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A New Approach to Solve Fuzzy Linear Equation $A \cdot X + B = C$

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Abstract: *Solving fuzzy equations has long been a problem in fuzzy set theory.Fuzzy equations were solved by using different standard methods. One of the well-known methods is the method of* $\alpha - cut$*. This article proposed a new and simple solution method to solve a fuzzy linear equation* $A \cdot X + B = C$*,without using* $\alpha - cut$*. The results obtained are also compared with the known solutions and are found to be in good agreement. In this article, we also compare the solutions of different methods by plotting them into graphical representation.*
Keywords: *Density Function, Fuzzy Number, Fuzzy Equation, Probability Distribution,* $\alpha - cut$ *method.*

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I. Introduction

Fuzzy set theory has been widely acclaimed as offering greater richness in applications than ordinary set theory. Lotfi A. Zadeh[1] first introduced the concept of fuzzy set and its applications in 1965. One area of fuzzy set theory in which fuzzy numbers and arithmetic operations on fuzzy numbers play a fundamental role is fuzzyequations. Fuzzy equations were investigated by Dubois and Prade [2]. Sanchez [3] put forward a solution of fuzzy equation by using extended operations. Accordingly various researchersBuckley [4], Wasowski [5], Biacino and Lettieri [6] have proposed different methods for solving the fuzzy equations. After this a lot research papers have appeared proposing solutions of various types of fuzzy equations viz. algebraic fuzzy equations, a system of fuzzy linear equations, simultaneous linear equations with fuzzy coefficients etc. using different methods ([see e.g. Jiang [7], Buckley and Qu [8]). Buckley and Qu in [9], [10], [11] defined the concept of solving fuzzy linear equations $A \cdot X + B = C$, where A, B, C and X are fuzzy numbers, by using the method of $\alpha - cut$and their work has been influential in the study of fuzzy linear systems.

Mazarbhuiya [12], Baruah [13], Chou [14], defined the arithmetic operations of fuzzy numbers without using the method of $\alpha - cuts$. In this article, we present a new approach of solving fuzzy linear equation $A \cdot X + B = C$, without utilizing the standard method. Our procedure is based on fuzzy arithmetic without $\alpha - cut$. We useddistribution function and complementary distribution function to solve the equation with numerical examples.

The paper is arranged as follows. In Section 2,we discuss about the definitions and notations used in this article. In Section 3,we have methodically reviewed the classical methods, extension principle, $\alpha - cut$ and interval arithmetic of solving fuzzy linear equations with numerical examples and their graphs. In Section 4,we introduce a new solution technique of $A \cdot X + B = C$,without using $\alpha - cut$where we get the solution$X_{W.C.}$ and compare the new solution with the old solution by graphical representation. All the graphs are plotted by using the computer programming MATLABfunction. In section 5, we give a brief conclusion of our task and lines for future.

II. Definitions and Notations

A function$A: X \longrightarrow [0,1]$is called a fuzzy seton X, where$X$is a nonempty set of objects called referential set and$[0, 1]$(the unit interval) is called valuation set and$\forall x \in X;\ A(x)$represents the grade of membership of x.
An $\alpha - cut,\ A^{\alpha}$of a fuzzy set A is an ordinary set of elements with membership not less than $\alpha$for $0 \leq \alpha \leq 1$. This means, $A^{\alpha} = \{x \in X:\ A(x) \geq \alpha\}$. [15]

The height of A is denoted by $h(A)$ and is defined as $h(A) = \sup\{A(x): x \in X\}$.$A$is called normal fuzzy set iff$h(A) = \sup\{A(x)\} = 1$. [15]
A fuzzy set is said to be convex if all its $\alpha - cuts$ are convex sets. [16]
A fuzzy number is a convex normal fuzzy set A define on the real line such that $A(x)$ is piecewise continuous.
The support of a fuzzy set $A$is denoted by supp$(A)$and is defined as the set of elements with membership nonzero *i.e.*,supp$(A) = \{x \in X: A(x) > 0\}$. [15]

A fuzzy number A, denoted by a triad $[a, b, c]$ such that $A(a) = 0 = A(c)$and $A(b) = 1$, where$A(x)$ for $x \in [a, b]$is called the left reference function and for $x \in [b, c]$is called right reference function. The left reference function is right continuous monotone and non-decreasing whereasright reference function is left continuous, monotone and non-increasing. The above definition of a fuzzy number is called L-R fuzzy number [17].

If Z is a continuous random variable with density function $f(z)$, then distribution of Z , denoted by $\mu(z)$, is defined by $\mu(z) = Prob[Z \leq z] = \int_{-\infty}^{z} f(z)\, dz$ and thecomplementary distribution of Z,denoted by $\bar{\mu}(z)$, is defined by $\bar{\mu}(z) = 1 - Prob[Z \leq z] = 1 - \int_{-\infty}^{z} f(z)\, dz$. [18]

III. Solving Fuzzy Linear Equation $A \cdot X + B = C$ by using the Method of $\alpha - cut$

LetA, B and C be the triangular fuzzy numbers and let= $[a_1, a_2, a_3]$; $B = [b_1, b_2, b_3]$; $C = [c_1, c_2, c_3]$.

Then $A \cdot X + B = C$ … … … (3.1) is a fuzzy linear equation.

X, if it exists, will be a triangular shaped fuzzy number. Let $X \approx [x_1, x_2, x_3]$, then $X = \frac{C-B}{A}$is not a solution of equation (3.1).

3.1.Classical Method [9]

We say that this method defines the solutionX_c. For any$\alpha \in [0,1]$,

Let $A^{\alpha} = [a_1(\alpha), a_2(\alpha)]$; $B^{\alpha} = [b_1(\alpha), b_2(\alpha)]$; $C^{\alpha} = [c_1(\alpha), c_2(\alpha)]andX_c^{\alpha} = [x_1(\alpha), x_2(\alpha)]$ denote, respectively the $\alpha - cuts$ ofA, B, C and X in the given equation (3.1). Substituting these into equation (3.1),we find

$$[a_1(\alpha), a_2(\alpha)][x_1(\alpha), x_2(\alpha)] + [b_1(\alpha), b_2(\alpha)] = [c_1(\alpha), c_2(\alpha)] \quad \ldots \ldots (3.1.1)$$

Example 3.1: Let $A = [8, 9, 10]$; $B = [-3, -2, -1]$; $C = [3, 5, 7]$

Therefore$A^{\alpha} = [8 + \alpha, 10 - \alpha]$; $B^{\alpha} = [-3 + \alpha, -1 - \alpha]$; $C^{\alpha} = [3 + 2\alpha, 7 - 2\alpha]$

Since $A > 0$ and $C > 0$, we must have $X_c > 0$ and equation (3.1.1) gives

$$[(8 + \alpha)x_1(\alpha) + (-3 + \alpha), (10 - \alpha)x_2(\alpha) + (-1 - \alpha)] = [3 + 2\alpha, 7 - 2\alpha]$$

Implies that$x_1(\alpha) = \frac{6+\alpha}{8+\alpha}andx_2(\alpha) = \frac{8-\alpha}{10-\alpha}$.

We see that $x_1(\alpha)$ is increasing (its derivative is positive),$x_2(\alpha)$ is decreasing (derivative is negative) and $x_1(1) = \frac{7}{9} = x_2(1)$. The solution $X_c$exists with $\alpha - cut$, $X_c^{\alpha} = \left[\frac{6+\alpha}{8+\alpha}, \frac{8-\alpha}{10-\alpha}\right]$.

The fuzzy membership function of $X_c$is given by,

$$X_c(x) = \begin{cases} 36x - 27, & \frac{3}{4} \leq x \leq \frac{7}{9} \\ 36 - 45x, & \frac{7}{9} \leq x \leq \frac{4}{5} \\ 0, & x \leq \frac{3}{4} \text{ or } x \geq \frac{4}{5} \end{cases}$$

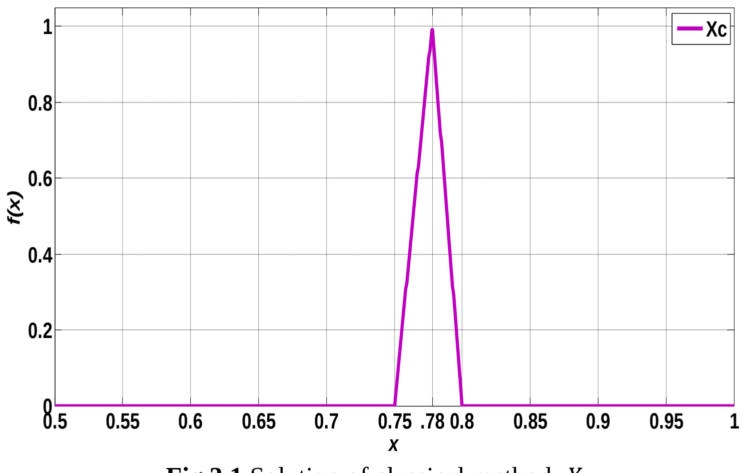

Fig 3.1. Solution of classical method, X_c

3.2.Extension Principal [10]

If X_e is the value of equation (3.1) by using extension principal, then

$$X_e(x) = max\,\{\pi\,(a\,,b\,,c)\mid \frac{(c\,-\,b)}{a} = x\} \qquad \dots\dots(3.2.1)$$

where

$$\pi\,(a\,,b\,,c) = \,min\,\{A(a), B(b), C(c)\} \qquad \dots\dots(3.2.2)$$

Since the expression $\frac{c\,-\,b}{a}$, $a \neq 0$, is continuous in a, b, c we know how to find α – $cut$of X_e.

$$x_{e1}(\alpha) = min\left\{\frac{c\,-\,b}{a}\mid a\,\in\,A^{\alpha}\,,b\,\in\,B^{\alpha},c\,\in\,C^{\alpha}\right\} \qquad \dots\dots(3.2.3)$$

$$x_{e2}(\alpha) = max\left\{\frac{c\,-\,b}{a}\mid a\,\in A^{\alpha}\,,b\,\in B^{\alpha}\,,c\,\in C^{\,\alpha}\right\} \qquad \dots\dots(3.2.4)$$

Therefore $X_e^{\alpha} = [x_{e1}(\alpha), x_{e2}(\alpha)]$ $\qquad \dots\dots(3.2.5)$

Example 3.2:Given that $A = [8\,,9\,,10]$; $\quad B = [-3\,,-2\,,-1]$; $\quad C = [3\,,5\,,7]$.

$\therefore C^{\alpha} - \,B^{\alpha} = [4 + 3\alpha\,, 10 - 3\alpha\,]and\frac{1}{A^{\alpha}} = \left[\frac{1}{10-\,\alpha}\,,\ \frac{1}{8+\,\alpha}\right]$

Now

$$x_{e1} = \left\{\frac{4\,+3\alpha}{10-\,\alpha}\right\}\text{and}\,x_{e2} = \left\{\frac{10-3\alpha}{8\,+\alpha}\right\}$$

Hence

$$X_e^{\alpha} = \left[\frac{4\,+\,3\alpha}{10\,-\,\alpha}\,,\frac{10\,-\,3\alpha}{8\,+\,\alpha}\right]$$

The fuzzy membership function of $X_e$is given by

$$X_e(x) = \begin{cases} \dfrac{45x - 18}{17}\,, & \dfrac{2}{5} \leq x \leq \dfrac{7}{9} \\ \dfrac{45 - 36x}{17}\,, & \dfrac{7}{9} \leq x \leq \dfrac{5}{4} \\ 0\ \ ,\ x \leq \dfrac{2}{5}\ or\ x \geq \dfrac{5}{4} & \end{cases}$$

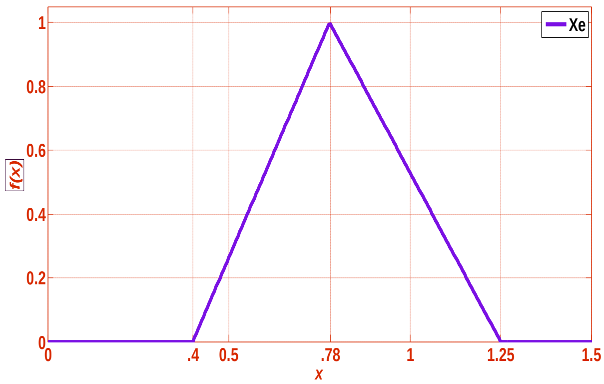

Fig 3.2. Solution X_e

3.3.Interval Arithmetic [10]

Another way to evaluate equation (3.1) is to use α – $cuts$ and interval arithmetic, we get the solution X_I where

$$X_I^{\alpha} = \frac{C^{\alpha}\,-\,B^{\alpha}}{A^{\alpha}} \qquad \dots\dots(3.3.1)$$

to be simplified by the interval arithmetic.

Example 3.3:Given that $\quad A = [8\,,9\,,10]$; $\quad B = [-3\,,-2\,,-1]$; $\quad C = [3\,,5\,,7]$.

$$\therefore \frac{C^{\alpha}\,-\,B^{\alpha}}{A^{\alpha}} = \left[\frac{4\,+\,3\alpha}{10\,-\,\alpha}\,,\qquad \frac{10\,-\,3\alpha}{8\,+\,\alpha}\right]$$

Here we see that the solutions of equation (3.1) by extension principle X_e and by interval arithmetic X_I are same. That is $X_e = X_I$.
The fuzzy membership function of X_I is given by

$$X_I(x) = \begin{cases} \frac{45x - 18}{17}, & \frac{2}{5} \leq x \leq \frac{7}{9} \\ \frac{45 - 36x}{17}, & \frac{7}{9} \leq x \leq \frac{5}{4} \\ 0 \ , & x \leq \frac{2}{5} \ or \ x \geq \frac{5}{4} \end{cases}$$

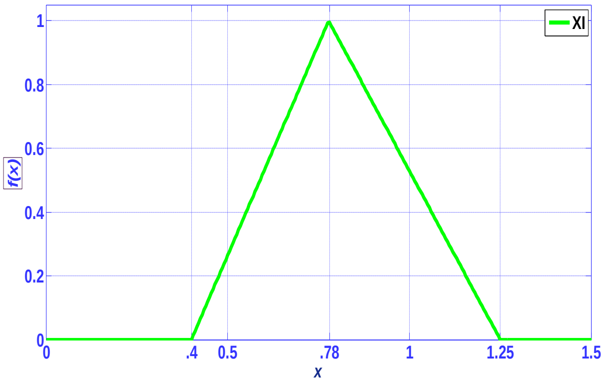

Fig 3.3. Solution X_I

IV. Solving Fuzzy Linear Equation $A \cdot X + B = C$ Without Using $\alpha - cut$

In this section we are going to solve the simple fuzzy linear equation $A \cdot X + B = C$ by the method of without using $\alpha - cut$. A new way to evaluate equation (3.1) is to use distribution functionand complementary distributionfunction and we get the solution which we denoted by $X_{W.C.}$. Here we have followed [12], [13], [14]. Chou [14] has forwarded a method of finding the fuzzy membership function for a triangular fuzzy numbers X. We now define a fuzzy number $X = [a, b, c]$ with membership function

$$X(x) = \begin{cases} L(x), & a \leq x \leq b \\ R(x), & b \leq x \leq c \\ 0, & otherwise \end{cases} \qquad \ldots\ldots(4.1)$$

Where $L(x)$ being continuous non-decreasing function in the interval $[a, b]$ and $R(x)$ being a continuous non -increasing function in the interval $[b, c]$ with $L(a) = R(c) = 0$and $L(b) = R(b) = 1$. Where $L(x)$ is left reference function and $R(x)$ is right reference function of the concerned fuzzy number. In this discussion, we are going to demonstrate the easiness of applying this method in evaluating the arithmetic of fuzzy numbers if start from the simple assumption that the left reference function is a distribution function, and similarly the right reference function is a complementary distribution function. Accordingly, the functions $L(x)$ and $(1 - R(x))$ would have to be associated with densities

$$\frac{d}{dx} L(x) \text{ in } [a, b] \quad \text{and} \quad \frac{d}{dx}\big(1 - R(x)\big) \text{ in } [b, c].$$

Now consider $X = [a, b, c]$ and $Y = [p, q, r]$ be two triangular fuzzy numbers.
Suppose $Z = X * Y = [a * p, b * q, c * r]$ be the fuzzy number of $X * Y$, where$*$ denotes addition$(+)$ and multiplication$(\cdot)$.

Let $$X(x) = \begin{cases} L(x), & a \leq x \leq b \\ R(x), & b \leq x \leq c \\ 0, & otherwise \end{cases} ; \quad Y(y) = \begin{cases} L(y), & p \leq y \leq q \\ R(y), & q \leq y \leq r \\ 0, & otherwise \end{cases}$$

Where $L(x)$ and $L(y)$ are the left reference functions and $R(x)$ and $R(y)$ are the right reference functions respectively. We assume that $L(x)$ and $L(y)$ are distribution functions and $R(x)$ and $R(y)$ are complementary distribution functions. Accordingly, there would exist some density functions for the distribution functions $L(x)$and $(1 - R(x))$. Let

$$f(x) = \frac{d}{dx}\big(L(x)\big),\ a \le x \le b \quad \text{and} \quad g(x) = \frac{d}{dx}\big(1 - R(x)\big),\ b \le x \le c$$

$$m(t) = -1 \text{ or } -\frac{1}{t^2} \text{ for subtraction and division respectively.}$$

We start with equating $L(x)$ with $L(y)$, and$R(x)$ with $R(y)$. And so, we obtain

$$y = \varphi_1(x) \quad \text{and} \quad y = \varphi_2(x) \ \text{respectively.}$$

Let $z = x * y$, so we have $z = x * \varphi_1(x)$and $z = x * \varphi_2(x)$

so that $x = \psi_1(z)$ and $x = \psi_2(z)$ (say).

Replacing x by $\psi_1(z)$ in $f(x)$, and$\psi_2(z)$ in $g(x)$, we get

$$f(x) = \eta_1(z) \text{and} g(x) = \eta_2(z) \quad \text{(say).}$$

Now let

$$\frac{dx}{dz} = \frac{d}{dz}\big(\psi_1(z)\big) = m_1(z) \quad \text{and} \quad \frac{dx}{dz} = \frac{d}{dz}\big(\psi_2(z)\big) = m_2(z)$$

Therefore, the fuzzy arithmetic without $\alpha - cut$ would exists as follows:

Addition:

$$(X+Y)(x) = \begin{cases} \displaystyle\int_{a+p}^{x} \eta_1(z) m_1(z)\,dz\,, & a+p \le x \le b+q \\ 1 - \displaystyle\int_{b+q}^{x} \eta_2(z) m_2(z)\,dz\,, & b+q \le x \le c+r \\ 0\,, & otherwise \end{cases}$$

Subtraction:

$$(-Y)(y) = \begin{cases} \displaystyle\int_{-r}^{y} \eta_2(t) m(t)\,dt\,, & -r \le y \le -q \\ 1 - \displaystyle\int_{-q}^{y} \eta_1(t) m(t)\,dt\,, & -q \le y \le -p \\ 0\,, & otherwise \end{cases}$$

Multiplication:

$$(X \cdot Y)(x) = \begin{cases} \displaystyle\int_{ap}^{x} \eta_1(z) m_1(z)\,dz\,, & ap \le x \le bq \\ 1 - \displaystyle\int_{bq}^{x} \eta_2(z) m_2(z)\,dz\,, & bq \le x \le cr \\ 0\,, & otherwise \end{cases}$$

Division:

$$(Y^{-1})(y) = \begin{cases} \displaystyle\int_{r^{-1}}^{y} \eta_2(t) m(t)\,dt\,, & r^{-1} \le y \le q^{-1} \\ 1 - \displaystyle\int_{q^{-1}}^{y} \eta_1(t) m(t)\,dt\,, & q^{-1} \le y \le p^{-1} \\ 0\,, & otherwise \end{cases}$$

Example 4.1:Given that $A = [8\,, 9\,, 10]$; $B = [-3\,, -2\,, -1]$; $C = [3\,, 5\,, 7]$.

Fuzzy membership function of $A,\ B\ and\ C$ are

$$A(x) = \begin{cases} x - 8\,, & 8 \le x \le 9 \\ 10 - x\,, & 9 \le x \le 10 \\ 0\,, & x \le 8 \ or\ x \ge 10 \end{cases}$$

$$B(x) = \begin{cases} x+3, & -3 \le x \le -2 \\ -1-x, & -2 \le x \le -1 \\ 0, x \le -3 \text{ or } x \ge -1 \end{cases}$$

$$C(x) = \begin{cases} \frac{x-3}{2}, & 3 \le x \le 5 \\ \frac{7-x}{2}, & 5 \le x \le 7 \\ 0, x \le 3 \text{ or } x \ge 7 \end{cases}$$

We have to find the fuzzy membership function of $X_{W.C.}$, that is

$$X_{W.C.} = \left(\frac{C-B}{A}\right)(x) \qquad \dots\dots (4.2)$$

For fuzzy membership function of$(C-B)(x)$:

$$C(x) = \begin{cases} \frac{x-3}{2}, & 3 \le x \le 5 \\ \frac{7-x}{2}, & 5 \le x \le 7 \\ 0, & x \le 3 \text{ or } x \ge 7 \end{cases} \qquad \dots\dots (4.3)$$

$$B(y) = \begin{cases} y+3, & -3 \le y \le -2 \\ -1-y, & -2 \le y \le -1 \\ 0, & y \le -3 \text{ or } y \ge -1 \end{cases} \qquad \dots\dots (4.4)$$

Suppose $Z = C - B$ or $Z = C + (-B)$. Now $-B = [1, 2, 3]$ be the fuzzy number of $(-B)$.
Let $t = -y$ so that $m(t) = \frac{dy}{dt} = -1$.
Then the density function $f(y)$ and $g(y)$ would be, say,

$$f(y) = \frac{d}{dy}(y+3) = 1 = \eta_1(t), \quad -3 \le y \le -2$$

$$g(y) = \frac{d}{dy}(1-(-1-y)) = 1 = \eta_2(t), \quad -2 \le y \le -1$$

Then the fuzzy membership function of $(-B)$is given by

$$\mu_{-B}(y) = \begin{cases} y-1, & 1 \le y \le 2 \\ 3-y, & 2 \le y \le 3 \\ 0, & y \le 1 \text{ or } y \ge 3 \end{cases} \qquad \dots\dots (4.5)$$

If $L(x)$ and $L(y)$ are the left reference function and $R(x)$ and $R(y)$ are the right reference functions respectively then we assume that $L(x)$ and $L(y)$ are distribution function and $R(x)$ and $R(y)$ are complementary distribution function.

$$\therefore L(x) = \frac{x-3}{2} \quad \text{and} \quad R(x) = \frac{7-x}{2}$$

$$L(y) = y - 1 \quad \text{and} \quad R(y) = 3 - y$$

There would exist some density functions for $L(x)$and $(1 - R(x))$.
Say,

$$f(x) = \frac{d}{dx}(L(x)) = \frac{d}{dx}\left(\frac{x-3}{2}\right) = \frac{1}{2}, \quad 3 \le x \le 5$$

and

$$g(x) = \frac{d}{dx}\big(1-R(x)\big) = \frac{d}{dx}\left(\frac{7-x}{2}\right) = \frac{1}{2}, \quad 5 \le x \le 7$$

Here

$$C + (-B) = [4, 7, 10]$$

Equating distribution function and complementary distribution function, we get

$$L(x) = L(y) \quad \text{and} \quad R(x) = R(y)$$

$$\frac{x-3}{2} = y - 1 \quad \text{and} \quad \frac{7-x}{2} = 3 - y$$

$$y = \frac{x-1}{2} = \varphi_1(x) \quad \text{and} \quad y = \frac{x-1}{2} = \varphi_2(x)$$

Let $z = x + y$, so we have

$$z = x + \varphi_1(x) = \frac{3x - 1}{2} \quad \text{and} \quad z = x + \varphi_2(x) = \frac{3x - 1}{2}$$

So that,

$$x = \frac{2z + 1}{3} = \psi_1(z) \quad \text{and} \quad x = \frac{2z + 1}{3} = \psi_2(z) \text{ respectively.}$$

Replacing x by $\psi_1(z)$ in $f(x)$and$\psi_2(z)$in $g(x)$, we obtain

$$f(x) = \frac{1}{2} = \eta_1(z) \quad \text{and} \quad g(x) = \frac{1}{2} = \eta_2(z)$$

Now let

$$m_1(z) = \frac{d}{dz}(\psi_1(z)) = \frac{2}{3} \quad \text{and} \quad m_2(z) = \frac{d}{dz}(\psi_2(z)) = \frac{2}{3}$$

Then fuzzy membership function of$(C - B)$ would be given by,

$$(C - B)(x) = \begin{cases} \int_4^x \eta_1(z)m_1(z)dz\,, 4 \le x \le 7 \\ 1 - \int_7^x \eta_2(z)m_2(z)dz\,, 7 \le x \le 10 \\ 0 \quad , \quad x \le 4 \text{ or } x \ge 10 \end{cases} \quad \ldots\ldots(4.6)$$

Hence we get from equation (4.6)

$$(C - B)(x) = \begin{cases} \frac{x - 4}{3} \;, \; 4 \le x \le 7 \\ \frac{10 - x}{3} \;, \; 7 \le x \le 10 \\ 0 \;, \; x \le 4 \text{ or } x \ge 10 \end{cases} \quad \ldots\ldots(4.7)$$

For fuzzy membership function of$\left(\frac{C-B}{A}\right)(x)$:Again suppose $Z = \frac{C-B}{A} = (C - B) \cdot (A^{-1})$.
Then fuzzy membership function of$\frac{1}{A} = \left[\frac{1}{10}, \frac{1}{9}, \frac{1}{8}\right]$is given by

$$\frac{1}{A}(y) = \begin{cases} 10 - \frac{1}{y}, \; \frac{1}{10} \le y \le \frac{1}{9} \\ \frac{1}{y} - 8 \;, \; \frac{1}{9} \le y \le \frac{1}{8} \\ 0 \;, \; y \le \frac{1}{10} \text{ or } y \ge \frac{1}{8} \end{cases} \quad \ldots\ldots(4.8)$$

Then by multiplication of fuzzy numbers $(C - B) = [4, 7, 10]$and$\frac{1}{A} = \left[\frac{1}{10}, \frac{1}{9}, \frac{1}{8}\right]$
That is

$$Z = \frac{C - B}{A} = \left[\frac{2}{5}, \frac{7}{9}, \frac{5}{4}\right]$$

If $L(x)$ and $L(y)$ are the left reference function and $R(x)$ and $R(y)$ are the right reference functions respectively then we assume that $L(x)$ and $L(y)$ are distribution function and $R(x)$ and $R(y)$ are complementary distribution function.

$$\therefore \; L(x) = \frac{x - 4}{3} \quad \text{and} \quad R(x) = \frac{10 - x}{3}$$

$$L(y) = 10 - \frac{1}{y} \quad \text{and} \quad R(y) = \frac{1}{y} - 8$$

There would exist some density functions of $L(x)$and $(1 - R(x))$, then

$$f(x) = \frac{d}{dx}(L(x)) = \frac{1}{3}\,, \quad 4 \le x \le 7$$

$$g(x) = \frac{d}{dx}(1 - R(x)) = \frac{1}{3}, \qquad 7 \le x \le 10$$

Equating distribution function and complementary distribution function, we obtain

$$y = \frac{3}{34 - x} = \varphi_1(x) \qquad \text{and} \qquad y = \frac{3}{34 - x} = \varphi_2(x)$$

$$z = x \cdot y = x \cdot \varphi_1(x) = \frac{3x}{34 - x} \qquad \text{and} \qquad z = x \cdot y = x \cdot \varphi_2(x) = \frac{3x}{34 - x}$$

So that,

$$x = \frac{34z}{z + 3} = \psi_1(z) \qquad \text{and} \qquad x = \frac{34z}{z + 3} = \psi_2(z) \quad \text{respectively.}$$

Replacing x by $\psi_1(z)$ in $f(x)$ and $\psi_2(z)$in $g(x)$, we obtain

$$f(x) = \frac{1}{3} = \eta_1(z) \qquad \text{and} \qquad g(x) = \frac{1}{3} = \eta_2(z)$$

Now let

$$m_1(z) = \frac{d}{dz}(\psi_1(z)) = \frac{102}{(z + 3)^2} \qquad \text{and} \qquad m_2(z) = \frac{d}{dz}(\psi_2(z)) = \frac{102}{(z + 3)^2}$$

Then fuzzy membership function of $Z = \frac{C-B}{A}$ would be given by,

$$\left(\frac{C - B}{A}\right)(x) = \begin{cases} \int_{\frac{2}{5}}^{x} \eta_1(z) m_1(z) dz\,, & \frac{2}{5} \le x \le \frac{7}{9} \\ 1 - \int_{\frac{7}{9}}^{x} \eta_2(z) m_2(z) dz\,, & \frac{7}{9} \le x \le \frac{5}{4} \\ 0 & , \ x \le \frac{2}{5} \ or \ x \ge \frac{5}{4} \end{cases} \qquad \dots\dots (4.9)$$

Hence we get the solution $X_{W.C.}$ from equation (4.9)and the fuzzy membership function of the solution $X_{W.C.}$ is given by

$$X_{W.C.}(x) = \begin{cases} \frac{10x - 4}{x + 3}, & \frac{2}{5} \le x \le \frac{7}{9} \\ \frac{10 - 8x}{x + 3}, & \frac{7}{9} \le x \le \frac{5}{4} \\ 0 & , x \le \frac{2}{5} \ or \ x \ge \frac{5}{4} \end{cases}$$

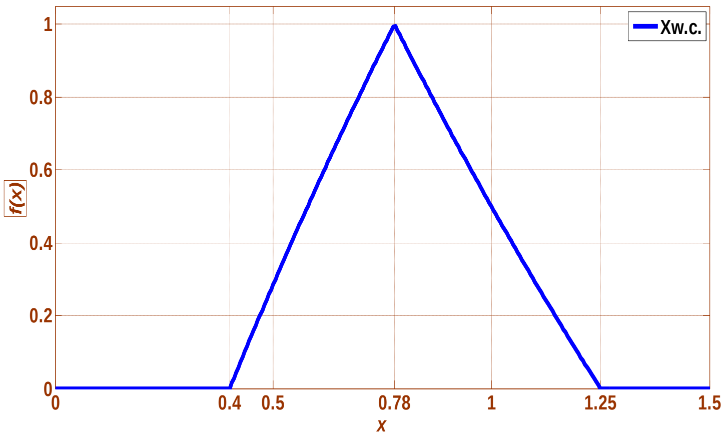

Fig 4.1. Solution $X_{W.C.}$

The comparison of four solutions in different methods of same problem is shown in the following figure:

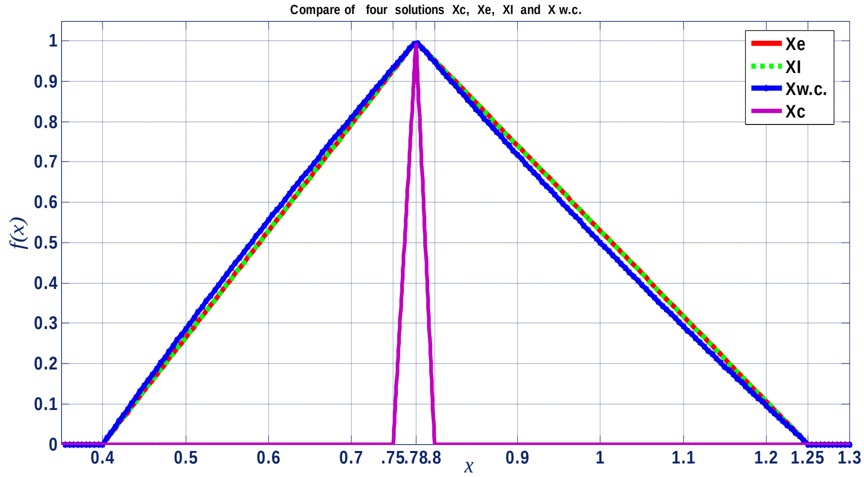

Fig 4.2. Solution $X_c, X_e, X_I, X_{W.C.}$

V. Conclusions

The classical methods, involving the extension principle and $\alpha - cuts$, are too restrictive because very often there is no solution or very strong conditions must be placed on the equations so that there will be a solution. This hampered the application of fuzzy set theory to algebra, economics, decision theory, physics, etc. These facts motivated us to develop a new solution technique. In this article we have acquainted with a new technique to solve fuzzy linear equation$A \cdot X + B = C$. We also compared our solution to solutions based on$\alpha - cuts$ and the extension principle, when they existed. In future we will try to formulate a full method to solve $A \cdot X + B = C$ without using $\alpha - cut$.

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