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AC Finite Binary Automata

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Abstract: *Associative Finite Binary Automaton, Commutative Finite Binary Automaton, AC Finite Binary Automaton have been introduced. Cross Product of Finite Binary Automatons has been defined. If $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two Finite Binary Automatons, then $B_1 \times B_2$ is also a finite binary automaton. If $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two Associative Finite Binary Automatons, then $B_1 \times B_2$ is also an associative finite binary automaton. If $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two commutative Finite Binary Automatons, then $B_1 \times B_2$ is also a commutative finite binary automaton. If $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two AC Finite Binary Automatons, then $B_1 \times B_2$ is also an AC commutative finite binary automaton.*

Keywords: *Finite Binary Automaton, Associative Finite Binary Automaton, Commutative Finite Binary Automaton, AC Finite Binary Automaton*

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I. Introduction

The theory of Automata plays an important role in many fields. It has become a part of computer science. It is very useful in electrical engineering. It provides useful techniques in a wide variety of applications and helps to develop a way of thinking.

II. Finite Automaton

2.1 Finite Automaton: A Finite Automaton is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where Q is a finite set of states, Σ is a finite set of inputs, q_0 in Q is the initial state, $F \subseteq Q$ is the set of final states and δ is the transition function mapping $Q \times \Sigma$ to Q.
If Σ^* is the set of strings of inputs, then the transition function δ is extended as follows :
For $w \in \Sigma^*$ and $a \in \Sigma$, $\delta' : Q \times \Sigma^* \to Q$ is defined by $\delta'(q,wa) = \delta(\delta'(q,w),a)$.
If no confusion arises δ' can be replaced by δ.

III. Finite Binary Automaton

3.1 Finite Binary Automaton: A Finite Binary Automaton B is a 6-tuple $(Q, *, \Sigma, \delta, q_0, F)$, where Q is a finite set of states, * is a mapping from $Q \times Q$ to Q, Σ is a finite set of integers, q_0 in Q is the initial state and $F \subseteq Q$ is the set of final states and δ is the transition function mapping from $Q \times \Sigma$ to Q defined by $\delta(q,n) = q^n$.
If Σ^* is the set of strings of inputs, then the transition function δ is extended as follows :
For $m \in \Sigma^*$ and $n \in \Sigma$, $\delta' : Q \times \Sigma^* \to Q$ is defined by $\delta'(q,mn) = \delta(\delta'(q,m),n)$.
If no confusion arises δ' can be replaced by δ.

3.2 Associative Finite Binary Automaton: A Finite Binary Automaton $B = (Q, *, \Sigma, \delta, q_0, F)$ is said to be an associative finite binary automaton if $p * (q * r) = (p * q) * r$, for all p,q,r in Q.

3.3 Commutative Finite Binary Automaton: A Finite Binary Automaton $B = (Q, *, \Sigma, \delta, q_0, F)$ is said to be a commutative finite binary automaton if $p * q = q * p$, for all p,q in Q.

3.3 AC Finite Binary Automaton: A Finite Binary Automaton $B = (Q, *, \Sigma, \delta, q_0, F)$ is said to be an AC Finite Binary Automaton if it is both associative and commutative

3.4 Cross Product of Finite Binary Automatons: Let $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ be any two Finite Binary Automatons. Then we define $B_1 \times B_2 = (Q, *, \Sigma, \delta, r_0, F)$, where $Q = Q_1 \times Q_2$, * is a mapping from $Q \times Q$ to Q defined by for $p,q \in Q = Q_1 \times Q_2$, where $p=(p_1,p_2)$, $q=(q_1,q_2)$, $p * q = (p_1 \Delta_1 q_1, p_2 \Delta_2 q_2)$ $\Sigma = \Sigma_1 \times \Sigma_2$, $r_0 = p_o \times q_o$ in Q is the initial state and $F = F_1 \times F_2 \subseteq Q$ is the set of final states and δ is the transition function mapping from $Q \times \Sigma$ to Q defined by $\delta((p,q,),n) = (p^n,q^n)$.

Proposition 3.4.1 : If $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two Finite Binary Automatons, then $B_1 \times B_2$ is also a finite binary automaton.

Proof: Let $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ be any two Finite Binary Automatons.

Consider $B_1 \times B_2$

Then by definition $B_1 \times B_2 = (Q, *, \Sigma, \delta, r_0, F)$,

where $Q = Q_1 \times Q_2$,

* is a mapping from $Q \times Q$ to Q defined by for $p,q \in Q = Q_1 \times Q_2$, where $p=(p_1,p_2)$, $q=(q_1,q_2)$,

$p * q = (p_1 \Delta_1 q_1, p_2 \Delta_2 q_2)$

$\Sigma = \Sigma_1 \times \Sigma_2$,

$r_0 = p_o \times q_o$ in Q is the initial state

$F = F_1 \times F_2 \subseteq Q$ is the set of final states

δ is the transition function mapping from $Q \times \Sigma$ to Q defined by $\delta((p,q,),n) = (p^n,q^n)$.

Therefore, $B_1 \times B_2$ is also a finite binary automaton.

Proposition 3.4.2 : Let $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ be any two Associative Finite Binary Automatons. Then $B_1 \times B_2$ is also an associative finite binary automaton.

Proof: Let $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ be any two Associative Finite Binary Automatons.

Consider $B_1 \times B_2$

By the Proposition 3.4.1 $B_1 \times B_2$ is also a finite binary automaton.

Let $p,q,r \in Q = Q_1 \times Q_2$, where $p=(p_1,p_2)$, $q=(q_1,q_2)$ $r=(r_1,r_2)$

$p * (q * r) = (p_1,p_2) * ((q_1,q_2) * (r_1,r_2))$

$= (p_1,p_2) * (q_1 \Delta_1 r_1, q_2 \Delta_2 r_2)$

$= (p_1 \Delta_1 (q_1 \Delta_1 r_1), p_2 \Delta_2 (q_2 \Delta_2 r_2))$

$= ((p_1 \Delta_1 q_1) \Delta_1 r_1, (p_2 \Delta_2 q_2) \Delta_2 r_2)$

$= ((p_1 \Delta_1 q_1), (p_2 \Delta_2 q_2)) * (r_1, r_2)$

$= ((p_1, p_2) * (q_1, q_2)) * (r_1, r_2)$

$= (p * q) * r$

Hence $B_1 \times B_2$ is an associative finite binary automaton.

Proposition 3.4.3 : Let $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ be any two commutative Finite Binary Automatons. Then $B_1 \times B_2$ is also a commutative finite binary automaton.

Proof: Let $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ be any two Commutative Finite Binary Automatons.

Consider $B_1 \times B_2$

Let $p,q \in Q = Q_1 \times Q_2$, where $p=(p_1,p_2)$, $q=(q_1,q_2)$

$p * q = (p_1,p_2) * (q_1,q_2)$

$= ((p_1 \Delta_1 q_1) , (p_2 \Delta_2 q_2))$

$= ((q_1 \Delta_1 p_1) , (q_2 \Delta_2 p_2))$ (since B_1 and B_2 are commutative)

$= (q_1, q_2) * (p_1, p_2)$

$= q * p$

$B_1 \times B_2$ is also a commutative finite binary automaton.

Proposition 3.4.4 : Let $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ be any two AC Finite Binary Automatons. Then $B_1 \times B_2$ is also an AC finite binary automaton.

Proof: It is clear from Propositions 3.4.1, 3.4.2, 3.4.3

Proposition 3.4.5 : Let $B = (Q, \Delta_1, \Sigma, \delta, p_0, F)$ be an AC Finite Binary Automaton. Then $\delta((a * b) , n) = \delta(a , n) * \delta(b , n)$, for any $a , b \in Q$ and $n \in \Sigma$.

Proof: Let $B = (Q, *, \Sigma, \delta, p_0, F)$ be an AC Finite Binary Automaton.

Let $a , b \in Q$ and let $n \in \Sigma$.

$\delta((a * b) , n) = (a * b)^n$

$= (a * b) * (a * b) * \ldots\ldots * (a * b)$

(n times)

$= a^n * b^n$ (since * is associate and commutative)

$= \delta(a , n) * \delta(b , n)$

IV. Conclusion

Automata theory is a developing area which helps the computer and electrical engineering. Finite binary automata is also useful in these fields. Researcher can develop these ideas and produce good results.

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