t+c} \tag{4}$$

$$p(t) = e^{(r-\alpha)t}.e^{c} \tag{5}$$

$$p(t) = Ae^{(r-\alpha)t},\ \ A = e^{c} \tag{6}$$

When t=0, we have

$$p(0) = p_o = A \tag{7}$$

Hence, the model equation now becomes

$$p(t) = p_o e^{(r-\alpha)t} \tag{8}$$

From the above model, let our proposed year

$$t = p_y - b_y, \tag{9}$$

Where, $p_y - b_y = present\ \ year - base\ \ year$

Substitute $t = p_y - b_y$ into the model equation above, we have

$$p(t) = p_o e^{(r-\alpha)\,p_y - b_y} \tag{10}$$

Where $p_o = population\ at\ the\ base\ year.$

Evaluation Of The Model

According to the American Central Intelligence Agency, the world fact book, the present reproductive and mortality rate of Nigeria are 38.03births/1000 population and 13.16deaths/1000 population respectively. The population being used for Niger state is using the above state to evaluate the growth model from equation (10) we have,

$$p(t) = p_o e^{(r-\alpha)\,p_y - b_y}$$

By using the model equation above, the predicted population is as follows:

$$P(2006) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2006-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(0)}$$
$$= 3{,}950{,}249$$

$$P(2007) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2007-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(1)}$$
$$= 4052855$$

$$P(2008) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2008-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(2)}$$
$$= 4158127$$

$$P(2009) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2009-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(3)}$$
$$= 4266132$$

$$P(2010) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2010-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(4)}$$
$$= 4376943$$

$$P(2011) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2011-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(5)}$$
$$= 4490633$$

$$P(2012) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2012-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(6)}$$
$$= 4607275$$

$$P(2013) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2013-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(7)}$$
$$= 4726947$$

$$P(2014) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2014-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(8)}$$
$$= 48497298$$
$$P(2015) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2015-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(9)}$$
$$= 4975698$$
$$P(2016) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2016-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(10)}$$
$$= 5104940$$
$$P(2017) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2017-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(11)}$$
$$= 5237538$$
$$P(2018) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2018-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(12)}$$
$$= 5373581$$
$$P(2019) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2019-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(13)}$$
$$= 5513158$$
$$P(2020) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2020-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(14)}$$
$$= 5656360$$
$$P(2021) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2021-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(15)}$$
$$= 5803282$$
$$P(2022) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2022-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(16)}$$
$$= 5954020$$
$$P(2023) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2023-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(17)}$$
$$= 6108673$$
$$P(2024) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2024-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(18)}$$
$$= 6267343$$
$$P(2025) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2025-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(19)}$$
$$= 6430136$$

$$P(2026) = 3{,}950{,}249\ell^{(0.3803-0.01316)(2026-2006)}$$
$$= 3{,}950{,}249\ell^{(0.025643)(20)}$$
$$= 6597156$$

III. Results

The population figures used in this paper is obtained from National Population Commission, Census 2006. This is shown in table below:

TABLE 1: Nigeria 2006 Census Figures

	Male	Female	Total	% of the Nation
South-East				
Abia	1,434,193	1,399,806	2,833,999	2.02%
Anambra	2,174,641	2,007,391	4,182,032	2.99%
Ebonyi	1,040,984	1,132,517	2,173,501	1.55%
Enugu	1,624,202	1,633,096	3,257,298	2.33%
Imo	2,032,286	1,902,613	3,934,899	2.81%
SubTotal	8,306,306	8,075,423	16,381,729	11.70%
South-South				
Akwa-Ibom	2,044,510	1,875,698	3,920,208	2.80%
Bayelsa	902,648	800,710	1,703,358	1.22%
Cross-River	1,492,465	1,396,501	2,888,966	2.06%
Delta	2,074,306	2,024,085	4,098,391	2.93%
Edo	1,640,461	1,577,871	3,218,332	2.30%
Rivers	2,710,665	2,474,735	5,185,400	3.70%
SubTotal	10,865,055	10,149,600	21,014,655	15.01%
South-West				
Ekiti	1,212,609	1,171,603	2,384,212	1.70%
Ogun	1,847,243	1,810,855	3,658,098	2.61%
Ondo	1,761,263	1,679,761	3,441,024	2.46%
Osun	1,740,619	1,682,916	3,423,535	2.45%
Oyo	2,809,840	2,781,749	5,591,589	3.99%
Lagos	4,678,020	4,335,514	9,013,534	6.44%
SubTotal	14,049,594	13,462,398	27,511,992	19.65%
North-Central				
Benue	2,164,058	2,055,186	4,219,244	3.01%
Kogi	1,691,737	1,566,750	3,258,487	2.33%
Kwara	1,220,581	1,150,508	2,371,089	1.69%
Nasarawa	945,556	917,719	1,863,275	1.33%
Niger	2,032,725	1,917,524	3,950,249	2.82%
Plateau	1,593,033	1,585,679	3,178,712	2.27%
SubTotal	9,647,690	9,193,366	18,841,056	13.46%
North-East				
Adamawa	1,606,123	1,561,978	3,168,101	2.26%
Bauchi	2,426,215	2,250,250	4,676,465	3.34%
Borno	2,161,157	1,990,036	4,151,193	2.97%
Gombe	1,230,722	1,123,157	2,353,879	1.68%
Taraba	1,199,849	1,100,887	2,300,736	1.64%
Yobe	1,206,003	1,115,588	2,321,591	1.66%
SubTotal	9,830,069	9,141,896	18,971,965	13.55%
North-West				
Jigawa	2,215,907	2,132,742	4,348,649	3.11%
Kaduna	3,112,028	2,954,534	6,066,562	4.33%
Kano	4,844,128	4,539,554	9,383,682	6.70%
Katsina	2,978,682	2,813,896	5,792,578	4.14%
Kebbi	1,617,498	1,621,130	3,238,628	2.31%
Sokoto	1,872,069	1,824,930	3,696,999	2.64%
Zamfara	1,630,344	1,629,502	3,259,846	2.33%
SubTotal	18,270,656	17,516,288	35,786,944	25.56%
Abuja(FCT)				
	740,489	664,712	1,405,201	1.00%
Nigeria				
Nigeria	71,709,859	68,293,683	140,003,542	100.00%

TABLE 2: PREDICTED VALUES OF POPULATION OF NIGER STATE

S/N	YEAR	PROJECTED POPULATION
0	2006	3950249
1	2007	4052855
2	2008	4158127
3	2009	4266132
4	2010	4376943
5	2011	4490633
6	2012	4607275
7	2013	4726947
8	2014	4849728
9	2015	4975698
10	2016	5104940
11	2017	5237538
12	2018	5373581
13	2019	5513158
14	2020	5656360
15	2021	5803282
16	2022	5954020
17	2023	6108673
18	2024	6267344
19	2025	6430136
20	2026	6597156

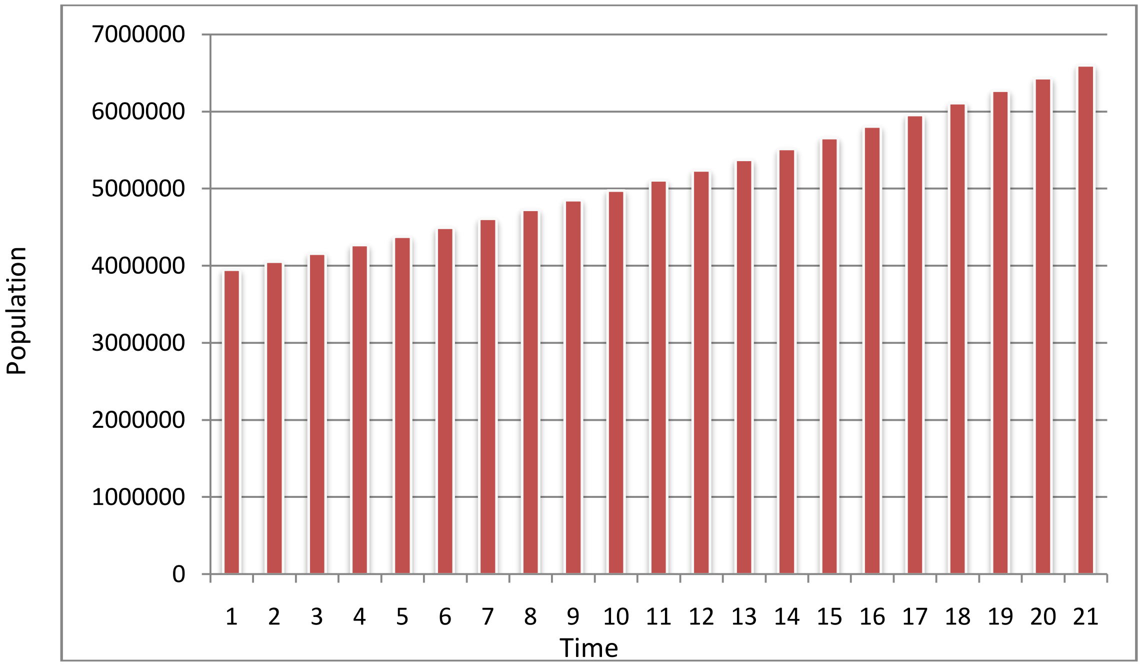

FIGURE 1: Graph of predicted population of Niger State for a period of 20 years

INTERPRETATION: the above graph shows the predicted population value against time. The values were computed using the Growth Model Equation in equation (10)

IV. Conclusion

In conclusion, the population projection gives a picture of what the future population may look like, based on knowledge of the past and taking for the future based on fertility, mortality and migrations. The exponential model predicted Niger state population to be 6,597156 in the year 2026. The population estimate can be used to update population data gather from the last census to approximate the current situation. Lastly, it can be seen in this paper that for any number of year specify as the proposed year for the growth model, the population growth and the corresponding value can be obtained. As seen, the model justifies the population growth and most importantly predicts the estimated amount of the population for the next 20 years.

V. Recommendation

The models of human population growth in this paper can be exploited to predict the population of people in Niger State over the next 20years. The model parameters r and α are of great importance to policy makers of any nation. The knowledge of r and α for the population of any given region helps to check unwanted population increase or decrease and the accuracy of the data also determines the reliability of the prediction results and the management decisions to be arrived at. Also this parameter can be exploited access the success of the method implemented over time. Lastly, to be able to manage the population growth of any nationsuccessfully, government policy makers and planners around the world should used population projection to measure future demand for food, water, energy and services and invest more on housing, education and medical supply to satisfy the increased demand.

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