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The Modified Adomian Decomposition Method for the Solution of Third Order Ordinary Differential Equations

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Abstract: *In this paper, we introduced the concept of the Modified Adomian Decomposition (MADM) in solving third-order ordinary differential equations. Some test problems were considered and the results are compared with the existing methods in the literature. The convergent of the MADM was so rapid and involved only few terms of the series and by far better than the Seven-Step Block method, Variation Iteration Method and Differential Transform method.*

***Keywords**: Ordinary differential equations, Modified Adomian Decomposition method, Initial value problems (IVP)*

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I. Introduction

Most phenomena in sciences, economics, management, engineering etc. can be modeled by differential and integral theories. Interestingly, solutions to most of the differential equations arising from such models do not have analytic solutions necessitating the development of numerical techniques.However, most of these methods require tedious analysis, large computational work, and lack of existing rule in the selection of initial value, scientifically unrealistic assumptions, perturbation, linearization and discretization of the variable which givesrise to rounding off errors causing loss of Physical nature of the problem.Some of the numerical methods used in literature include; variational Iteration method Kiymaz (2010),homotopy perturbation method Desai and Pradhan (2013).Seven-step block method.Okuboye (2015)The modified Adomian decomposition method proposed in this work is easy to apply; it requires no linearization or discretization, it is free of trial and error method in the selection of initial value. The method was first introduced by Adomian (1976) to solve nonlinear stochastic differential equations. It has been widely applied to numerous problems,Biazar et al. (2008) for Riccati differential equations.Opanuga et al (2014), Edeki et al (2014),Gbadeyan and Agboola (2012) used DTM to solve vibration problem. Two examples solved in this present work gave the exact solution and the resultof the third is compared with the exact solution in form of table.

II. Fundamentals Of The Modified Adomian Decomposition Method

The Adomian decomposition method provides the solution in an infinite series of components. The components $u_n(x), n \geq 0$are easilycomputed if the inhomogeneous term $f(x)$ in the differential equationconsists of a polynomial. However, if the function $f(x)$ consists of a combination of two or more of polynomials, trigonometric functions, hyperbolic functions, and others, the evaluation of the components $u_n(x), n \geq 0$ requires cumbersome work. Going by the work of Wazwaz (2011), a reliable modification of the Adomian decomposition method is presented. The modified decomposition method will facilitate the computational process and further accelerate the convergence of the series solution. The modified ADM depends mainly on splitting the function $f(x)$ into two parts.To give a clear description of the technique, we recall that the standard ADMadmits the use of the recurrence relation. Where the solution$u(x)$ is expressed by an infinite series of components defined before by

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \dots (1)$$

In view of the MADM, the components $u_n(x), n \geq 0$ can be easily evaluated.

The modified ADM introduces a slight variation to the recurrence relation that will lead to the determination of the components of $u(x)$ in an easier and faster manner. The function $f(x)$ can be set as the sum of two partial functions, namely $f_1(x) and f_2(x)$. Wazwaz (2011)
In other words, it can be written as

$$f(x) = f_1(x) + f_2(x) \dots (2)$$

An introduction is made to change the formation of the standard ADM, to minimize the size of calculations, we identified the zeroth component $u_0(x)$ by one part of $f(x)$, the other part of $f(x)$ can be added to the component $u_1(x)$ among other terms. The modified Adomian's Decomposition method (MADM) introduces the modified recurrence relation:

$$u_0(x) = f_1(x),$$

$$u_1(x) = f_2(x) + \lambda \int_0^x k(x,t)u_0\, dt$$

$$\vdots$$

$$u_{n+1}(x) = \lambda \int_0^x k(x,t)u_n(t)\, dt, \qquad n \geq 1 \qquad \dots (3)$$

Therefore

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots = \sum_{n=0}^{\infty} u_n(x) \dots (4)$$

III. Numerical Examples

We illustrate the method by the following problems

Problem 1

Consider the following third-orderordinary differential equation

$u''' = -u(t)$Okuboye (2015)

With the following initial conditions

$$u(0) = 1, u'(0) = -1, u''(0) = 1$$

The exact solution is

$$u(t) = e^{-t} \quad .$$

Integrating both sides three times and applying the n-fold integral formula yields

$$u(t) = 1 - t + \frac{t^2}{2} - \frac{1}{2}\int_0^t (t-x)^2\, u(x)dx$$

Applying the MADM gives

$$u_0(t) = 1$$

$$u_1(t) = -t + \frac{t^2}{2} - \frac{1}{2}\int_0^t (t-x)^2\, u_0(x)dx = -t + \frac{t^2}{2} - \frac{t^3}{6}$$

$$u_2(t) = -\frac{1}{2}\int_0^t (t-x)^2\, u_1(x)dx = \frac{t^4}{24} - \frac{t^5}{120} + \frac{t^6}{720}$$

$$\vdots$$

$$u_n(t) = -\frac{1}{2}\int_0^t (t-x)^2\, u_{n-1}(x)dx$$

Therefore the series solution is

$$u(t) = 1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} - \frac{t^5}{5!} + \frac{t^6}{5!} \dots = e^{-t}$$

Which coincides with the exact solution

Problem 2:

Consider the third-order ordinary differentialequation

$u'''(t) = e^t$, Okuboye (2015)

The initial conditions are:

$$u(0) = 3, u'(0) = 1, u''(0) = 5$$

With exact solution

$$u(t) = 2 + 2t^2 + e^t$$

Integrating both sides three times and applying the n-fold integral formula yields

$$u(t) = 2 + 2t^2 + e^t$$

Applying (2) gives

$$u_0(t) = 2$$
$$u_1(t) = 2t^2 + e^t$$

$u_2(t) = u_3(t) = u_4(t) = 0$

$$\vdots$$

Therefore the series solution is

$$u(t) = 2 + 2t^2 + e^t$$

Which coincides with the exact solution

Problem 3

Consider the following third-order ordinary differential equation
$u'''(t) + 4u(t) = t$, Zanariah (2012)

The initial conditions are:
$$u(0) = 3, u'(0) = 0, u''(0) = 1$$

With exact solution
$$u(t) = \frac{3}{16}(1 - \cos 2t) + \frac{1}{8}t^2$$

Integrating both sides three times and applying the n-fold integral formula yields
$$u(t) = \frac{1}{2}t^2 + \frac{1}{4}t^4 - 4\int_0^t (t-x)u(t)dx$$

Applying (2) gives
$$u_0(t) = \frac{1}{2}t^2$$
$$u_1(t) = \frac{1}{4}t^4 - 4\int_0^t (t-x)u_0(x)dx = \frac{1}{12}t^4$$

$u_2(t) = -4\int_0^t (t-x)u_1(x)dx = -\frac{1}{90}t^6$

$$\vdots$$
$$u_n(t) = -4\int_0^t (t-x)u_{n-1}(x)dx$$

Therefore the series solution is

$$u(t) = \frac{1}{2}t^2 + \frac{1}{12}t^4 - \frac{1}{90}t^6 \ldots$$

Table of Numerical result for problem 3

t	MADM(n=2)	EXACT SOLUTION	Abs. Error
0.1	0.005008322	0.004987517	2.08E-05
0.2	0.020132622	0.019801064	0.000332
0.3	0.045666900	0.043999572	0.001667
0.4	0.082087822	0.076867492	0.005220
0.5	0.130034722	0.117443318	0.012591
0.6	0.190281600	0.164557921	0.025724
0.7	0.263701122	0.216881161	0.046820
0.8	0.351220622	0.27297491	0.078246
0.9	0.453770100	0.331350393	0.122420
1	0.572222222	0.390527532	0.181695

IV. Conclusion

The Modified Adomian Decomposition Method is a promising method for the solution of initial value problems in the treatment of third-order differential equations. This was clearly seen in the test problems considered in this work. In the first two problems the results obtained coincide with the exact solution and the third converges faster and in better agreement with the exact solutions when compared, computational difficulties in other traditional methods were reduced.

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