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Some Statistical Properties of the Beta Exponentiated Gompertz Distribution

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Abstract: *This paper deals with a new five- parameter generalized version of the exponentiated Gompertz model which is called beta exponentiated Gompertz distribution. Gompertz, exponentiated Gompertz,beta Gompertz, exponentiated exponentiated Gompertz, exponential, exponentiated exponential, beta exponentiated exponential, beta exponential distributions are its sub models. We investigate various properties of this model such as mean, median, mode, quantiles, mean residual lifetime, mean deviations and Rényi entropy.The cumulative, density, hazard, survival,moment generating and rth moment expressions are derived. Also, we obtain the variation of the skewness and kurtosis measures. A behavior of the curves for distribution and hazard rate function are discusse.*

Keywords: *Beta Exponentiated Gompertz, Log-Concave,moment generating function, Quantile function,Rényi entropy*

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I. Introduction

The generalized Gompertz distribution was proposed by El-Gohary et al. [1] and the estimation of its parameters based on the progressively type-II censored sample was studied by Ahmed [2]. Further, Borges [3] developed a regression model for survival data in the presence of long-term survivors based on this distribution and developed an expectation maximization algorithm for determining the maximum likelihood estimates of its parameters. Martinez and Achcar [4] proposed a cure fraction model based on a modification of this model in the presence of right-censored data and covariates. Khan et al. [5] introduced the four parameters transmuted generalized Gompertz distribution, and they studied its statistical properties. A new bivariate generalized Gompertz distribution considered by El-Sherpieny et al. [6]. They discussed its properties and obtained the parameters estimators using the maximum likelihood method. Furthermore, Benkhelifa [7] presented a new four-parameter continuous distribution called the Marshall-Olkin extended generalized Gompertz distribution. The odd generalized exponential Gompertz distribution proposed by El-Damcese et al. [8]. They derived some of its mathematical properties. Ade et al. [9] presented the exponentiated generalized Gompertz Makeham distribution. They provided some comprehensive properties of this new distribution. Additionally, Cordeiro et al. [10] defined the exponentiated Gompertz generated family of distributions. They investigated the general structural properties of this new family and estimated the distribution's parameters.

Moreover, Abu-Zinadah and Aloufi [11] introduced the exponentiated Gompertz (EGpz) distribution and studied some of its characterizations. Abu-Zinadah [12] discussed a goodness-of-fit test for this distribution based on type II censored sampling. Also, Abu-Zinadah [13 and 14] derived the Bayes estimates of its shape parameter under type II censoring, and provided six methods of estimations for this parameter. Abu-Zinadah and Aloufi [15 and 16] considereddifferent method estimations for the three parameters of this model based on a complete and type-II censored samples and compared the performances of thes estimators. In addition, the estimators for its two shape parameters based on progressively type-II censored samples have been derived by Abu-Zinadah and Bakoban [17].

Over the last years, the class of beta generalized distributions has been received special attention by many researchers such as Jafari et al. [18] discussed some mathematical properties, the maximum likelihood estimation and the Fisher information matrix for the beta Gompertz distribution. Benkhelifa [19] introduced the beta generalized Gompertz distribution, provided its mathematical treatment, estimated the model parameters, and obtained the information matrix. Bakoban and Abu-Zinadah [20] considered the beta generalized inverted exponential distribution and investigated various properties of it also estimated the model parameters. Yousof et al. [21] proposed and studied a new class of continuous distributions called the beta Weibull-G family. Feroze and Elbatal [22] introduced a new distribution named beta exponentiated gamma distribution, and they discussed its properties. Siddiqui et al. [23] proposed the beta exponentiated Mukherjee-Islam distribution and studied some of its structural properties and obtained the maximum likelihood estimates of all parameters of this new distribution. Mead et al. [24] defined the beta exponential Fréchet distribution and derived some of its statistical properties and estimated the model parameters. Chhetri et al. [25] proposed and studied the beta transmuted Pareto distribution and discussed several mathematical properties and the method of maximum likelihood was proposed to estimate the distribution's parameters. Dikko et al. [26] introduced and investigated the beta –Burr type V distribution and provided some mathematical treatment of this distribution and estimated its parameters via the method of maximum likelihood estimation technique.

In this paper, we introduce a new generalization of the EGpz distribution as a life times model which is called the beta exponentiated Gompertz (BEGpz) distribution. The objective of this work is to study some mathematical properties of the proposed model which can be used in the analysis of survival.

The rest of this paper is organized as follows: In section 2, we provide the cumulative distribution, probability density, survival, and hazard functions of the BEGpz distribution with graphical illustrations. Some special sub models of BEGpz are presented in Section 3. Some statistical properties are derived in Section 4 such as quantiles, moments, moment generating function, mean residual, mean deviations and Rényi entropy. A behavior of the curves for distribution and hazard rate function are discussed in Section 5. Finally, some conclusions are given in Section 6.

II. The Model of BEGpz Distribution

The cumulative distribution function (cdf) of the BEGpz distribution with five parameters is given by

$$F(x;a,b,\lambda,\alpha,\theta) = \frac{1}{B(a,b)} \int_0^{\left(1-e^{-\lambda\left(e^{\alpha x}-1\right)}\right)^{\theta}} w^{a-1}\,(1-w)^{b-1}\;dw \;\; ,x>0, a,b,\lambda,\alpha,\theta>0, \qquad (1)$$

where a, b, λ and θ are called the shape parameters and α is scale parameter.By using binomail series expansion, we found:

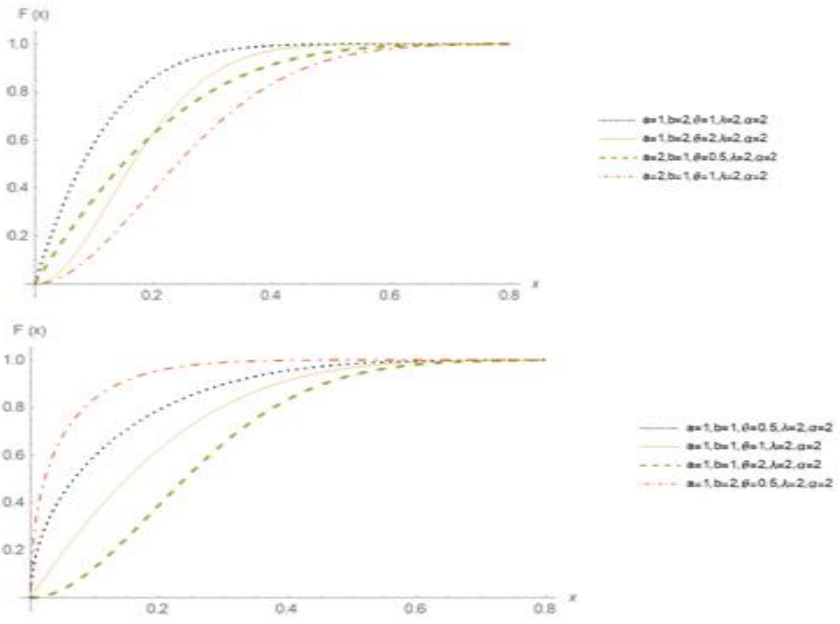
$$F(x;a,b,\lambda,\alpha,\theta) = \frac{1}{B(a,b)} \int_0^{\left(1-e^{-\lambda\left(e^{\alpha x}-1\right)}\right)^{\theta}} w^{a-1} \sum_{k=0}^{\infty} \binom{b-1}{k} (-1)^k w^k \quad dw. \qquad (2)$$

So,

$$F(x;a,b,\lambda,\alpha,\theta) = \sum_{k=0}^{\infty} w_k \frac{(-1)^k}{(a+k)} \left(1-e^{-\lambda\left(e^{\alpha x}-1\right)}\right)^{\theta(a+k)}, \qquad (3)$$

Where $w_k = \frac{\Gamma(a+b)}{\Gamma a \;\; \Gamma(b-k)\;\Gamma(k+1)}$.

Remark: The index k in the sum (2) stops at $(b-1)$, when b is positive integer.



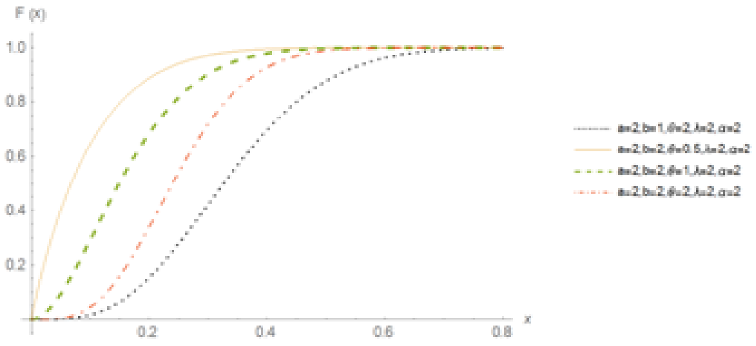


Figure 1:*curves of the cdf for BEGpz by used select values of parameters*

We can see the probability density function (pdf) of BEGpz distribution as follow:

$$f(x; a, b, \lambda, \alpha, \theta) = \frac{\theta\lambda\alpha\ e^{\alpha x - \lambda(e^{\alpha x} - 1)}}{B(a,b)} \left(1 - e^{-\lambda(e^{\alpha x} - 1)}\right)^{\theta a - 1} \left[1 - \left(1 - e^{-\lambda(e^{\alpha x} - 1)}\right)^{\theta}\right]^{b-1}. \quad (4)$$

By applying the same expantion method we used in (2) for pdf of the BEGpz distribution we obtain:

$$f(x; a, b, \lambda, \alpha, \theta) = \theta\lambda\alpha \sum_{j=0}^{\infty} w_j^* (-1)^j \ e^{\alpha x - \lambda(e^{\alpha x} - 1)[1+j]}, \quad (5)$$

where $w_j^* = \sum_{k=0}^{\infty} \frac{w_k\ \Gamma(\theta(a+k))}{\Gamma(\theta(a+k)-j)\ \Gamma(j+1)}$.

Remark: The index j in the sum (5) stops at $\theta(a+k) - 1$, when a and θ are positive integers.
When X is a random variable following the BEGpz distribution, it will be denoted by $X \sim BEGpz\ (a, b, \lambda, \alpha, \theta)$.

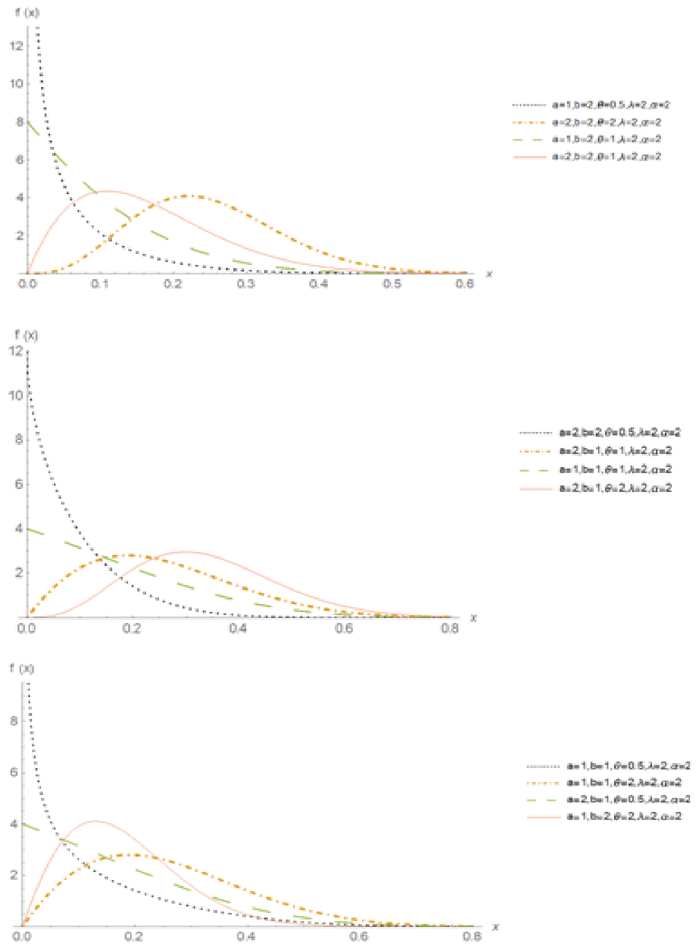


Figure 2:*curves of the pdf for BEGpz by used select values of parameters*

Some possible shapes of the BEGpz density function such as semi-symmetrical, right-skewed, reversed-J and unimodal have been observable from Figure 2. It is evident that the BEGpz distribution can be very useful for modeling data set with various shapes.

We can note that:

$$lim_{x\to 0^+} f(x) = \begin{cases} 0 & \theta a > 1 \\ \frac{\theta\lambda\alpha}{B(a,b)} & \theta a = 1 \\ \infty & \theta a < 1 \end{cases} \quad . \tag{6}$$

The survival function of the BEGpz distribution can be obtained as follows:

$$S(x; a, b, \lambda, \alpha, \theta) = 1 - \sum_{k=0}^{\infty} w_k \frac{(-1)^k}{(a+k)} \left(1 - e^{-\lambda(e^{\alpha x}-1)}\right)^{\theta(a+k)}. \tag{7}$$

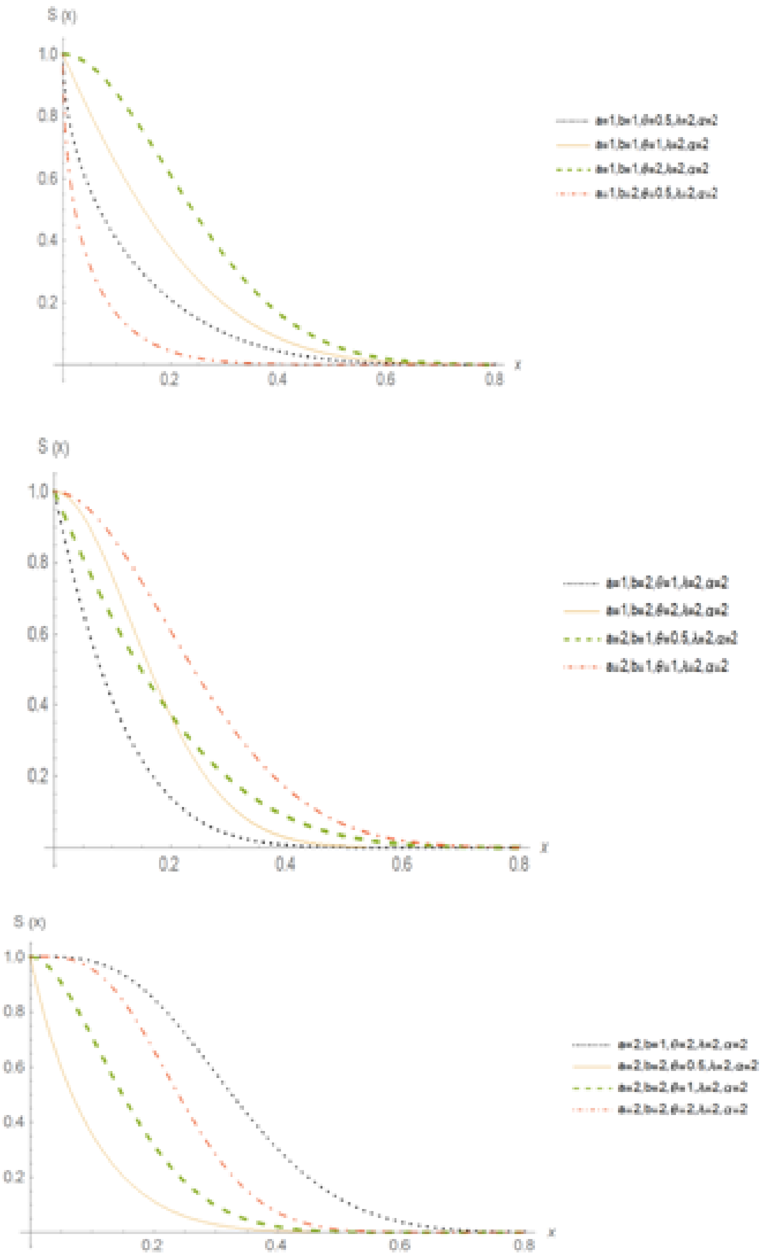


Figure 2:*curves of the survival function for BEGpz by used select values of parameter*

The hazard rate function of the BEGpzdistribution can be written

$$h(x;a,b,\lambda,\alpha,\theta)=\frac{\theta\lambda\alpha\ \sum_{j=0}^{\infty} w_j^{*}(-1)^{j}\ e^{\alpha x-\lambda\left(e^{\alpha x}-1\right)[1+j]}}{1-\sum_{k=0}^{\infty} w_k\frac{(-1)^k}{(a+k)}\left(1-e^{-\lambda\left(e^{\alpha x}-1\right)}\right)^{\theta(a+k)}}. \quad (8)$$

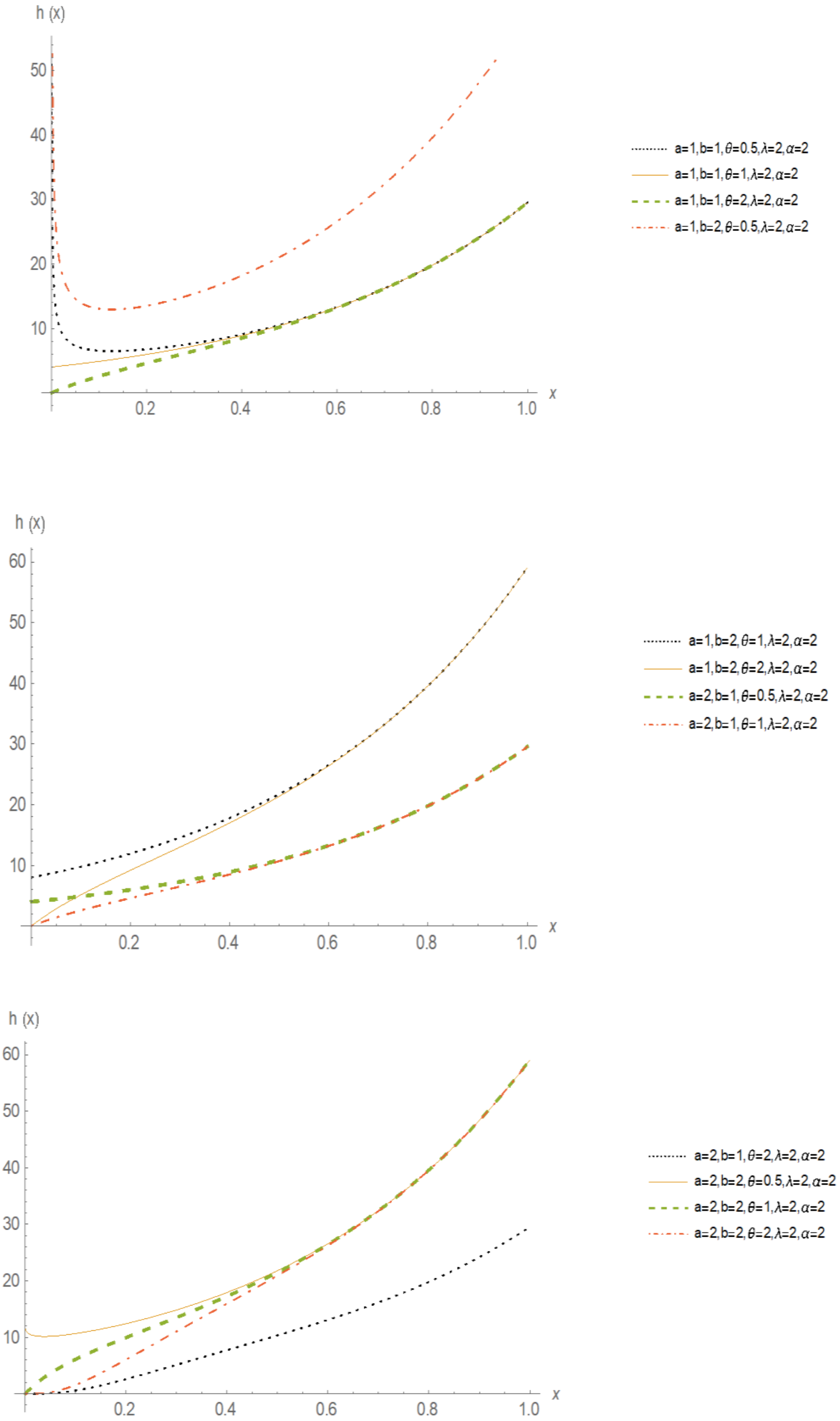


Figure 4:*curves of the hazard rate function for BEGpz by used select values of parameters*

Furthermore, figure 4 indicates that the BEGpz hazard rate function can have increasing and bathtub shaped depending on the parameter values.

We can note that:

$$\lim_{x\to 0^+} h(x) = \begin{cases} 0 & \theta a > 1 \\ \dfrac{\theta\lambda\alpha}{B(a,b)} & \theta a = 1 \\ \infty & \theta a < 1 \end{cases} .$$

III. Special Sub Models

Some well-known distributions are special cases of the BEGpz distribution as following:

- If $a = b = 1$ or $b = \theta = 1$, then we get exponentiated Gompertz distribution.
- If $a = b = \theta = 1$, then we get Gompertz distribution.
- If $b = 1$, then we get exponentiated exponentiated Gompertz distribution.
- If $b = 1$ and α tends to zero, then we get exponentiated exponentiated exponential distribution.
- If $\theta = 1$, then we get beta Gompertz distribution.
- If $a = b = \theta = 1$ and α tends to zero, then we get exponential distribution.
- If $(a = b = 1$ or $b = \theta = 1)$ and α tends to zero, then we get exponentiated exponential distribution.
- If α tends to zero, then we get beta exponentiated exponential distribution.
- If $\theta = 1$ and α tends to zero, then we get beta exponential distribution.

IV. Statistical Properties

In this section, we present some statistical properties of the BEGpz distribution as follow:
The quantile function of the BEGpz distribution is given by

$$Q(q) = F^{-1}(q) = \frac{1}{\alpha} log\left[1 - \frac{1}{\lambda} log\left[1 - \left[I_q^{-1}(a,b)\right]^{\frac{1}{\theta}}\right]\right], \quad 0 < q < 1, \tag{9}$$

were $I_q^{-1}(a,b)$ denotes the inverse of the incomplete beta function with parameters a and b. The quartile Q_1, Q_2 (median), and Q_3 of the BEGpz distribution correspond to the values $q = 0.25, 0.50$ and 0.75 respectively. Therefore, it is easy to simulate the BEGpz random variable X. Let V be a beta random variable with parameters $a, b > 0$. Then, the random variable X given by

$$X = \frac{1}{\alpha} log\left[1 - \frac{1}{\lambda} log\left[1 - V^{\frac{1}{\theta}}\right]\right], \quad 0 < q < 1, \tag{10}$$

wich is follows the BEGpz distribution.We can generate a random variable X having the BEGpz distribution from (10) when the parameters are known.

Bowley's skewness is based on quartiles, Kenney and keeping [27] calculated it as follows

$$skew_B = \frac{Q\left(\frac{3}{4}\right) - 2Q\left(\frac{1}{2}\right) + Q\left(\frac{1}{4}\right)}{Q\left(\frac{3}{4}\right) - Q\left(\frac{1}{4}\right)}, \tag{11}$$

Moors' kurtosis, Moors [28] is based on octiles via the form

$$kurt_M = \frac{Q\left(\frac{7}{8}\right) - Q\left(\frac{5}{8}\right) - Q\left(\frac{3}{8}\right) + Q\left(\frac{1}{8}\right)}{Q\left(\frac{6}{8}\right) - Q\left(\frac{2}{8}\right)}, \tag{12}$$

where *Q(.)* represents the quantile function defined in (9).

The rth moment of the BEGpz distribution can be written as

$$\mu_r' = \theta\lambda\alpha \sum_{i=0}^{\infty} w_i^{**}(-1)^{r+1}\left[\frac{1}{\alpha(1+i)}\right]^{r+1} \Gamma(r+1), \quad r = 1,2,..., \tag{13}$$

where $w_i^{**} = \sum_{j=0}^{\infty} w_j^* (-1)^{j+i} \frac{[\lambda(1+j)]^i}{i!} e^{\lambda(1+j)}$.

Therefore, the first four moments of the BEGpz distribution are given by

$$\mu_1' = E[X] = \mu = \theta\lambda\alpha \sum_{i=0}^{\infty} \frac{w_i^{**}}{\alpha^2(1+i)^2}, \qquad \mu_2' = E[X^2] = -2\theta\lambda\alpha \sum_{i=0}^{\infty} \frac{w_i^{**}}{\alpha^3(1+i)^3},$$

$$\mu_3' = E[X^3] = 6\,\theta\lambda\alpha \sum_{i=0}^{\infty} \frac{w_i^{**}}{\alpha^4(1+i)^4}\,, \qquad \mu_4' = E[X^4] = -24\,\theta\lambda\alpha \sum_{i=0}^{\infty} \frac{w_i^{**}}{\alpha^5(1+i)^5}\,.$$

Additionally, the variance of the BEGpz distribution can be found as:

$$\sigma^2 = \mu_2' - \mu^2 = -2\theta\lambda\alpha \sum_{i=0}^{\infty} \frac{w_i^{**}}{\alpha^3(1+i)^3} - \mu^2\,.$$

The coefficients of skewness and kurtosis for the BEGpz distribution based on rth moments are given by

$$skew(X) = \frac{E[(x-\mu)^3]}{\sigma^3} = \frac{\mu_3' - 3\mu\,\mu_2' + 2\,\mu^3}{\sigma^3}\,, \tag{14}$$

$$kurt(X) = \frac{E[(x-\mu)^4]}{\sigma^4} = \frac{\mu_4' - 4\mu\,\mu_3' + 6\,\mu^2\mu_2' - 3\,\mu^4}{\sigma^4}\,, \tag{15}$$

respectivaly.

Moreover, we derived the moment generating functhion $M_x(t)$of the BEGpz distribution as follows:

$$M_x(t) = \theta\lambda \sum_{i=0}^{\infty} \frac{w_i^{**} Beta\left(i+1, \frac{t}{\alpha}+1\right)}{(-1)}\,.$$

The mode for the BEGpz distribution can be found by differentiating *f(x)* with respect to x; thus, (4) gives

$$\begin{aligned}\frac{df(x)}{dx} = f(x)\Big(\alpha(1-\lambda e^{\alpha x}) &+ \alpha\lambda e^{\alpha x} e^{-\lambda(e^{\alpha x}-1)}\Big[(\theta a - 1)\left(1 - e^{-\lambda(e^{\alpha x}-1)}\right)^{-1} \\ &- \theta(b-1)\left(1 - \left(1 - e^{-\lambda(e^{\alpha x}-1)}\right)^{\theta}\right)^{-1}\left(1 - e^{-\lambda(e^{\alpha x}-1)}\right)^{\theta-1}\Big]\Big).\end{aligned} \tag{16}$$

By equating (16) with zero, we get

$$\begin{aligned}\alpha(1-\lambda e^{\alpha x}) + \alpha\lambda e^{\alpha x} e^{-\lambda(e^{\alpha x}-1)}\Big[(\theta a - 1)\left(1 - e^{-\lambda(e^{\alpha x}-1)}\right)^{-1} \\ - \theta(b-1)\left(1 - \left(1 - e^{-\lambda(e^{\alpha x}-1)}\right)^{\theta}\right)^{-1}\left(1 - e^{-\lambda(e^{\alpha x}-1)}\right)^{\theta-1}\Big] = 0\,,\end{aligned} \tag{17}$$

Then, the mode of the BEGpz distribution can be found numerically by solving (17).

In Table 1, we present the mode, median, mean, skewness and kurtosis of the BEGpz distribution for various values of a, b, λ, α and θ by using the equations (9, 11, 12, 13, 17).

Table 1: Different measures of the BEGpz distribution.

			λ=2 , α=2				
a	b	θ	Mode	Median	Mean	Skewness	Kurtosis
1	1	0.5	0	0.067196	0.114813	0.346698	0.824328
		1	0	0.148782	0.180664	0.167911	0.426616
		2	0.189927	0.239349	0.258156	0.090014	0.237925
1	2	0.5	0	0.021935	0.048962	0.444016	1.125367
		1	0	0.079905	0.103173	0.207781	0.530703
		2	0.130001	0.164496	0.178296	0.095092	0.252404
2	1	0.5	0	0.148782	0.180664	0.167911	0.426616
		1	0.189927	0.239349	0.258156	0.090014	0.237925
		2	0.298955	0.325928	0.338016	0.061151	0.162862
2	2	0.5	0	0.067196	0.091619	0.239682	0.608806
		1	0.108774	0.148782	0.164251	0.112515	0.295657
		2	0.220327	0.239349	0.248048	0.059925	0.160379

We note from above table that:

- The BEGpz distribution is unimodal when $\theta a > 1$ and positively skewed.
- The skewness and kurtosis of the BEGpz distribution are decreasing when θa is increasing.
- The median and mean of the BEGpz distribution become largest for large values of θa.

- The mode of the BEGpz distribution is increasing when θa is increasing and $\theta a > 1$.
- The median and mean of the BEGpz distribution become largest for small values of b when θa fixed.
- The mode of the BEGpz distribution is decreasing function of b when θa fixedand $\theta a > 1$.

The mean residual lifetime (MRL) function of the BEGpzdistribution can given by

$$m(t) = \frac{\mu + I_c(t) - t}{S(t)}, \tag{18}$$

where

$$I_c(t) = \int_0^t F(x)\, dx$$
$$= \sum_{k=0}^{\infty} w_k \frac{(-1)^k}{(a+k)} \int_0^t \left(1 - e^{-\lambda(e^{\alpha x}-1)}\right)^{\theta(a+k)} dx.$$

Then by using binomial series expansion, we have

$$I_c(t) = \sum_{j=0}^{\infty} v_j^* \int_0^t e^{-\lambda j\,(e^{\alpha x})}\, dx\,,$$

where $v_j^* = \sum_{k=0}^{\infty} \binom{\theta(a+k)}{j} \frac{w_k (-1)^{k+j} e^{\lambda j}}{(a+k)}$.

Remark: If θ and a are an integers the index j stops at $\theta(a+k)$.

By using the series expansion for $e^{-\lambda j\,(e^{\alpha x})}$, $I_c(t)$ can be written as

$$I_c(t) = \sum_{i=0}^{\infty} v_i^{**} \left(\frac{1}{i\alpha}\right) [e^{i\alpha t} - 1], \tag{19}$$

where $v_i^{**} = \sum_{j=0}^{\infty} v_j^* \frac{(-1)^i (\lambda j)^i}{i!}$.

Finally, MRL for the BEGpz distribution can be written in the form

$$m(t) = \frac{1}{S(t)} \left[\mu + \sum_{i=0}^{\infty} v_i^{**} \left(\frac{1}{i\alpha}\right) [e^{i\alpha t} - 1] - t\right]. \tag{20}$$

The maen deviation from the mean and the median are respectively, can be defined by

$$D(\mu) = \int_0^{\infty} |x - \mu|\, f(x)\, dx = 2\mu\, F(\mu) - 2 \int_0^{\mu} x\, f(x)\, dx = 2\, I_c(\mu) \tag{21}$$

and

$$D(m) = \int_0^{\infty} |x - m|\, f(x)\, dx = \mu - 2 \int_0^{m} x\, f(x)\, dx = \mu - m + 2\, I_c(m), \tag{22}$$

where μ is the mean (13), m is the median (9), F(x) is the cdf (3) and $I_c(.)$ defined in (19) for the BEGpz distribution.

The Rényi entropy of a random variable X~BEGpz$(a, b, \lambda, \alpha, \theta)$defined by

$$\xi_R(\tau) = \frac{1}{1-\tau} \log[\xi(\tau)],$$

Where

$\xi(\tau) = \int_0^{\infty} f^{\tau}(x)\, dx$ and $f(x)$ given in (4).

Substituting from (4) into the last equation yields

$$\xi(\tau) = \frac{\theta^{\tau} \lambda^{\tau} \alpha^{\tau}}{\left(B(a,b)\right)^{\tau}} \int_0^{\infty} e^{\tau \alpha x} e^{-\tau\lambda(e^{\alpha x}-1)} \left(1 - e^{-\lambda(e^{\alpha x}-1)}\right)^{\tau(\theta a - 1)} \left[1 - \left(1 - e^{-\lambda(e^{\alpha x}-1)}\right)^{\theta}\right]^{\tau(b-1)} dx \tag{23}$$

By applying the exponential series expansion and the binomial expansion in (23) leads to

$$\xi(\tau) = \frac{\theta^{\tau} \lambda^{\tau} \alpha^{\tau-1}}{\left(B(a,b)\right)^{\tau}} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \binom{\tau(b-1)}{i} \binom{\tau(\theta a - 1) + \theta i}{j} (-1)^{i+j+1} \frac{[\lambda(\tau+j)]^k}{k!} Beta(\tau, k+1). \tag{24}$$

Remark:

- The index i in the sum (24) stops at $\tau(b-1)$, when b and τ are positive integers.
- The index j in the sum (24) stops at $\tau(\theta a - 1) + \theta i$, when a, θ and τ are positive integers.

V. Behavior of the Curves for Distribution and Hazard Rate Function

We find out some important result about the curves of the BEGpz density in following theorem:

Theorem 1:

a. The density of the BEGpz is log- concave when any of the following cases are satisfy:
   1. $\theta a > 1.$
   2. $\theta a = 1, \theta > a \; and \; b \geq 1.$
   3. $\theta a = 1 \; and \; \theta = a = 1.$
   4. $\theta a = 1, \theta < a \; and \; b \leq 1.$

b. The density of the BEGpz is log- convex if $x < x_0$ and log- concave if $x > x_0$, where the x_0 is the solution of (25) with respect to x:

$$\lambda\alpha^2 e^{\alpha x_0}\left(-1+\frac{(\theta a-1)e^{-\lambda(e^{\alpha x0}-1)}}{1-e^{-\lambda(e^{\alpha x0}-1)}}\left((1-\lambda\, e^{\alpha x_0})-\frac{\lambda\, e^{\alpha x_0-\lambda(e^{\alpha x0}-1)}}{1-e^{-\lambda(e^{\alpha x0}-1)}}\right)\right. \tag{25}$$

$$-\frac{(b-1)\,\theta\, e^{-\lambda(e^{\alpha x0}-1)}\left(1-e^{-\lambda(e^{\alpha x0}-1)}\right)^{\theta-1}}{1-(1-e^{-\lambda(e^{\alpha x0}-1)})^{\theta}}\left((1-\lambda\, e^{\alpha x_0})\right.$$

$$+\lambda\,(\theta-1)\, e^{\alpha x_0-\lambda(e^{\alpha x0}-1)}\left(1-e^{-\lambda(e^{\alpha x0}-1)}\right)^{-1}$$

$$\left.\left.+\frac{\lambda\,\theta\, e^{\alpha x_0-\lambda(e^{\alpha x0}-1)}\left(1-e^{-\lambda(e^{\alpha x0}-1)}\right)^{\theta-1}}{1-(1-e^{-\lambda(e^{\alpha x0}-1)})^{\theta}}\right)\right)=0,$$

when any of the following cases are satisfy:
   1. $\theta a < 1.$
   2. $\theta a = 1, \theta > a \; and \; b < 1.$
   3. $\theta a = 1, \theta < a \; and \; b > 1.$

Proof

By differentiating the logarithm of (4) twice with respect to x, then the theorem has been proven.

Theorem 2:

The hazard rate functionof the BEGpz distribution given by (8), is

a. Increasing if any of the following cases are satisfy:
   1. $\theta a > 1.$
   2. $\theta a = 1, \theta > a$ and $b \geq 1.$
   3. $\theta a = 1$ and $\theta = a = 1.$
   4. $\theta a = 1, \theta < a$ and $b \leq 1.$

b. Bathtub shaped with change point x_0 , defined in (25),if $\theta a < 1$.

Proof

By using previous theorem1 and lemma 2 in Abu-Zinadah and Aloufi [11] the proof of this theorem is verified.

Theorem 3:

The mean residual lifetime function of BEGpz distribution which given by (20), is:

a. Decreasing if any of the following cases are satisfy:
   1. $\theta a > 1.$
   2. $\theta a = 1, \theta > a$ and $b \geq 1.$
   3. $\theta a = 1$ and $\theta = a = 1.$
   4. $\theta a = 1, \theta < a$ and $b \leq 1.$

b. Upside-down bathtub shaped with a unique change point t_0 , $0 < t_0 < x_0 < \infty$, $x_0$defined by (25) when $\mu >$ 1and $\theta a < 1$.

Proof

By using theorem 2 and lemma 3 in Abu-Zinadah and Aloufi [11] the proof of this theorem has been done.

VI. Conclusion

A new generalization for the EGpz distribution is proposed which is called BEGpz distribution. Some statistical properties of this distribution have been discussed. Several measures of this distribution have been derived. We found important results about the behavior of the curves for density, hazard rate and mean residual lifetime functions.

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