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Improved Second-Degree Lindley Distribution And Its Applications

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Abstract: *In this paper a new one-parameter lifetime distribution named "Improved Second-Degree Lindley Distribution" which is a modification of Lindley Distribution, with an increasing hazard rate for modeling lifetime data has been suggested. Its first four moments about origin and mean have been deduced and expressions for mean, variance, coefficient of variation, skewness, kurtosis and index of dispersion have been obtained. Various mathematical and statistical properties of the proposed distribution including its survival function, hazard rate function, mean deviations, and Bonferroni and Lorenz curves have been discussed. Estimation of its parameter has been obtained using the method of maximum likelihood and the method of moments. The applications and goodness of fit of the distribution have been discussed with three real lifetime data sets and the fit has been compared with other one-parameter lifetime distributions including Sujatha, Akash, Lindley and exponential distributions.*

Keywords: *lifetime distributions, Sujatha distribution, Akash distribution, Lindley distribution, mathematical and statistical properties, estimation of parameter, goodness of fit.*

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I. Introduction

The analysis and modeling of lifetime data play a crucial role in all branches of applied sciences including engineering, medicine, economics and insurance. There are a number of continuous distributions for modeling lifetime data such as exponential, Lindley, gamma, log-normal and Weibull. Of these, exponential, Lindley and Weibull distributions gained popularity in modeling lifetime data as compared to gamma and log-normal distributions since their survival functions do not require numerical integration. Besides, in the recent past a number of new class of lifetime distributions have evolved in statistical literature, which are in general extensions or modifications or generalizations of Lindley distribution that was used in the context of fiducial and Bayesian statistics (Lindley 1958). The range of such distributions include Power Lindley (Ghitany etal,2013), Two parameters Lindley (Shankar et al,2013), Inverse Lindley (Sharma et al,2015), Generalized inverse Lindley (Sharma et al, 2015), Extended power Lindley (Alkarni, 2015), Extended inverse Lindley (Alkarni, 2015), Aakash distribution (Shankar, 2015), Sujatha distribution (Shankar, 2016) and Amarendra distribution (Shankar, 2016) and each of these distributions has its own advantages and disadvantages in modeling lifetime data.

As a continuation of these models, in this paper, as an extension of Lindley distribution, we have proposed a new one parameter continuous distribution namely ImprovedSecond-Degree Lindley Distribution (ISLD) and it has been shown that it is better than exponential, Lindley, Akash and Sujatha distributions for modeling life time data. We have also discussed various statistical properties including its shape, moment generating function, moments, skewness and kurtosis, hazard rate function, mean and variance, mean deviations, Bonferroni and Lorenz curves, of this new distribution have been discussed. Finally, the maximum likelihood estimation and method of moments for estimating its parameter including the goodness of fit of the proposed distribution using maximum likelihood estimation has been given for data sets and the fit is compared with ones that obtained by other distributions.

II. Improvedsecond-Degree Lindley Distribution

The probability density function (p.d.f) and the cumulative density function (c.d.f) of Lindley distribution (1958) are given by

$$f_1(x;\lambda)=\frac{\lambda^2}{\lambda+1}(1+x)e^{-\lambda x}; \qquad x>0,\ \lambda>0 \tag{2.1}$$

$$F_1(x;\lambda)=1-\left[1+\frac{\lambda x}{\lambda+1}\right]e^{-\lambda x}; \qquad x>0,\ \lambda>0 \tag{2.2}$$

A detailed discussion of Lindley distribution, its mathematical properties, estimation of parameter and application showing advantages of Lindley distribution over exponential distribution can be formed in Ghitnay et al (2008).

A modified version of Lindley distribution by the name Akash distribution was given by Shankar, R (2015) with probability distribution function and cumulative density function as

$$f_2(x;\lambda)=\frac{\lambda^2}{\lambda^2+1}(1+x^2)e^{-\lambda x}; \quad x>0,\ \lambda>0 \tag{2.3}$$

and
$$F_2(x;\lambda)=1-\left[1+\frac{\lambda x(\lambda x+2)}{\lambda^2+2}\right]e^{-\lambda x}; \qquad x>0,\ \ \lambda>0 \tag{2.4}$$

A detailed discussion of this distribution and important mathematical properties shall be found in Shankar. R (2015).Yet another modification of Lindley distribution by the name Sujatha distribution was introduced by Shankar (2016) with probability density function and cumulative density function as

$$f_3(x;\lambda)=\frac{\lambda^3}{\lambda^2+\lambda+2}(1+x+x^2)e^{-\lambda x}; \quad x>0,\ \lambda>0 \tag{2.5}$$

and
$$F_3(x)=1-\left[1+\frac{\lambda x(\lambda x+\lambda+2)}{\lambda^2+\lambda+2}\right]e^{-\lambda x}; \qquad x>0,\ \ \lambda>0 \tag{2.6}$$

and a detailed discussion on this distribution shall be formed in Shankar (2016).

As a continuation of these distributions, we have proposed a new distribution by the name '*ImprovedSecond-Degree Lindley Distribution'* with probability density function and cumulative density function as

$$f(x;\lambda)=\frac{\lambda^3}{\lambda^2+2\lambda+2}(1+x)^2\,e^{-\lambda x}; \qquad x>0,\ \ \lambda>0 \tag{2.7}$$

$$F(x)=1-\left[1+\frac{\lambda^2x^2+2(\lambda^2+\lambda)x}{\lambda^2+2\lambda+2}\right]e^{-\lambda x}; \qquad x>0,\ \ \lambda>0 \tag{2.8}$$

and a detailed discussion including its mathematical properties are as follows.

The graphs of the p.d.f and c.d.f of ISLD (2.7) and (2.8) for different values of λ are shown in figures 1and 2and a detailed discussion including its mathematical properties we as follows.

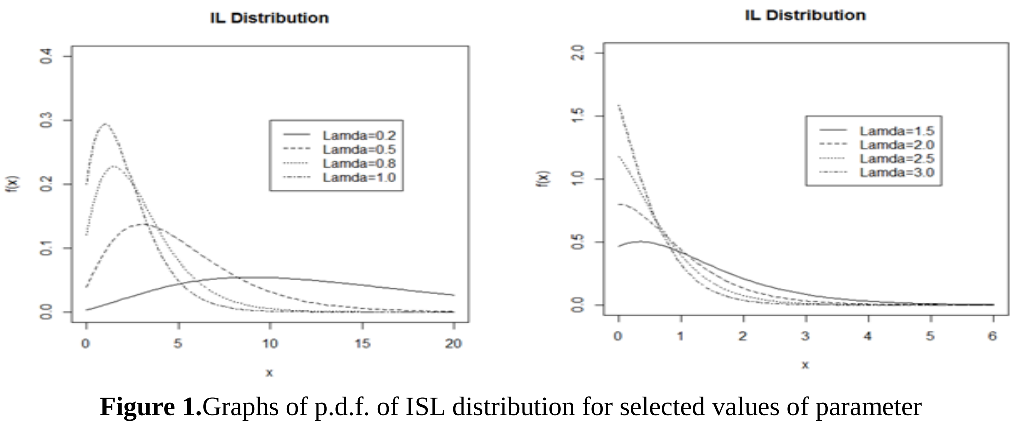

Figure 1. Graphs of p.d.f. of ISL distribution for selected values of parameter

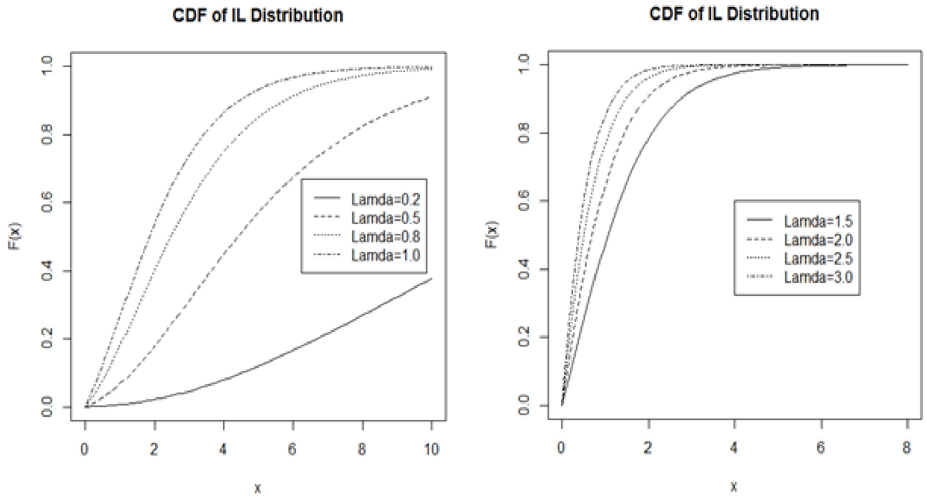

Figure 2. Graphs of c.d.f. of ISL distribution for selected values of parameter

1. Moment Generating Function and expression for moments:

$$M_X(t) = \frac{\lambda^3}{\lambda^2+2\lambda+2}\int_0^{\infty}(1+x)^2 e^{-(\lambda-t)}dx$$

$$= \frac{\lambda^3}{\lambda^2+2\lambda+2}\left[\frac{1}{\lambda-t}+\frac{2}{(\lambda-t)^2}+\frac{2}{(\lambda-t)^3}\right]$$

$$= \frac{\lambda^3}{\lambda^2+2\lambda+2}\left[\frac{1}{\lambda}\sum_{k=0}^{\infty}\left(\frac{t}{\lambda}\right)^k+\frac{2}{\lambda^2}\sum_{k=0}^{\infty}\binom{k+1}{k}\left(\frac{t}{\lambda}\right)^k+\frac{2}{\lambda^3}\sum_{k=0}^{\infty}\binom{k+2}{k}\left(\frac{t}{\lambda}\right)^k\right]$$

$$M_X(t) = \sum_{k=0}^{\infty}\frac{\left[\lambda^2+2\lambda(k+1)+(k+1)(k+2)\right]}{\lambda^2+2\lambda+2}\left(\frac{t}{\lambda}\right)^k \qquad (3.1)$$

The r^{th} moment about origin μ_r' obtained as the coefficient of$t^r/r!$in $M_X(t)$

$$\therefore \quad \mu_r' = \frac{r!\left[\lambda^2+2\lambda(r+1)+(r+1)(r+2)\right]}{\lambda^r(\lambda^2+2\lambda+2)} \qquad r = 1,2,3,4,\ldots$$

The first four moments about origin of MLD are thus obtained as

$$\mu_1' = \frac{\lambda^2+4\lambda+6}{\lambda(\lambda^2+2\lambda+2)} \qquad (3.2)$$

$$\mu_2' = \frac{2(\lambda^2+6\lambda+12)}{\lambda^2(\lambda^2+2\lambda+2)} \qquad (3.3)$$

$$\mu_3' = \frac{6(\lambda^2+8\lambda+20)}{\lambda^3(\lambda^2+2\lambda+2)} \qquad (3.4)$$

$$\mu_4' = \frac{24(\lambda^2+10\lambda+30)}{\lambda^4(\lambda^2+2\lambda+2)} \qquad (3.5)$$

Using the relationship between moments about mean and the moments about origin, the moments about mean of IL distribution (2.7) are obtained as,

$$\mu_2 = \frac{(\lambda^4 + 8\lambda^3 + 24\lambda^2 + 24\lambda + 12)}{\lambda^2(\lambda^2 + 2\lambda + 2)^2} \tag{3.6}$$

$$\mu_3 = \frac{2(\lambda^6 + 12\lambda^5 + 54\lambda^4 + 100\lambda^3 + 108\lambda^2 + 72\lambda + 24)}{\lambda^3(\lambda^2 + 2\lambda + 2)^3} \tag{3.7}$$

$$\mu_4 = \frac{3(3\lambda^8 + 48\lambda^7 + 304\lambda^6 + 944\lambda^5 + 1816\lambda^4 + 2304\lambda^3 + 1920\lambda^2 + 960\lambda + 240)}{\lambda^4(\lambda^2 + 2\lambda + 2)^4} \tag{3.8}$$

III. Mean, Variance And Other Associated Measures

The mean of the distribution (μ) is given by the expression

$$\mu = \frac{\lambda^2 + 4\lambda + 6}{\lambda(\lambda^2 + 2\lambda + 2)} \tag{4.1}$$

And the variance of the distribution (σ^2) is given by

$$\sigma^2 = \frac{\lambda^4 + 8\lambda^3 + 24\lambda^2 + 24\lambda + 12}{\lambda^2(\lambda^2 + 2\lambda + 2)^2} \tag{4.2}$$

The coefficient of variation (C.V) for ISLD is given by

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\lambda^4 + 8\lambda^3 + 24\lambda^2 + 24\lambda + 12}}{\lambda^2 + 4\lambda + 6} \tag{4.3}$$

Based on the first four moments of ISLD, the measures of skewness A(λ) and kurtosis K(λ) of ISLD can be obtained as,

$$A(\lambda) = \frac{\mu_3(\lambda) - 3\mu_1(\lambda).\mu_2(\lambda) + 2\mu_1^3(\lambda)}{\left[\mu_2(\lambda) - \mu_1^2(\lambda)\right]^{3/2}} \tag{4.4}$$

$$K(\lambda) = \frac{\mu_4(\lambda) - 4\mu_1(\lambda).\mu_3(\lambda) + 6\mu_1^2(\lambda).\mu_2(\lambda) - 3\mu_1^4(\lambda)}{\left[\mu_2(\lambda) - \mu_1^2(\lambda)\right]^2} \tag{4.5}$$

Index of dispersion for ISLD is given by,

$$\gamma(\lambda) = \frac{\lambda^4 + 8\lambda^3 + 24\lambda^2 + 24\lambda + 12}{\lambda(\lambda^2 + 2\lambda + 2)(\lambda^2 + 4\lambda + 6)} \tag{4.6}$$

IV. Survival Function And Hazard Rate Function:

If X is a continuous random variable with p.d.f $f(x)$ and c.d.f $F(x)$, then the Survival function $S(x)$ and hazard rate function $h(x)$ (also known as the failure rate function) of X are respectively given by,

$$S(x) = 1 - F(x)$$

$$= \left[\frac{\lambda^2(1+x)^2 + 2\lambda x + 2(\lambda + 1)}{\lambda^2 + 2\lambda + 2}\right] e^{-\lambda x} \quad \text{for } \lambda > 0,\ x > 0 \tag{5.1}$$

and

$$h(x) = \frac{f(x)}{1-F(x)}$$
$$= \frac{\lambda^3 (1+x)^2}{\lambda^2 (1+x)^2 + 2\lambda x + 2(\lambda+1)} \qquad (5.2)$$

It can be verified easily that

$$h(0) = \frac{\lambda^3}{\lambda^2 + 2\lambda + 2} = f(0)$$

The graphs of h (x) of ISLD for different values of parameter are shown in figure

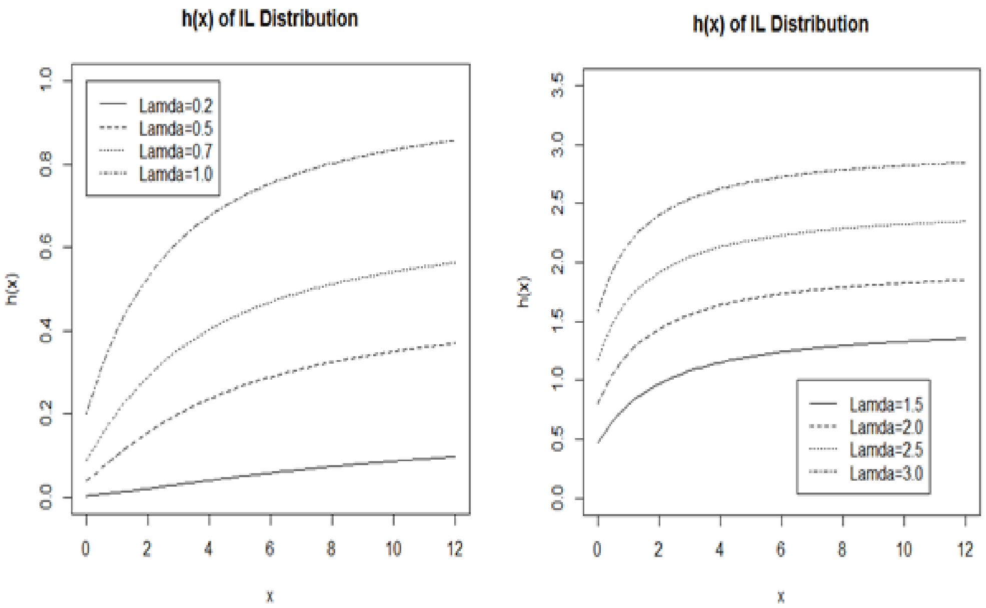

Figure 3. Graphs of *h*(*x*) of IL distribution for selected values of parameter

It shall be noted that *h(x)* is a monotonically increasing function of *x* and λ.

V. Deviations From Mean

The mean deviation about mean in a distribution is a measure of the amount of scatter in a population. It is defined for a continuous distribution with random variable X as follows:

$$\delta(X) = \int_0^{\infty} |x-\mu| \, f(x)\,dx \qquad where \;\; \mu = E(x) \qquad (6.1)$$

$$= \int_0^{\mu} (\mu - x)\, f(x)dx + \int_{\mu}^{\infty} (x-\mu) f(x)dx \qquad (6.2)$$

which on simplification results in

$$\delta(X) = 2\mu\, F(\mu) - 2\int_0^{\mu} x\, f(x)\,dx \qquad (6.3)$$

Now to evaluating second term on the right hand side we have

$$\int_0^{\mu} x\, f(x)\, dx = \int_0^{\mu} x \left[\frac{\lambda^3}{\lambda^2 + 2\lambda + 2} (1+x)^2\, e^{-\lambda x} \right] dx \qquad (6.4)$$

On integration the right hand side reduces to

$$\int_0^{\mu} x\, f(x)\, dx = \frac{2\left[\lambda^3(\mu^3 + 2\mu^2 + \mu) + \lambda^2(3\mu^2 + 4\mu + 1) + \lambda(6\mu + 4) + 6\right] e^{-\lambda\mu}}{\lambda(\lambda^2 + 2\lambda + 2)} \qquad (6.5)$$

Again using,

$$F(x) = 1 - \left[1 + \frac{\lambda^2 x^2 + 2(\lambda^2 + \lambda)x}{\lambda^2 + 2\lambda + 2} \right] e^{-\lambda x} \qquad x > 0; \quad \lambda > 0 \qquad (6.6)$$

In equation (6.1), it reduces to

$$\delta(X) = \frac{2\left[\lambda^2(\mu^2 + 2\mu + 1) + 4\lambda(\mu + 1) + 6\right]}{\lambda(\lambda^2 + \lambda + 2)} e^{-\lambda\mu} \qquad (6.7)$$

This is the expression for mean deviation about the mean of the ISLD distribution.

VI. Bonferroni And Lorenz Curves Related Indices:

Bonferroni curve and Bonferroni index (Bonferroni, 1930) have assumed great importance not only in economics to study income and poverty but also in several fields likedemography, medicine, and insurance.

The importance and application of Bonferroni and Lorenz curves and related indices are well acknowledges in several fields like income, wealth, insurance, reliability, demography and medicine. The Bonferroni and Lorenz curves, for a continuous random variable *X* with probability density function *f(x)* and cumulative distribution function *F(x)*, are defined by

$$B(p) = \frac{1}{p\mu} \int_0^{p} F^{-1}(t)\, dt \qquad (7.1)$$

$$L(p) = \frac{1}{\mu} \int_0^{p} F^{-1}(t)\, dt \qquad (7.2)$$

and corresponding Bonferroni and Gini indices are defined by

$$B = 1 - \int_0^{1} B(p)\, dp \qquad (7.3)$$

$$G = 1 - 2\int_0^{1} L(p)\, dp \qquad (7.4)$$

Now substituting $t = F(x)$ in (7.1) and (7.2) results in

$$B(p) = \frac{1}{p\mu} \int_0^{q} x\ f(x)\, dx \qquad (7.5)$$

which simplifies to

$$B(p) = \frac{1}{p\mu}\left[\mu - \int_q^\infty x\, f(x)\, dx\right] \qquad (7.6)$$

Similarly

$$L(p) = \frac{1}{\mu}\int_0^q x\, f(x)\, dx \qquad (7.7)$$

which simplifies to

$$L(p) = \frac{1}{\mu}\left[\mu - \int_q^\infty x\, f(x)\, dx\right] \qquad (7.8)$$

Where $\mu = E(x)$ and $q = F^{-1}(p)$

Using probability density function of ISL distribution (2.7), we get

$$\int_q^\infty x\, f_4(x)\, dx = \frac{\left[\lambda^3(q^3+2q^2+q)+\lambda^2(3q^2+4q+1)+2\lambda(3q+2)+6\right]e^{-\lambda q}}{\lambda(\lambda^2+2\lambda+2)} \qquad (7.9)$$

$$B(p) = \frac{1}{p}\left\{1 - \frac{\left[\lambda^3(q^3+2q^2+q)+\lambda^2(3q^2+4q+1)+2\lambda(3q+2)+6\right]e^{-\lambda q}}{(\lambda^2+4\lambda+6)}\right\} \qquad (7.10)$$

and

$$L(p) = 1 - \frac{\left[\lambda^3(q^3+2q^2+q)+\lambda^2(3q^2+2q+1)+2\lambda(3q+2)+6\right]e^{-\lambda q}}{(\lambda^2+4\lambda+6)} \qquad (7.11)$$

Using (7.10) and (7.11) in (7.3) and (7.4), The Bonferroni and Gini indices of ISLD are obtained as

$$B = 1 - \frac{\left\{\lambda^3(q^3+2q^2+q)+\lambda^2(3q^2+4q+1)+2\lambda(3q+2)+6\right\}e^{-\lambda q}}{\lambda^2+4\lambda+6} \qquad (7.12)$$

and

$$G = -1 + \frac{\left\{\lambda^3(q^3+2q^2+q)+\lambda^2(3q^2+4q+1)+2\lambda(3q+2)+6\right\}e^{-\lambda q}}{\lambda^2+4\lambda+6} \qquad (7.13)$$

VII. Maximum Likelihood Estimation

The ISLD is given by

$$f(x;\lambda) = \frac{\lambda^3}{\lambda^2+2\lambda+2}(1+x)^2\, e^{-\lambda x} \qquad x>0,\ \lambda>0$$

Let $\{x_1, x_2, x_3, \ldots, x_n\}$ an iid sample from ISLD population

Then the likelihood function of ISLD is,

$$L = \prod_{i=1}^{n}\left\{\frac{\lambda^3(1+x_i)^2\ e^{-\lambda x_i}}{\lambda^2+2\lambda+2}\right\} \qquad (8.1)$$

$$= \frac{\lambda^{3n} \prod_{i=1}^{n} (1+x_i)^2 \, e^{-\lambda \sum x_i}}{(\lambda^2 + 2\lambda + 2)^n} \tag{8.2}$$

The corresponding log-likelihood function is

$$\log L = 3n \log \lambda - n \log(\lambda^2 + 2\lambda + 2) + 2\sum_{i=1}^{n} \log (1+x_i) - \lambda \sum x_i \tag{8.3}$$

$$\frac{\partial}{\partial \lambda}(\log L) = \frac{3n}{\lambda} - \frac{n(2\lambda+2)}{\lambda^2 + 2\lambda + 2} - \sum x_i \tag{8.4}$$

Now,

$$\frac{\partial}{\partial \lambda}(\log L) = 0$$

$$\Rightarrow \; n\left[\frac{3}{\lambda} - \frac{2\lambda+2}{\lambda^2 + 2\lambda + 2}\right] = \sum x_i \tag{8.5}$$

which reduces to a cubic equation in λ and $\overline{X}$ as,

$$\overline{X}\lambda^3 + (2\overline{X} - 1)\lambda^2 + 2(\overline{X} - 2)\lambda - 6 = 0 \tag{8.6}$$

(8.6) can be solved for its solution using any available mathematical software and $\frac{d^2}{d\lambda^2}(\log L)$ shall be used for different values of λ to maximize the likelihood function.

VIII. Method Of Moments Estimation Of The Parameter

The method of moment's estimator is obtained by equating the mean of ISLD to the corresponding sample mean. The equation to estimate λ is the same as the one given by the equation (8.6) and the estimate shall be obtained by solving the equation (8.6).

Advantages and goodness of fit:

In this section, we present examples that illustrate the superiority and applicability of ISLD as compared to Sujatha, Lindley and Exponential distributions using two data sets.

Data set 1: It is a lifetime data relating to relief times (in minutes) of 20 patients that received an analgesic reported by Gross and Clark (1975, Page No: 105).

1.1	1.4	1.3	1.7	1.9	1.8	1.6	2.2	1.7	2.7
4.1	1.8	1.5	1.2	1.4	3.0	1.7	2.3	1.6	2.0

Data Set 2: The following data represent the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20 mm (Bader and Priest, 1982):

1.312 1.314 1.479 1.552 1.700 1.803 1.861 1.865 1.944 1.958 1.966 1.997 2.006 2.021 2.027 2.055 2.063 2.098 2.140 2.179 2.224 2.240 2.253 2.270 2.272 2.274 2.301 2.301 2.359 2.382 2.382 2.426 2.434 2.435 2.478 2.490 2.511 2.514 2.535 2.554 2.566 2.570 2.586 2.629 2.633 2.642 2.648 2.684 2.697 2.726 2.770 2.773 2.800 2.809 2.818 2.821 2.848 2.880 2.954 3.012 3.067 3.084 3.090 3.096 3.128 3.233 3.433 3.585 3.585

Data set 3: The data set is from Lawless (1982). The data given arose in tests on endurance of deep groove ball bearings. The data are the number of million revolutions before failure for each of the 23 ball bearings in the life tests and they are:

17.88	28.92	33.00	41.52	45.60	48.80	51.84	51.96	54.12	55.56	67.80
68.44	68.64	68.88	84.12	93.12	98.64	105.12	105.84	127.92	128.04	173.40

The goodness of fit of ISLD, Sujatha, Lindley and exponential distributions are compared using -2*ln*L, AIC (Akaike Information Criterion), AICC (Akaike Information Criterion Corrected), BIC (Bayesian Information Criterion), and K-S Statistics (Kolmogorow – Smirnov Statistics). The values are computed for above mentioned two data sets and presented in Table-1. The formulae used for computation of AIC, AICC, BIC and K-S statistics are,

$$AIC = -2\ln L + 2k$$

$$AICC = AIC + \frac{2k(k+1)}{(n-k-1)}$$

$$BIC = -2\ln L + k\ln n$$

and

$$D = \sup_x \left| F_n(x) - F_0(x) \right|$$

Where k = The number of parameters

n = The sample size

$F_n(x)$ = The empirical cumulative distribution function.

The following table displays -2lnL, AIC, AICC, BIC and K-S Statistic values obtained for all three data sets

Tabel 1

Data	Model	Parameters	-2lnL	AIC	AICC	BIC	K-S Statistic
1	**ISLD**	**1.1232**	**56.39**	**58.39**	**58.59**	**59.39**	**0.305**
	Sujatha	1.1367	57.50	59.50	59.72	60.49	0.309
	Aakash	1.1569	59.52	61.52	61.74	62.51	0.320
	Lindley	0.8161	60.50	62.50	62.72	63.49	0.341
	Exponential	0.5263	65.67	67.67	67.90	68.67	0.389
2	**ISLD**	**0.9270**	**219.92**	**221.92**	**221.98**	**224.15**	**0.357**
	Sujatha	0.9361	221.61	223.61	223.67	225.84	0.319
	Aakash	0.9647	224.28	226.28	226.34	228.51	0.348
	Lindley	0.6590	238.38	240.38	240.38	242.61	0.390
	Exponential	0.4079	261.74	263.74	263.74	265.97	0.434
3	**ISLD**	**0.0410**	**227.29**	**229.29**	**229.48**	**230.43**	**0.109**
	Sujatha	0.0431	227.31	229.31	229.50	230.45	0.098
	Aakash	0.0415	227.16	229.16	229.35	230.30	0.1074
	Lindley	0.0273	231.47	233.47	233.66	234.61	0.149
	Exponential	0.0138	242.87	244.87	245.06	246.01	0.263

The best fit of the distribution is the distribution which corresponds to the lower values of -2lnL, AIC, AICC, BIC and K-S Statistics. It is clear from the Table 1 that Improved Second-Degree Lindley Distribution (ISLD) provides better fit than exponential, Lindley, Aakash and Sujatha, distributions for data sets 1 and 2 while for data set 3, ISLD provides a good fit like Sujatha and Aakash distributions.

IX. Conclusion

In this paper, a new lifetime data modeling, one parameter continuous distribution named "Improved Second-Degree Lindley Distributions (ISLD)" has been introduced and various statistical properties of the distribution have been discussed. It has been shown that the proposed ISLD provides better fit in modeling life time data than exponential, Lindley, Aakash and Sujatha, distributions using three different data sets. The

Improved Second-Degree Lindley Distribution shall serve as a better alternative to Sujatha and Aakash distributions.

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